Capital death in the world market

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We study the gross domestic product (GDP) *per capita* together with the market capitalization (MCAP) *per capita* as two indicators of the effect of globalization. We find that g, the GDP *per capita*, as a function of m, the MCAP *per capita*, follows a power law with average exponent close to 1/3. In addition, the Zipf ranking approach confirms that the m for countries with initially lower values of m tends to grow more rapidly than for countries with initially larger values of m. If the trends over the past 20 years continue to hold in the future, then the Zipf ranking approach leads to the prediction that in about 50 years, all countries participating in globalization will have comparable values of their MCAP *per capita*. We call this economic state "capital death," in analogy to the physics state of "heat death" predicted by thermodynamic arguments.

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I. INTRODUCTION

Recently, many studies have reported that economic and financial data exhibit properties, such as nonlinearity and complexity, to a much larger extent than is commonly believed [1–9]. Generally, finance and economics are closely related. Barro *et al.* study long-term data for 30 countries prior to 2006, identifying 232 stock-market "crashes" with multiyear real returns of −25% or less and 100 depressions with multiyear macroeconomic declines of 10% or more [10]. They report that 22% of the time a stock-market crash in a nonwar environment will be followed by a depression with a macroeconomic decline of at least 10%. This finding that not every market crash is followed by a depression supports the statement by the Nobel economist Paul Samuelson that "Wall Street indexes predicted nine out of the last five recessions!" On the other hand, 67% of the time a depression of 10% or greater will be followed by a stock-market crash with a return of −25% or worse. Thus the largest depressions are likely to be accompanied by stock-market crashes but there is an obvious asymmetry in this example. It is more likely that a stock-market crash will be followed by a depression than vice versa [10].

There are many other interactions between finance and economics. For example, macroeconomic data associated with gross domestic product (GDP)—its components and inflations—have been used in early-warning models designed to predict financial crises [11]. Another example is the use of the relationship between the total value of all publicly traded companies, quantified by the total market capitalization (MCAP), and the GDP—described by the famous investor Warren Buffet as "probably the best single measure of where valuations stand at any given moment" [12]. The assumption that total MCAP and GDP should follow each other is rational. While the total estimated MCAP of a country is based on future expectations, the GDP is the market value of all final goods and services produced in a given year. Since the market "cannot fool all of the people all of the time"—to borrow a phrase from Abraham Lincoln—it can be overpriced for only a finite period of time. Thus we expect that a country's asset expectations (MCAP) and realized assets (GDP) must eventually follow each other.

There are two prevailing approaches to the theory of economic growth [13–19] that relate capital and output, the neoclassical growth model [20] and the endogenous growth model [21–23]. In the most popular neoclassical growth model—the Solow-Swan growth model—economic growth depends on increases in labor and capital and on technical innovation. The macroproduction function of the Solow-Swan model $y = Ak^{\alpha}$ relates the basic economic elements: y is the total production per worker, A is technical innovation, k is the physical capital per worker, and α is a constant. In endogenous growth theory models [21–25], investment in human capital, innovation, and education—in addition to physical capital—all significantly contribute to economic growth.

A key prediction of neoclassical growth models is that the income levels of poor countries and rich countries tend to converge if they have similar characteristics. This prediction has triggered a huge number of empirical studies addressing the question of cross-country income convergence [26,27]. Sala-i-martin asked, "Is the degree of income inequality across economies increasing or decreasing with time?" [26] He reported that at the world level richer countries seem to grow more quickly than poorer countries. In contrast, analyzing countries' GDP *per capita* from the period 1980–1999 weighted by their population size, he found that worldwide inequality appears to decrease over time [28].

GDP and physical capital are closely related, so it would seem obvious to study convergence in both of them. The production function of the GDP depends not only on physical capital but also on human capital and several other economic variables [21–25]. If the GDP depends on many variables, a convergence in the GDP *per capita* does not necessarily imply the same convergence in all *per capita* variables in the production function (e.g., some variables may converge and some may diverge). Though the Solow-Swan neoclassical model predicts that both the growth rate of capital *per capita* and the growth rate of output *per capita* depend equally on initial conditions, and thus that the convergence coefficient β

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is the same for output *per capita* as for capital *per capita*, empirically they may be converging or diverging at different rates. Our goal is to determine how each of the variables in the production function is driving (or not driving) the convergence of GDP *per capita*.

