



Econophysics: can statistical physics contribute to the science of economics?

L.A.N. Amaral, P. Cizeau, P. Gopikrishnan, Y. Liu, M. Meyer, C.-K. Peng, H.E. Stanley¹

Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215, USA

Abstract

We address a current question in econophysics: Are fluctuations in economic indices correlated? To this end, we analyze 1-minute data on a stock index, the Standard and Poor index of the 500 largest stocks. We extend the 6-year data base studied by Mantegna and Stanley by including the 13 years 1984–1996 inclusive, with a recording frequency of 15 seconds. The total number of data points in this 13 years period exceed 4.5 million, which allows for a very detailed statistical analysis. We find that the fluctuations in the volatility are correlated, and that the correlations are well described by a power law. We also briefly describe some recent scaling results in economics, specifically some surprising features that appear to be common to the growth rates of business firms, countries, research budgets, and bird populations. © 1999 Elsevier Science B.V. All rights reserved.

1. Introduction

We begin by pointing out that economics, as a science, is not well understood. This seems to be almost generally accepted; in fact, the cover article of the 23 August 1997 issue of *The Economist* was actually entitled “The Puzzling Failure of Economics”.

What does this new field of econophysics have to offer to any solution to this puzzle? We begin with an assumption that I and my coworkers first encountered years ago when we worked on critical phenomena: “*Everything depends on everything else*”. A careful analysis of any system involves studying the propagation of correlations from one unit of the system to the next. We learned that these correlations propagate both directly and indirectly.

Our approach is to begin empirically, with real data that we can analyze in some detail. In economics, we have available to us a great deal of real data and, since

we have at our disposal the tools of computational physics and the computing power to carry out any number of approaches, this abundance of data is to our great advantage. For us, studying the economy means studying a wealth of data on a well-defined complex system. Today we are going to look at some examples of scale-invariant correlations that are of interest to social scientists.

At one time, it was imagined that the “scale-free” phenomena are relevant to only a fairly narrow slice of physical phenomena [1]. However, the range of systems that apparently display power law and hence scale-invariant correlations has increased dramatically in recent years, ranging from base pair correlations in noncoding DNA [2,3], lung inflation [4,5] and interbeat intervals of the human heart [6–9] to complex systems involving large numbers of interacting subunits that display “free will”, such as city growth [10–12], animal behaviour [14–19], and even economics [20,21]. In particular, economic time series, as, e.g., stock market indices or currency ex-

¹ E-mail: hes@miranda.bu.edu.

change rates depend on the evolution of a large number of strongly interacting systems far from equilibrium, and belong to the class of a complex evolving systems. Thus, the statistical properties of financial markets have attracted the interests of many physicists [22–33]. Methods originating in statistical physics have been proven useful in analyzing financial indices. They are also used to construct new models for the pricing of derivatives and the assessment of the involved risk [25].

The recent availability of very high frequency data allows to study economic time series with a high accuracy on a wide range of time scales varying from less than 1 minute up to more than 10 years. Consequently, a large number of methods known from statistical physics have been applied to characterize the time evolution of stock prices and foreign exchange rates [22–26,30–33]. It turns out that the distributions of the increments of economic time series, both in stock market indices and foreign currency exchange rates, are nearly symmetric and have strong “leptokurtic” wings [34,26]. Index increments as a function of time show only weak correlations on short time scales below 10 minutes [34,35], which seemingly makes them fundamentally different from well known examples of complex dynamic systems in physics such as, e.g., turbulent flow where power law correlations on long time scales are commonly observed [36].

The situation is different for the volatility, i.e., the market fluctuations averaged on a suitable time interval. There is long time persistence much larger than the correlation time in volatility [37]. Volatility is the key input of virtually all option pricing models, including the classic Black and Scholes [38] and Cox, Ross, and Rubinstein [39] binomial models that are based on estimates of the asset’s volatility over the remaining life of the option. So to understand the dynamics of the volatility has very important practical reason.

Here, we quantify long range power law correlations in the volatility of the S&P 500 stock index and report an occurrence of a cross-over phenomena of this long range correlation. Furthermore, we discuss the distribution of the volatility, and show that it can be fitted very well by a log-normal distribution.