Studying the convergence of physical capital directly requires that physical capital data are available. Although GDP data are available for a large number of countries and over long periods of time, the availability of physical capital data is much more limited. Because we lack these data, we perform convergence analysis not on physical capital (value of already realized goods) but on market capitalization (value based on future expectations), for which the data are available for many countries. Note that even these data on market capitalization are not available for as many countries as the GDP—e.g., data on market capitalization are not available for North Korea. In our study we distinguish between countries that provide market capitalization data and those that do not. If a country reports market capitalization, it has a stock exchange and has no barriers obstructing the flow of capital. We can think of this flow of capital in economics in analogy to the flow of heat in thermodynamics.

Basic thermodynamics teaches us that if two bodies, initially at different temperatures and isolated from each other, are brought together, ultimately the hot body will warm up the cold one, while the cold body will cool down the hot one. This process will terminate when the two bodies reach the same temperature and achieve thermal equilibrium. In economics we have a similar situation. Different countries have different levels of development and economists commonly quantify the level of development by GDP. If an economy is to be explained in physics terms, the level of development is quantified by the temperature [29,30]: the larger the GDP of a given country, the higher the temperature. Describing GDP by temperature, the first phase of globalization starts, in physics terms, by removing the barriers between the countries. In economics terms, removing the barriers is equivalent to a substantial reduction in tariffs between countries as with NAFTA or the European Union.

While there are clearly similarities between physics and economics, there are also differences. In thermodynamics, bodies are usually relatively simple objects described by a relatively small number of degrees of freedom. Countries, in contrast, are commonly defined by a large number of variables and, as they are guided by humans, can be very unpredictable. So the "thermodynamics of economics" is fraught with even more levels of stochasticity than thermodynamics itself. For example, people in different countries have different skills and educational levels and thus seem like more complicated objects than bodies in physics. Thus when the barriers among countries are removed, as physicists we expect the money flow to go mostly from rich to poor countries, but predicting the speed of the heat (money) transfer and the final equilibrium is difficult due to the complex nature of human interaction. Here we note that in economics, as in physics, flow may go in both directions but one is predominant until equilibrium is reached.

For the 68 countries that have published both GDP and MCAP over the 17-year period 1994–2010, here defined as countries "participating in globalization"—primarily the "high-income" and "middle-income" countries—we regress the market capitalization *per capita*, m, and GDP *per capita*, g, and find that m and g follow approximate power laws, with an average exponent close to $1/3$. We only have 17 data points for each country, so in fitting m and g, any number of functions can do just about as well as a power law, but we are interested in the exponent, α, of a power-law fit. While the 17-year longitudinal range of the data is a limitation of the analysis, applying the Zipf ranking approach over this time span we see a strong trend that the growth rate of m in countries with an initially smaller value of m tends to increase more rapidly than in countries with an initially larger value of m, implying the existence of market capitalization convergence among the countries participating in globalization.

II. DATA AND METHODS

We study market capitalization and population over the 17-year period 1994–2010 using a World Bank database [31]. Though more longitudinal data would be preferable, using a time span that was any longer would result in a sharp drop-off in the number of countries for which the data would be available.

We use the classical regression approach for convergence analysis of GDP *per capita* data as described by Martin [26], in which each country contributes equally to the regression. In order to test whether GDP *per capita*, g, grows more rapidly with time for the poorer or richer countries, we perform a regression between the annualized growth rate of g for country i between time t and time $t+\Delta T$, $\gamma_{i,t,t+\Delta T}$, and the logarithm of $g_{i,t}$, *per capita* GDP of country i at time t,

$$\gamma_{i,t,t+\Delta T} \equiv \frac{\ln(g_{i,t+\Delta T}/g_{i,t})}{\Delta T} = \beta_0 - \beta \ln(g_{i,t}) + \epsilon_{i,t}. \quad (1)$$

The value β then represents the speed of GDP *per capita* convergence for that data set. A positive β value indicates convergence, in which countries with a smaller initial *per capita* GDP tend to have a higher growth rate over the time period than countries with a larger initial GDP *per capita*. A negative β value indicates divergence of GDP *per capita*.

Data on market capitalization by country are not as readily available as GDP data, but the World Bank database contains annual market capitalization data going back to 1988. We study the years 1994–2010 in order to take into account a number of countries for which the data on MCAP were not available in the late 1980s and early 1990s. In the years 1994–2010, there are complete GDP and MCAP data for 68 countries.

III. RESULTS

A. Classical convergence study

We look at GDP *per capita* growth rates averaged over the 17-year period 1994–2010 ($\Delta T = 16$). We perform a convergence regression analysis for the separate 68-country sets reporting both GDP and market capitalization over that period and those just reporting GDP. Figure 1 shows that for countries that report market capitalization, the GDP *per capita* [solid (red) line] exhibits convergence with

$$\beta = 0.018 \pm 0.002, \quad (2)$$

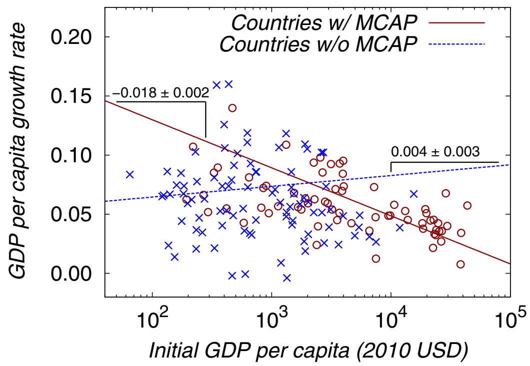

FIG. 1. (Color online) Annualized growth rates of GDP *per capita* for countries that report market capitalization [solid (red) line] and those that do not [dotted (blue) line] over the 17-year period 1994–2010. The trend lines, weighted by population, show that those countries with reported market capitalization (and thus more likely to be participating in globalization) have been converging, whereas those without appear to be diverging, though not at a statistically significant rate.

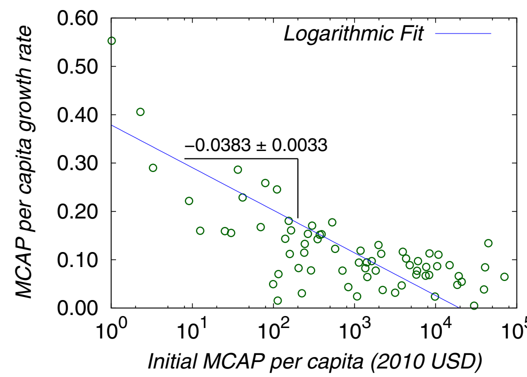

FIG. 2. (Color online) Annualized growth rates for the MCAP *per capita* over the 17-year period 1994–2010 plotted versus the initial MCAP *per capita* in 1994 on a linear-log scale. In the fit shown, the growth rate for each country is weighted by the population of the respective country in 1994, so a country with a population of 100 million is equivalent to 100 countries with populations of 1 million and the same *per capita* data. Note that the MCAP *per capita* grows more rapidly with time for those countries with a smaller initial MCAP *per capita*.

as opposed to countries not participating in globalization [dotted (blue) line], which appear to exhibit divergence, though not to a statistically significant degree. Note that countries reporting market capitalization are primarily "high-income" and "middle-income" countries and thus the results in Fig. 1 agree with what Lucas reports—that "middle-income" countries tend to have the highest growth [22].

We use the same regression approach to *per capita* market capitalization data that we used in the convergence analysis of *per capita* GDP data. To test whether MCAP *per capita*, m, grows more rapidly with time for countries with a larger or a smaller initial MCAP *per capita*, we perform a regression between $\gamma_{i,t,t+\Delta T}$, the annualized growth rate of $m_{i,t}$, the MCAP of economy i between t and $t + \Delta T$,

$$\gamma_{i,t,t+\Delta T} \equiv \frac{\ln(m_{i,t+\Delta T}/m_{i,t})}{\Delta T} = \beta_0 - \beta \ln(m_{i,t}) + \epsilon_{i,t}, \quad (3)$$

and the logarithm of $m_{i,t}$. Figure 2 shows, for the 68 countries that report market capitalization, a positive regression exponent β in Eq. (1),

$$\beta = 0.038 \pm 0.003, \quad (4)$$

which means that, like GDP *per capita*, MCAP *per capita* exhibits a convergence, i.e., countries with an initially smaller m experience a greater increase in m than countries with a larger m. If we compare Eqs. (2) and (4), however, we find that the convergence coefficient β calculated for the growth rate of capital *per capita* is much stronger than the corresponding convergence coefficient for GDP *per capita*.