2. Quantification of correlations in S&P 500

2.1. Data description and detrending

The S&P 500 index, an index of the New York Stock Exchange, consists of the 500 largest companies in the US. It is a market-value weighted index (stock price times number of shares outstanding), with each stock’s weight in the index proportionate to its market value. The S&P 500 index is one of the most widely used benchmarks of US equity performance. Our data cover 13 years (from Jan. 1984 to Dec. 1996) with a recording frequency of 15 seconds interval. The total number of data points in this 13 years period exceed 4.5 million, which allows for a very detailed statistical analysis.

The S&P 500 index $Z(t)$ from 1984 to 1996 tends to increase constantly on a semi-log graph except during crashes, e.g., October 1987 and May 1990. Since the standard deviation of $Z(t + \Delta t) - Z(t)$ is proportional to the price level, we take the logarithmic of the index as everyone does. We define the forward change

$$G(t) \equiv \log_e Z(t + \Delta t) - \log_e Z(t), \quad (1)$$

where Δt is the time-lag (set to 1 minute in the correlation study).

We only count the number of minutes during the opening hours of the stock market, and remove the nights, weekends and holidays from the data set, i.e., the closing and the next opening of the market is continuous.

The absolute value of $G(t)$ describes the amplitude of the fluctuation. $|G(t)|$ is, by definition, always positive, and there are no obvious global trends visible, which is due to the logarithmic difference, i.e., the relative increment on the original index $Z(t)$. The large values of $Z(t)$ correspond to the crashes and big rallies of the index. It is known in the financial literature that the volatility varies in time [40], as expected the $|G(t)|$ quantity also fluctuates in time.

It is known that there exists intra-day patterns in NYSE and S&P 500 index data, one simple explanation is that there are many information traders active near the open and many liquidity traders active near the close [41]. We find the similar intra-day pattern in our S&P 500 index data set. The intra-day pattern

$$A(t) \equiv \frac{\sum_{i=1}^N |G(t_{i,\text{same}})|}{N}, \quad (2)$$

where N is the total trading days over the 13-year period and t_{same} is the same time of each day ($N = 3309$ in our study). In order to avoid the artificial correlation caused by this daily oscillation, $G(t)$ signal is normalized by the intra-day pattern

$$g(t) \equiv G(t)/A(t), \quad (3)$$

i.e., each data point divided by the intra-day pattern of its corresponding time during the day.

2.2. Methods to calculate correlations

We have three methods to quantify the correlations. The direct method to study the correlation property is the correlation function estimation, which is defined as

$$C(\tau) \equiv \frac{\langle G(t)G(t+\tau) \rangle - \langle G(t) \rangle^2}{\langle G^2(t) \rangle - \langle G(t) \rangle^2}, \quad (4)$$

where τ is the time-lag. The problem with the correlation function estimation is that it depends on the estimated average value of the time series. Since it is difficult to calculate the true average value, the correlation function can only give us a qualitative estimation [42].

Another method to calculate the correlation functions is the traditional power spectrum analysis. Since this method can only apply to linear and stationary (or strictly periodic) time series, although it could give quantitative measures, we still need other method to confirm its results.

We applied the third method – termed *detrended fluctuation analysis* (DFA) [43,7] – to quantify the correlation exponent. The advantages of DFA over conventional methods (e.g., spectral analysis and Hurst analysis) are that it permits the detection of long-range correlations embedded in a nonstationary time series, and also avoids the spurious detection of apparent long-range correlations that are an artifact of nonstationarities. This method has been validated on control time series that consist of long-range correlations with the superposition of a nonstationary external trend [43]. The DFA method has also been successfully applied to detect long-range correlations in highly complex heart beat time series [7,44], and other physiological signals [45,46].

A detailed description of the DFA algorithm appears elsewhere [43,7]. Briefly, the $|g(t)|$ time series (with N data) is first integrated,

$$y(t) \equiv \sum_{i=1}^t |g(i)|. \quad (5)$$

Next the integrated time series is divided into boxes of equal length, n . In each box of length n , a least squares line is fitted to the data (representing the *trend* in that box). The y coordinate of the straight line segments is denoted by $y_n(t)$. Next we detrend the integrated time series, $y(t)$, by subtracting the local trend, $y_n(t)$, in each box. The root-mean-square fluctuation of this integrated and detrended time series is calculated by

$$F(n) = \sqrt{\frac{1}{N} \sum_{t=1}^N [y(t) - y_n(t)]^2}. \quad (6)$$

This computation is repeated over all time scales (box sizes) to provide a relationship between $F(n)$, the average fluctuation as a function of box size. In our case, the box size n ranged from 10 min to 10^5 min, the upper bound of n is determined by the actual data length. Typically, $F(n)$ will increase with box size n . A linear relationship on a double log graph indicates the presence of power law (fractal) scaling. Under such conditions, the fluctuations can be characterized by a scaling exponent α , the slope of the line relating $\log F(n)$ to $\log n$.

For exactly self-similar process, as, e.g., fractional Brownian motion, the DFA exponent α is related to the power spectrum exponent β through the relation $\alpha = (1 + \beta)/2$ [42]. The calculation of $F(n)$ can distinguish four types of behavior.

- (1) Uncorrelated time series give rise to uncorrelated random walks described by $F(n) \sim n^\alpha$ with $\alpha = 1/2$, as expected from the central limit theorem. Power spectrum would be flat with $\beta = 0$.
- (2) Markov processes with a characteristic correlation length t_0 , gives $C(\tau) \sim \exp(-\tau/t_0)$. For $t < t_0$, it is the Brownian process with $\alpha = 1.5$ and corresponding $\beta = 2$, nonetheless the asymptotic behavior for sufficiently large t with $\alpha = 1/2$ would be unchanged from the purely random case.
- (3) In the presence of long-range correlations with no characteristic time scale, the scaling property would be a power law function with $\alpha \neq 1/2$ and $\beta \neq 0$.

2.3. Correlation results of S&P 500 index

Using correlation function estimation, we find that the correlation function of $g(t)$ decays exponentially with a characteristic time of the order of 1–10 min, but the absolute value $|g(t)|$ does not. This result is consistent with previous studies on several economic series [35,37,47].

The power spectrum calculation of $|g(t)|$ shows that the data fit not one but rather two separate power laws: for $f > f_x$ the power law exponent is $\beta_1 = 0.31$, while for $f < f_x$ the exponent $\beta_2 = 0.90$ is three times larger; here f_x is called the crossover frequency.

DFA method confirms our power spectrum results. From the behavior of the power spectrum, we expect that the DFA method will also predict two distinct regions of power law behavior, with exponents $\alpha_1 = 0.66$ and $\alpha_2 = 0.95$ for t less than or greater than a characteristic time scale $t_x \equiv 1/f_x$, where we have used the relation

$$\alpha = (1 + \beta)/2. \quad (7)$$

The data yield $\alpha_1 = 0.66$, $\alpha_2 = 0.93$, thereby confirming the consistency of the power spectrum and DFA methods. Also the crossover time is very close to the result obtained from the power spectrum, with

$$t_x \approx 1/f_x \approx 600 \text{ min} \quad (8)$$

about 1.5 trading days.

To test whether this correlation is due to the distribution function, we shuffled each point of the $|g(t)|$ time series randomly. The shuffling operation keeps the distribution of $|g(t)|$ unchanged, but kills the correlations in the time series totally if there are any. DFA measurement of this randomly shuffled data does not show any correlations and gives exponent $\alpha = 0.50$. This tells us that the long-range correlations are actually due to the dynamics of the economic system and not simple due to the distribution.

The observed long range correlation and the crossover behavior noted above is from the entire 13-year period, so it is natural to enquire whether it will still hold for periods smaller than 13 years. Therefore, we choose a sliding window (with size 1 year) and calculate both exponents α_1 and α_2 within this window as the window is dragged, down the data set with one month step. We find that the value of α_1 is very ‘stable’ (independent of the position of the window)

fluctuating around the mean value $2/3$. Surprisingly, however, the variation of α_2 is much greater, showing sudden jumps when very volatile periods enter or leave the time window.

We studied several standard mathematical models, such as fractional Brownian motion [42,48] and fractional ARIMA processes [49], commonly used to account for long-range correlation in a time series and found that none of them can reproduce the large fluctuation of α_2 .

3. The volatility distribution of S&P 500

The volatility is a measure of the mean fluctuation of a market price over a certain time interval T . The volatility is of practical importance since it quantifies the risk related to assets [25]. As shown above, unlike price changes that are correlated only on very short time scales [35] (a few minutes), the absolute values of price changes (which are closely related to the volatility) show correlations on time scales up to many years [37,47,50,51].