B. Zipf convergence study

It is commonly believed that a capital transfer from developed countries to developing countries will trigger rapid growth in the wealth of developing countries. Assuming that capital convergence and a final state in which all countries have comparable *per capita* capital—in which all $m_{i,t}$ worldwide are equally distributed—is possible, when will this final state occur? The Zipf ranking approach [32] can suggest the answer. In the Zipf approach, when the variables $m_{i,t}$ are sorted from largest to smallest, an equal distribution of $m_{i,t}$ corresponds to a 0 slope [33]. Zipf distributions are widely employed across many fields of science [34–40]. In economics, the Zipf distribution characterizes firm sizes, where the Zipf exponent is unchanging over a 10-year period [41]. In contrast, the Zipf rank distribution applied to bankruptcy risk reveals that the Zipf exponent changes during times of crisis [42].

Applying the Zipf approach to the 68 countries over the 17-year period 1994–2010, we rank the $m_{i,t}$ according to value, from largest to smallest, and plot the data as a function of rank. In this "Zipf ranking" approach, capital convergence exists (i.e., all capital is equal) if the slope is 0. Figure 3(a) shows a Zipf ranking that reveals that the parameter β' calculated for each year exhibits a decreasing functional dependence with time. In particular, the Zipf ranking plot of $m_{i,t}$ exhibits an exponential functional form. The functional dependence suggests a decrease in world capital differences. As the slope decreases (parameter β'), world capital equality increases. This result is confirmed by analyzing the Gini coefficient and Theil index over this 17-year period for the 68 countries with available data. For GDP, the Gini coefficient decreases from 0.75 to 0.63, and the country-level contribution to the Theil index from 1.13 to 0.73. For MCAP, the Gini coefficient decreases from 0.84 to 0.71 and the country-level contribution to the Theil index from 1.55 to 1.05. This process of decreasing world capital differences is not homogeneous, however, because this result holds only for countries participating in globalization. Note that our analysis includes only countries reporting both GDP and MCAP for every year since 1994.

According to the second law of thermodynamics, the entropy of the universe continually increases and asymptotically approaches a state in which all energy is evenly distributed. The final thermodynamic state of the universe characterized by one uniform temperature, commonly called heat death, occurs only

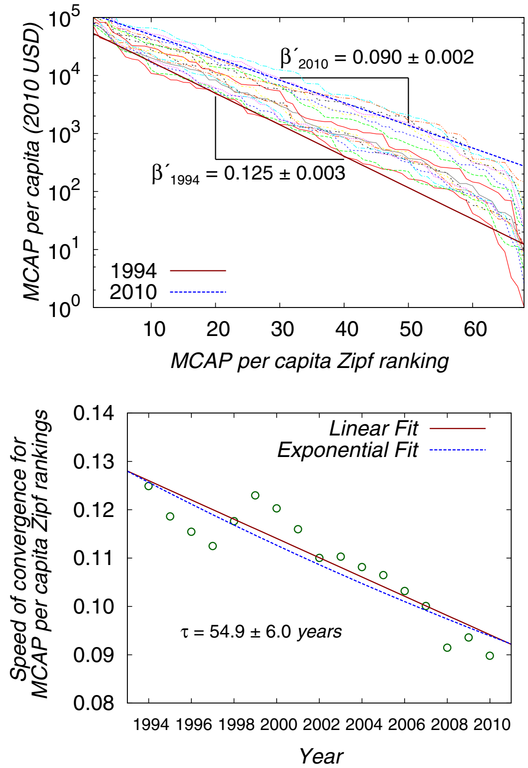

FIG. 3. (Color online) MCAP *per capita*. Capital convergence: decrease in the worldwide capital inequality measured by the Zipf plot of $m_{i,t}$ versus rank for different years. (a) The semilog Zipf plot for 17 years of data follows an exponential-decay curve with decreasing exponential parameter β, which is a signal for the worldwide capital convergence. (b) The Zipf slope versus year calculated for each year follows a decreasing functional dependence in time. The smaller the slope (exponential parameter β'), the larger the capital equality. We quantify the decrease in β' by calculating the mean lifetime of capital inequality, $\tau = 54.9 \pm 6.0$ years, through an exponential fit weighted by the goodness of fit for each β' value.