The same data set of the S&P 500 index of the New York stock exchange is explored here to study the volatility distribution. This data set has been extended by 7 years the data set previously analyzed in [26].

We calculate the logarithmic increments $G(t)$ in Eq. (1), where $G(t)$ is the relative price change $\Delta Z/Z$ in the limit $\Delta t \rightarrow 0$. Here we set $\Delta t = 30$ min, well above the correlation time of the price increments; and we obtain similar results for other choices of Δt (larger than the correlation time).

As we show in the correlation discussion, there is a strong ‘U-shape’ market activity over the day. To remove artificial correlations resulting from this intra-day pattern of the volatility [52,53,55,54], we normalized $|G(t)|$ by $A(t)$ as shown in Eq. (3).

We obtain the volatility at a given time by averaging $|g(t)|$ over a time window $T = n \cdot \Delta t$ with some integer n ,

$$v_T(t) \equiv \frac{1}{n} \sum_{t'=t}^{t+n-1} |g(t')|. \quad (9)$$

The volatility fluctuates strongly showing a marked maximum for the 1987 crash. Generally periods of high volatility are not independent but tend to ‘cluster’. This clustering is especially marked around the

1987 crash, which is accompanied by precursors (possibly related to oscillatory patterns [25]). Clustering occurs also at other times (e.g., during the second half of 1990), while there are extended periods where the volatility remains at a rather low level (e.g., in 1985 and 1993).

When we consider a scaled probability distribution $P(v_T)$ for several values of T , the data for different averaging windows collapse to one curve. Remarkably, the scaling form is log-normal, not Gaussian. In the limit of very long averaging times, one expects that $P(v_T)$ becomes Gaussian, since the central limit theorem holds also for correlated series [42], with a slower convergence than for non-correlated processes [29, 56]. However, a log-normal fits the data better than a Gaussian.

Correlations can be accurately quantified using detrended fluctuation analysis [43]. The analysis reveals power-law behavior independent of the T value chosen with an exponent $\alpha \cong 0.9$ in agreement with the value found for the absolute price increments (see Section 2).

4. Discussion of empirical results on finance

In this study, we have used the DFA method to display correlation in the volatility of S&P 500 index. We find that the volatility is highly correlated, and that the correlation is remarkably long range, indeed, over 5 decades. Moreover, the quantitative scaling of the correlation follows the power law form observed in numerous phenomena which have a self-similar or “fractal” origin.

We have also found that the probability distribution of the S&P 500 volatility can be well described by a log-normal function. This functional shape does not depend on the averaging time interval T used to calculate volatility $v_T(t)$. The log-normal shape of the distribution is consistent with a multiplicative process [57] for the volatility [27]. However, a multiplicative behavior would be surprising, because efficient market theories [35] assume that the price reflects all current information that could anticipate future events and the price changes, $G(t)$, are caused by incoming new informations about an asset. Since such information-induced price changes are additive

in $G(t)$, they should not give rise to multiplicative behavior of the volatility.

To account for the time dependence of the volatility and its long-range correlations, ARCH [58], GARCH [59] models and related approaches [49] have been developed, which assume that the volatility depends on time and on the past evolution of the index. It may be also worthwhile to test these models with regard to the volatility distribution $P(v_T)$.

5. Scale invariance in economics

Economics is different than finance, and we have also looked at economic data. Specifically, in collaboration with a card-carrying economist, Michael Salinger – we studied the possibility that all the companies in a given economy might interact, more or less, like an Edwards–Anderson spin glass. As in an Edwards–Anderson spin glass, each spin interacts with another spin – but not with the same coupling and not even with the same sign.

If the sales in a given company x decreases by, e.g., 10%, it will have repercussions in the economy. Some of the repercussions will be favorable – company y , which competes with x , may experience an increase in market share. Others will be negative – service industries that provide personal services for company x employees may experience a drop-off in sales as employee salaries will surely decline. There are positive and negative correlations for almost any economic change. Can we view the economy as a complicated Ising system or spin glass?

To approach this interesting bit of statistical “poetry” and make sense of it, we first located and secured a database that lists the actual size of every firm in the United States. With this information, we did an analysis to determine how the distribution of firm size changes from one year to the next. We then made a histogram for each of three characteristic firm sizes. The largest firms have a very narrow distribution – plausible because the percentage of size change from year to year for the largest firms cannot be that great. On the other hand, a tiny company or a garage-based start-up can radically increase (or decrease) in size from year to year. The histograms have a width determined by the size of the firm. When this width is plotted on the y axis of log-log paper as a function of the size of

the firm on the x axis, the data are approximately linear over 8 orders of magnitude, from the tiniest firms in the database to the largest. The width scales as the firm size to an exponent β , with $\beta \approx 1/6$ [60]. We can therefore normalize the growth rate and show that all the data collapse on a single curve – demonstrating the scaling of this measure of firm size.

Why does this occur? We are working on that. If we model this firm structure as an approximate Cayley tree, in which each subunit of a firm reacts to its directives from above with a certain probability distribution. This model, developed primarily by Sergey Buldryev, seems to be consistent with the critical exponent $-1/6$ [61]. More recently, Amaral et al. [62] have proposed a microscopic model, and Takayasu [32] has extended the empirical results to a wide range of countries, and developed still another model.

It is not impossible to imagine that there are some very general principles of complex organizations at work here, because similar empirical laws appear to hold for data on a range of systems that at first sight might not seem to be so closely related. For example, instead of studying the growth rate of firms one can study the growth rates of countries by analyzing the ratio of the GDP of a country in one year compared to its value in the previous year. It appears that the country GDP behaves the same way as the size of a firm [63,64]. Very recently, a data base comprising research budgets of universities were analyzed in the same way, with similar results [65].

Instead of the population of a firm at time t (measured in number of employees) one might analyze the population $N_s(t)$ of a species s in successive years. Such data exist for a 30-year period for every species sighted in North America, and very recently Keitt and Stanley [18,19] have analyzed this database using the same sort of techniques used to describe long-term data sets on economics and finance. They find statistical properties that are remarkably similar, and consistent with the idea that “every bird species interacts with every other bird species”, just as the economic analysis supports the notion that “every firm interacts with every other firm”. This empirical result is not without interest, since it serves to cast doubt on models of bird population (and of economic systems) in which one partitions the entire data set into strongly-interacting and weakly-interacting subsets,

and then ignores or oversimplifies the interactions in the weakly-interacting subset.

6. Conclusions

Is the point of this talk just to show that a lot of different systems appear to develop scale-invariant correlations? If so, how do we understand this empirical fact?

Bak’s idea that systems self-organize themselves such that they are in effect near a critical point is an appealing unifying principle. Near a critical point, there is a delicate balance between the exponentially-growing number of different one-dimensional paths connecting any two faraway subunits and the exponentially-decaying correlations along each one-dimensional path (this concept is illustrated, e.g., in Fig. 9.4 of Ref. [1]). If the control parameter (say coupling constant) is too small, the correlations die out so fast along each one-dimensional path that subunits far from one another are not well correlated. However, at a critical point, the exponentially-large number of paths connecting each pair of subunits is sufficient to balance out the exponential decay along each path and the “correction factor” wins out – this correction factor is the power law that governs the total number of one-dimensional paths connecting two distant subunits. The exponent in this correction factor depends primarily on the system dimension, and not at all on the actual arrangement of the subunits (lattice or no-lattice).

Could it be that somehow social systems push themselves up “up to the limit” – just as a sandpile is pushed to the limit before an avalanche starts, an image that has attracted recent attention in the debate between “self-organized criticality” and “plain old criticality” (see, e.g., Vespignani and Zapperi [66] and references therein)? For example, in economics every subunit plays according to rules and pushes itself up against the limits imposed by these rules. But social systems display a variety of rich forms of “order”, far richer than we anticipate from studies of ferromagnets and antiferromagnets (see, e.g., some of the papers appearing in Knobler et al. [67]). Could such orderings arise from the complex nature of the interactions? Or from the range of different “sizes” of the constituent subunits as, e.g., one finds ordering in sandpiles when

sand particles of two different grain sizes are dropped onto a heap – see, e.g., Refs. [66–69]. These are questions that occupy us now, and questions I would be delighted to discuss with any of the conference participants.

Acknowledgments

We thank D. Canning, S. Havlin, R. Mantegna, M. Salinger, and S. Zapperi for very helpful discussions through the course of this work, and DFG, NIH, and NSF for financial support.