if the universe continues to exist until it does occur. There is no prediction, even approximate, when this final state might occur. Using this thermodynamic analogy, globalization assumes that all barriers between countries are removed, which would imply that capital can transfer in all directions and that this capital transfer is predominantly from rich to poor countries. While this is predicted by classical economic theory, an historical observation known as the Lucas paradox has shown this to not be the case. However, with increasing global technological and cultural parity it is expected that this paradox should vanish. If countries with an initially smaller *per capita* capital finally reach the level of countries with an initially larger *per capita* capital, when can we expect the occurrence of this final state where all *per capita* capitals are evenly distributed? When does "income flow death" ("capital flow death") occur?

Figure 3(b) shows the changes in the Zipf slope parameter β' for each year between 1994 and 2010. This enables us to estimate when capital flow death—where all $m_{i,t}$ worldwide are evenly distributed—will occur.

We fit β' versus year with a linear functional dependence in which the slope a quantifies the rate of change in the annual

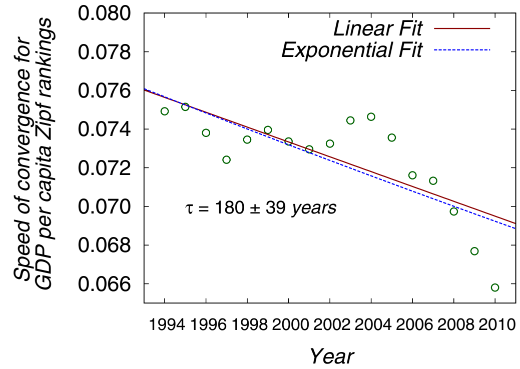

FIG. 4. (Color online) GDP *per capita*. Decrease in the world income inequality measured by the Zipf plot of *per capita* GDP versus rank for different years. (a) The Zipf slope versus year follows a decreasing functional dependence in time. The smaller the slope (exponential parameter β'), the larger the worldwide income equality.

per capita capital convergence,

$$a = -0.0020 \pm 0.0002. \tag{5}$$

If we assume that β' vs year will continue to follow a straight line, using the equation $\beta' = 0.09 - a\Delta t = 0$, we find that after $\approx$45 years the world will reach a state of capital flow death; i.e., the capital *per capita* will be distributed evenly across countries participating in globalization. This result is qualitatively similar to the analysis done for income GDP *per capita* data recently established for EU members [33]. In addition to the best linear fit, we also show, for comparison, the best exponential fit $\exp(-\text{year}/\tau)$ with

$$\tau \approx 55 \text{ years}, \tag{6}$$

or a half-life of approximately 38 years, indicating that every 38 years or so the inequality gap is cut in half.

Figure 4 shows the results of a convergence analysis of GDP *per capita* for the same set of countries and the same year interval as shown in Fig. 3. The Zipf ranking shown in Fig. 4 reveals that the parameter β' calculated for each year exhibits a decreasing functional dependence with time. If we once again assume that β' vs year continues to follow a straight line, using the equation $\beta' = 0.066 - a\Delta t = 0$, where $a = 0.0004$, we find that after $\approx$165 years the world will reach a state of GDP *per capita* death where GDP *per capita* will be distributed evenly across all countries participating in globalization. If we again assume exponential decay, we find a mean lifetime of $\approx$180 years. The *per capita* equalization results of an analysis of MCAP differ from those of an analysis of GDP, in agreement with Eqs. (2) and (4). We note that the past eight years' of GDP data indicate a date of wealth equalization that agrees more closely with the market capitalization data.

C. Modeling MCAP convergence

The neoclassical growth model mentioned above also utilizes a heat death concept. This model predicts that the income levels of poor countries will tend to converge towards the income levels of rich countries as long as they have similar characteristics, e.g., similar saving rates. In other words, the lower the starting level of GDP *per capita* or *per capita* capital, the faster the growth rate.