References

- [1] H.E. Stanley, *Introduction to Phase Transitions and Critical Phenomena* (Oxford University Press, Oxford, 1971).
- [2] C.-K. Peng, S.V. Buldyrev, A.L. Goldberger, S. Havlin, F. Sciortino, M. Simons, H.E. Stanley, *Nature* 356 (1992) 168.
- [3] A. Arneodo, E. Bacry, P.V. Graves, J.F. Mugy, *Phys. Rev. Lett.* 74 (1995) 3293.
- [4] B. Suki, A.-L. Barabási, Z. Hantos, F. Peták, H.E. Stanley, *Nature* 368 (1994) 615.
- [5] A.-L. Barabási, S.V. Buldyrev, H.E. Stanley, B. Suki, *Phys. Rev. Lett.* 76 (1996) 2192.
- [6] C.-K. Peng, J. Mietus, J.M. Hausdorff, S. Havlin, H.E. Stanley, A.L. Goldberger, *Phys. Rev. Lett.* 70 (1993) 1343.
- [7] C.-K. Peng, S. Havlin, H.E. Stanley, A.L. Goldberger, *Chaos* 5 (1995) 82.
- [8] P.Ch. Ivanov, M.G. Rosenblum, C.-K. Peng, J. Mietus, S. Havlin, H.E. Stanley, A.L. Goldberger, *Nature* 383 (1996) 323.
- [9] P.Ch. Ivanov, M.G. Rosenblum, C.-K. Peng, J. Mietus, S. Havlin, H.E. Stanley, A.L. Goldberger, *Physica A* 249 (1998) 587.
- [10] H.A. Makse, S. Havlin, H.E. Stanley, *Nature* 377, 608 (1995); H.A. Makse, J. Andrade, M. Batty, S. Havlin, H.E. Stanley, *Modeling urban growth patterns with correlated percolation*, *Phys. Rev. E* (accepted).
- [11] D. Zanette, S. Manrubia, *Phys. Rev. Lett.* 79 (1997) 323.
- [12] P. Ball, *The Self-Made Tapestry: Pattern Formation in Nature* (Oxford University Press, Oxford, 1998).
- [13] M.H.R. Stanley, L.A.N. Amaral, S.V. Buldyrev, S. Havlin, H. Leschhorn, P. Maass, M.A. Salinger, H.E. Stanley, *Nature* 379 (1996) 804.
- [14] I. Peterson, *Sci. News* 150 (1996) 104.
- [15] I. Peterson, *The Jungles of Randomness* (Viking, New York, 1997).
- [16] G.M. Viswanathan, V. Afanasyev, S.V. Buldyrev, E.J. Murphy, P.A. Prince, H.E. Stanley, *Nature* 381 (1996) 413.
- [17] G.M. Viswanathan, S.V. Buldyrev, S. Havlin, M.G. da Luz, E. Raposo, H.E. Stanley, *Optimizing success in random foraging: a physical approach*, *Phys. Rev. Lett.* (submitted).
- [18] T. Keitt, H.E. Stanley, *Nature* 393 (1998) 257.
- [19] T. Keitt, H.E. Stanley, *Lévy flights and $1/f$ noise in high abundance populations*, Preprint.
- [20] B.B. Mandelbrot, *Fractals and Scaling in Finance: Discontinuity, Concentration, Risk* (Springer, New York, 1997).