A simple production function widely employed by economists is the Cobb-Douglas double-power-law function, which is an example of the use of double power laws in economics [16,43],

$$Y(t) = AK^{\alpha}L^{1-\alpha}, \tag{7}$$

where A is the level of technology, and α is a constant for which $0 < \alpha < 1$. We can easily transform the previous expression into $y = Ak^{\alpha} = Af(k)$, where $k = K/L$ and $y = Y/L$, representing capital per labor and output per labor, respectively. In the Solow-Swan model, the derivative of the growth rate of k with respect to k is [16]

$$\frac{\partial \gamma_k}{\partial k} = \frac{\partial \frac{\dot{k}}{k}}{\partial k} \propto [f'(k) - f(k)/k]/k < 0. \tag{8}$$

Because $\alpha < 1$, this is a negative value. This result implies that smaller values of k are associated with larger values of γ_k or, alternatively, economies understood in terms of *per capita* capital with lower capital *per capita* tend to grow more rapidly in *per capita* terms.

We use a Cobb-Douglas production function to show that the growth rate of capital *per capita* (k) is proportional to the growth rate of output *per capita* (y), where

$$\gamma_y = \alpha\gamma_k. \tag{9}$$

Similarly, a Cobb-Douglas production function shows that smaller values of y are associated with larger values of γ_y or, alternatively, economies understood in terms of *per capita* output with lower output *per capita* tend to grow more rapidly in *per capita* terms.

As mentioned above, because we lack physical capital data, we instead perform convergence analysis for market capitalization. Market capitalization is not capital. At the country level, it is the total value of the outstanding shares of all publicly traded companies in each country. As such, increased market capitalization for a country implies either an overall increase in value of listed companies or newly listed companies. Large increases in market capitalization likely indicate the latter, though in either case, increased market capitalization implies a growing economy. When new companies are being listed on stock markets, there is greater opportunity for investment from outside countries and greater opportunity for capital flow. So while market capitalization and capital are two very different concepts, we may expect that increases in market capitalization should be tied to increases in capital and may serve as a reasonable proxy. The main reasons we are interested in looking at market capitalization specifically is that (a) the data are more available than physical capital data, especially for less developed countries, and (b) the data are high-frequency and thus more useful than physical capital data for dynamic studies. If we assume that the physical capital $\mathcal{K}_{i,t}$ of a country i grows in a constant proportion to market capitalization, $\mathcal{M}_{i,t}$, then

$$\mathcal{M}_{i,t} = b_i\mathcal{K}_{i,t}, \tag{10}$$

where b_i is the constant for country i.

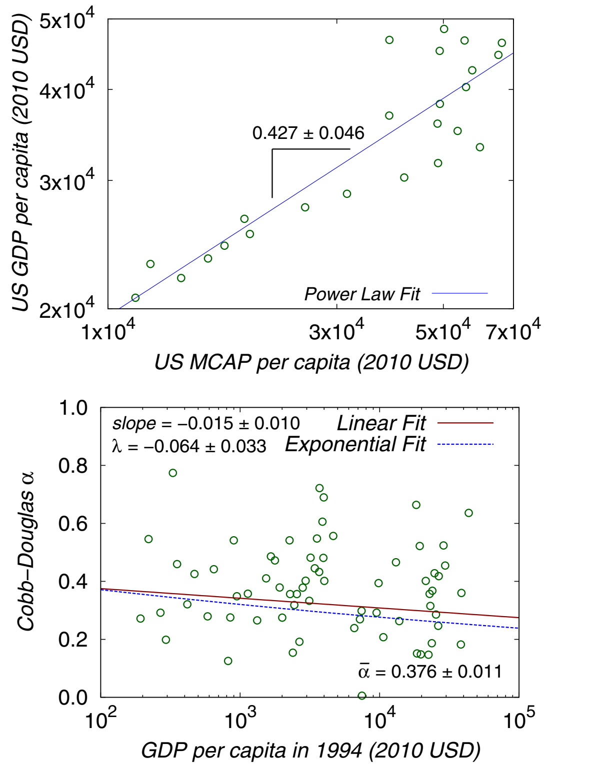

FIG. 5. (Color online) (a) GDP *per capita* versus MCAP *per capita* for the United States. (b) GDP *per capita* and MCAP *per capita* for a country i we approximate by the power law $g_i \propto m_i^{\alpha}$ with exponent α, which substantially changes across countries but has an average $\alpha = 1/3$ ($\alpha = 0.38 \pm 0.15$).

We next assume that a relationship between output and capital takes the form of a Cobb-Douglas production function,

$$g_{i,t} \propto k_{i,t}^{\alpha}, \tag{11}$$

where $g_{i,t}$ is GDP *per capita* and $k_{i,t}$ is physical capital *per capita* in country i. Note that studies done on a few developed countries yield $\alpha = 1/3$ (see Refs. [44–46]).

Using Eq. (10) we obtain

$$g_{i,t} \propto m_{i,t}^{\alpha}, \tag{12}$$

where $m_{i,t}$ is the MCAP *per capita*. Figure 5(a) shows a regression of Eq. (12) for US data and yields slope 0.43 ± 0.05. To test our assumption (10) at a worldwide level, we perform a regression of Eq. (12) for each country i. Figure 5(b) shows the regression between α and the initial GDP *per capita*, and we find an insignificant relationship. Note that α varies substantially with GDP *per capita* because the estimated time series are very short (17 years). The average α,

$$\alpha = 0.38 \pm 0.15, \tag{13}$$

obtained from the previous plot agrees with the $\alpha = 1/3$ value obtained from the regression between physical capital and GDP data for a small sample of rich countries [45]. Our result for a large number of countries, $\langle\alpha\rangle \approx 1/3$, supports our assumption that market capitalization is a reasonable proxy for physical capital. However, it should be noted that as we are limited to 17 data points per country, this analysis should be continued as more data become available. This new economic regularity is not obvious because physical capital values indicate assets already in place, and market capitalization values indicate expectations about future realization. In addition to our time-series analysis of GDP *per capita*, g, and MCAP *per capita*, m, we also do a cross-section analysis by calculating for each year t a power-law exponent α in Eq. (12) between g and m. Summing up over all countries, we calculate α for each year and obtain the average α, $\langle\alpha\rangle = 0.3 \pm 0.01$, again not far from 1/3.

IV. DISCUSSION

In conclusion, we report a new scaling result. When we regress m, the MCAP *per capita*, and g, the GDP *per capita*, we find that m and g follow an approximate power law with an average exponent close to 1/3. This value agrees with the power law obtained in previous studies that regress *per capita* physical capital, k, and g. The similarity in scaling results between m and g, and between k and g, implies that market capitalization can possibly serve as a proxy for physical capital K, though it should again be noted that this analysis should be continued as more data become available, as we are limited to 17 data points per country. This relationship would seem obvious since physical capital refers to factors of production, such as machinery, buildings, and computers—already realized goods—and market capitalization refers to expectations of future market behavior, and our market expectations tend to ultimately follow already realized goods.

If we assume (i) that our linear data extrapolations into the future will prove to be accurate and (ii) that political configurations will remain stable, we can calculate that all countries will experience "capital death," i.e., that all countries will have a *per capita* market capitalization of an approximate equal value, in about 50 years. On the other hand, we may speculate that such events as the rioting in France, Spain, and Greece during the 2010–2012 period may be harbingers of larger social conflicts in the future that may slow economic globalization.