- [21] R.N. Mantegna, H.E. Stanley, *Physics investigation of financial markets*, in: *Proceedings of the International School of Physics “Enrico Fermi”, Course CXXXIV*, F. Mallamace, H.E. Stanley (Eds.) (IOS Press, Amsterdam, 1997); *Econophysics* (Cambridge University Press, under contract).
- [22] P. Bak, K. Chen, J.A. Scheinkman, M. Woodford, *Ricerca Economica* 47 (1993) 3; J.A. Scheinkman, J. Woodford, *Amer. Econ. Rev.* 84 (1994) 417.
- [23] M.H.R. Stanley, L.A.N. Amaral, S.V. Buldyrev, S. Havlin, H. Leschhorn, P. Maass, M.A. Salinger, H.E. Stanley, *Nature* 379 (1996) 804.
- [24] M. Levy, H. Levy, S. Solomon, *Economics Letters* 45 (1994) 103; M. Levy, H. Levy, S. Solomon, *J. Phys. I* 5 (1995) 1087.
- [25] J.-P. Bouchaud, D. Sornette, *J. Phys. I (France)* 4 (1994) 863; D. Sornette, A. Johansen, J.-P. Bouchaud, *J. Phys. I (France)* 6 (1996) 167.
- [26] R.N. Mantegna, H.E. Stanley, *Nature* 376 (1995) 46; *Phys. Rev. Lett.* 73 (1994) 2946.
- [27] S. Ghoshghaie, W. Breymann, J. Peinke, P. Talkner, Y. Dodge, *Nature* 381 (1996) 767; see also R.N. Mantegna, H.E. Stanley, *Nature* 383 (1996) 587; *Physica A* 239 (1997) 255; A. Arneodo et al., Preprint.
- [28] M. Levy, N. Persky, S. Solomon, *Int. J. High Speed Comput.* 8 (1996) 93; M. Levy, S. Solomon, *Int. J. Mod. Phys. C* 4 (1996) 7.
- [29] M. Potters, R. Cont, J.-P. Bouchaud, *Financial markets as adaptive systems*, Preprint.
- [30] H. Takayasu, H. Miura, T. Hirabayashi, K. Hamada, *Physica A* 184 (1992) 127.
- [31] T. Hirabayashi, H. Takayasu, H. Miura, K. Hamada, *Fractals* 1 (1993) 29.
- [32] H. Takayasu, K. Okuyama, *Fractals* 6 (1998) 67.
- [33] P.R. Krugman, *The Self-Organizing Economy* (Blackwell Publishers, Cambridge, 1996).
- [34] B.B. Mandelbrot, *J. Business* 36 (1963) 393.
- [35] E.-F. Fama, *J. Finance* 25 (1970) 383.
- [36] A.N. Kolmogorov, *Local structure of turbulence in fluid for very large Reynolds numbers*, in: *Transl. in Turbulence*, S.K. Friedlander, L. Topper (Eds.) (Interscience Publishers, New York, 1961) 151–155.
- [37] Z. Ding, C.W.J. Granger, R.F. Engle, *J. Empirical Finance* 1 (1983) 83.
- [38] F. Black, M. Scholes, *J. Political Econ.* 81 (1973) 637.
- [39] J. Cox, S. Ross, M. Rubinstein, *J. Financial Econ.* 7 (1979) 229.
- [40] T. Bollerslev, R.Y. Chou, K.F. Kroner, *J. Econometrics* 52 (1992) 5; G.W. Schwert, *J. Finance* 44 (1989) 1115; A.R. Gallant, P.E. Rossi, G. Tauchen, *Rev. Financial Stud.* 5 (1992) 199;