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[1] D. D. Gatti, M. Gallegati, and D. Mignacca, Stud. Nonlin. Dynam. Econometr. **3**, 3 (1998).
[2] C. A. Hidalgo, B. Klinger, and A.-L. Barabasi, Science **317**, 482 (2007).
[3] M.-A. Serrano, M. Boguna, and A. Vespignani, J. Econ. Interact. Coordinat. **2**, 111 (2007).
[4] D. Garlaschelli, T. D. Matteo, T. Aste, G. Caldarelli, and M. I. Loffredo, Eur. Phys. J. B **57**, 159 (2007).
[5] M. Ausloos and R. Lambiotte, Physica A **382**, 16 (2007).
[6] F. Clementi, T. D. Matteo, M. Gallegati, and G. Kaniadakis, Physica A **387**, 3201 (2008).
[7] C. A. Hidalgo and R. Hausmann, Proc. Natl. Acad. Sci. USA **106**, 10570 (2009).
[8] A. Pluchino, A. Rapisarda, and C. Garofalo, Physica A **389**, 467 (2010).
[9] A. Tacchella, M. Cristelli, G. Caldarelli, A. Gabrielli, and L. Pietronero, Sci. Rep. **2**, 723 (2012).
[10] R. J. Barro and J. F. Ursua, NBER Working Paper No. 14760 (2009).
[11] L. Allesi and C. Detken, ECB Working Paper 1039 (2009).
[12] W. Buffet and C. Loomis, Fortune **144**(12), 80 (2001).
[13] R. J. Barro, Q. J. Econ. **106**, 407 (1991).
[14] R. J. Barro and X. X. Sala-i-Martin, J. Polit. Econ. **100**, 223 (1992).
[15] D. Quah, Scand. J. Econ. **95**, 427 (1993).
[16] R. J. Barro and X. X. Sala-i-Martin, *Economic Growth* (MIT Press, Cambridge, MA, 1994).
[17] A. B. Bernard and S. N. Durlauf, J. Econometr. **71**, 161 (1996).
[18] A. B. Atkinson and A. Brandolini, J. Econ. Lit. **39**, 771 (2001).
[19] B. Milanovic, Econ. J. **112**, 51 (2002).
[20] R. M. Solow, Q. J. Econ. **70**, 65 (1956).
[21] P. M. Romer, J. Polit. Econ. **94**, 1002 (1986).
[22] R. E. Lucas, Jr., J. Monet. Econ. **22**, 3 (1988).
[23] R. E. Lucas, Jr., Am. Econ. Rev. **80**, 92 (1990).
[24] J. Benhabib and M. M. Spiegel, J. Monet. Econ. **34**, 143 (1994).
[25] N. G. Mankiw, D. Romer, and D. N. Weil, Q. J. Econ. **107**, 407 (1992).
[26] X. X. Sala-i-Martin, Econ. J. **106**, 1019 (1996).
[27] S. N. Durlauf, J. Econ. **100**, 65 (2001).
[28] X. X. Sala-i-Martin, Q. J. Econ. **121**, 351 (2006).
[29] A. A. Dragulescu and V. M. Yakovenko, Eur. Phys. J. B **17**, 723 (2000).
[30] A. A. Dragulescu and V. M. Yakovenko, Eur. Phys. J. B **20**, 585 (2001).
[31] http://data.worldbank.org.
[32] G. K. Zipf, *Human Behavior and the Principle of Least Effort* (Addison-Wesley, Reading, MA, 1949).
[33] J. Shao, P. C. Ivanov, B. Urosevic, H. E. Stanley, and B. Podobnik, Europhys. Lett. **94**, 48001 (2011).
[34] M. H. R. Stanley, S. V. Buldyrev, S. Havlin, R. Mantegna, M. A. Salinger, and H. E. Stanley, Econ. Lett. **49**, 453 (1995).
[35] X. Gabaix, Q. J. Econ. **114**, 739 (1999).
[36] Y. Fujiwara, Physica A **337**, 219 (2004).
[37] B. P. J. Shao, P. Ch. Ivanov, and H. E. Stanley, Eur. Phys. J. B **56**, 157 (2007).
[38] X. Gabaix, Annu. Rev. Econ. **1**, 255 (2009).
[39] A. M. Petersen, B. Podobnik, D. Horvatic, and H. E. Stanley, Europhys. Lett. **90**, 38006 (2010).
[40] B. Podobnik, D. Horvatic, D. Y. Kennet, and H. E. Stanley, Sci. Rep. **2**, 678 (2012).
[41] R. L. Axtell, Science **293**, 1818 (2001).
[42] B. Podobnik, D. Horvatic, A. M. Petersen, B. Urosevic, and H. E. Stanley, Proc. Natl. Acad. Sci. USA **107**, 18325 (2010).
[43] P. H. Douglas, J. Polit. Econ. **84**, 5 (1976).
[44] E. F. Denison, *Supplement Paper 13* (Committee for Economic Development, New York, 1962).
[45] A. Madisson, *Phases of Capitalist Development* (Oxford University Press, Oxford, UK, 1982).
[46] D. W. Jorgensen, F. M. Gollop, and B. M. Fraumeni, *Productivity and U.S. Economic Growth* (Harvard University Press, Cambridge, MA, 1987).