- B. Le Baron, *J. Business* 65 (1992) 199;
 K. Chan, K.C. Chan, G.A. Karolyi, *Rev. Financial Stud.* 4 (1991) 657.
- [41] A. Admati, P. Pfleiderer, *Rev. Financial Studies* 1 (1988) 3.
 [42] J. Beran, *Statistics for Long-Memory Processes* (Chapman & Hall, NY, 1994).
 [43] C.-K. Peng, S.V. Buldyrev, S. Havlin, M. Simons, H.E. Stanley, A.L. Goldberger, *Phys. Rev. E* 49 (1994) 1684.
 [44] N. Iyengar, C.-K. Peng, R. Morin, A.L. Goldberger, L.A. Lipsitz, *Am. J. Physiol.* 271 (1997) R1078.
 [45] J.M. Hausdorff, P.L. Purdon, C.-K. Peng, Z. Ladin, J.Y. Wei, A.L. Goldberger, *J. Appl. Physiol.* 80 (1996) 1448.
 [46] J.M. Hausdorff, C.-K. Peng, *Phys. Rev. E* 54 (1996) 2154.
 [47] M.M. Dacorogna, U.A. Muller, R.J. Nagler, R.B. Olsen, O.V. Pictet, *J. Intern. Money Finance* 12 (1993) 413.
 [48] B.B. Mandelbrot, J.W. van Ness, *SIAM Rev.* 10 (1968) 422.
 [49] C.W.J. Granger, Z. Ding, *J. Econometrics* 73 (1996) 61.
 [50] Y. Liu, P. Cizeau, M. Meyer, C.-K. Peng, H.E. Stanley, *Physica A* 245 (1997) 437.
 [51] P. Cizeau, Y. Liu, M. Meyer, C.-K. Peng, H.E. Stanley, *Physica A* 245 (1997) 441.
 [52] R.A. Wood, T.H. McInish, J.K. Ord, *J. Finance* 40 (1985) 723.
 [53] L. Harris, *J. Financial Econ.* 16 (1986) 99.
 [54] P.D. Ekman, *J. Futures Markets* 12 (1992) 365.
 [55] A. Admati, P. Pfleiderer, *Rev. Financial Stud.* 1 (1988) 3.
 [56] R. Cont, *cond-mat/9705075*.
 [57] A. Bunde, S. Havlin, in: *Fractals and Disordered Systems*, A. Bunde, S. Havlin (Eds.) (Springer, Heidelberg, 2nd ed., 1996).
 [58] R.F. Engle, *Econometrica* 50 (1982) 987.
 [59] T. Bollerslev, *J. Econometrics* 31 (1986) 307.
 [60] S.V. Buldyrev, L.A.N. Amaral, S. Havlin, H. Leschhorn, P. Maass, M.A. Salinger, H.E. Stanley, M.H.R. Stanley, *J. Phys. I (France)* 7 (1997) 635;
 S.V. Buldyrev, H. Leschhorn, P. Maass, H.E. Stanley, M.H.R. Stanley, L.A.N. Amaral, S. Havlin, M.A. Salinger, Scaling behavior in economics: empirical results and modeling of company growth, in: *Proc. Intern. School of Physics ‘Enrico Fermi’*, Course CXXXIV, F. Mallamace, H.E. Stanley (Eds.) (IOS Press, Amsterdam, 1997).
 [61] S.V. Buldyrev, L.A.N. Amaral, S. Havlin, H. Leschhorn, P. Maass, M.A. Salinger, H.E. Stanley, M.H.R. Stanley, *J. Phys. I (France)* 7 (1997) 635.
 [62] L.A.N. Amaral, S.V. Buldyrev, S. Havlin, M.A. Salinger, H.E. Stanley, *Phys. Rev. Lett.* 80 (1998) 1385.
 [63] Y. Lee, L.A.N. Amaral, D. Canning, M. Meyer, H.E. Stanley, *Phys. Rev. Lett.* 81 (1998) 3275.
 [64] D. Canning, L.A.N. Amaral, Y. Lee, M. Meyer, H.E. Stanley, A power law for scaling the volatility of GDP growth rates with country size, *Econ. Lett.* 20 (1998) 30.
 [65] V. Plerou, L.A.N. Amaral, P. Gopikrishnan, M. Meyer, H.E. Stanley, *Research at Ivory Tower Universities: a competitive enterprise?*, Preprint.
 [66] A. Vespignani, S. Zapperi, *Phys. Rev. Lett.* 78 (1997) 4793.
 [67] C.M. Knobler, A. Robledo, H.E. Stanley, *Statistical Mechanics in the Physical, Biological, and Social Sciences: Festschrift in Honor of Benjamin Widom on the occasion of his 70th Birthday* (Elsevier, Amsterdam, 1997) [Published as a special issue of *Physica A* 244 (1 Sept. 1997)].
 [68] H.A. Makse, P. Cizeau, H.E. Stanley, *Phys. Rev. Lett.* 78 (1997) 3298.
 [69] H.A. Makse, S. Havlin, P.R. King, H.E. Stanley, *Nature* 386 (1997) 379.
 [70] H.A. Makse, P. Cizeau, S. Havlin, P.R. King, H.E. Stanley, Spontaneous Self-Stratification Without Shaking: ‘Potatoes from Mashed Potatoes’ [Proc. 1997 NATO ASI], in: *Granular Matter*, H.J. Herrmann (Ed.) (Kluwer, Dordrecht, 1998).
 [71] H.A. Makse, P. Cizeau, H.E. Stanley, *Physica A* 249 (1998) 391;
 H.A. Makse, S. Havlin, P.R. King, H.E. Stanley, in: *Proc. Minerva Conf.*, *Phil. Mag.* 77 (1998) 1341.
 [72] H.A. Makse, R.C. Ball, H.E. Stanley, S. Warr, *Phys. Rev. E* 58 (1998) 3357.