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Multiplex network analysis of employee performance and employee social relationships

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HIGHLIGHTS

- Superimposed multiplex network (SMN) and unfolded multiplex network (UMN) are proposed to measure employees' social relationships.
- Whole network datasets from three firms are collected for empirical research.
- The different categories of relationship are mutually embedded.
- Multiplex network model provides a richer explanation of employee performance than the single layer network model.
- Employees with high centrality in a weighted UMN are more likely to perform well.

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ABSTRACT

In human resource management, employee performance is strongly affected by both formal and informal employee networks. Most previous research on employee performance has focused on monolayer networks that can represent only single categories of employee social relationships. We study employee performance by taking into account the entire multiplex structure of underlying employee social networks. We collect three datasets consisting of five different employee relationship categories in three firms, and predict employee performance using degree centrality and eigenvector centrality in a superimposed multiplex network (SMN) and an unfolded multiplex network (UMN). We use a quadratic assignment procedure (QAP) analysis and a regression analysis to demonstrate that the different categories of relationship are mutually embedded and that the strength of their impact on employee performance differs. We also use weighted/unweighted SMN/UMN to measure the predictive accuracy of this approach and find that employees with high centrality in a weighted UMN are more likely to perform well. Our results shed new light on how social structures affect employee performance.

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1. Introduction

In recent years network analysis methods [1] have been applied to a wide variety of fields, including economics, sociology, demography, and management [2–5]. The application of network analysis to economic management issues has recently become a research hotspot [6–8].

In firm management, improving employee performance is a critical issue, and understanding the major factors that influence employee performance is essential to both managers and researchers [3,9–14]. Only when we understand all of the factors that affect individual performance can we understand, evaluate, and manage employee underperformance. Current studies of the organizational structure of firms concentrate on the formation of links between different firms and the impact of firm clusters on the overall performance of a firm. What is lacking is an examination of internal firm structure in terms of the interaction among employees, including formal interactions at work and informal interactions in non-work settings. An analysis of internal firm structure from the perspective of employee interaction can be a new approach to exploring firm organizational phenomena, can expand our understanding of the internal interaction structure of firms, and can provide new methods of analyzing the performance of both individual employees and the overall organization.

In a social network of employees, nodes are employees and edges between nodes are employee interactions. Irrespective of network type, the location of an employee in the network strongly affects their ability to obtain information and become aware of opportunities [10,11]. Employees increase their competitive advantage and improve their performance through connections with other employees. Different locations in the network provide different levels of power, and an employee in the center of a network will have access to a higher level of information and to more resources [3,12], and this enhances their performance and competitiveness.

Using network theory and methods of analysis many scholars have analyzed the empirical relationship between network structure and individual performance [9]. Most empirical studies have found that individuals in a central network position have a higher social status and more power in the organization [13], and that this improves their performance. Sparrowe et al. [14] found a positive correlation between network centrality and individual job performance, and that employees with a central position in the network have a higher level of performance and more enthusiasm than employees at the periphery of the network. Ahuja et al. [15] found that, in the organizational structure of a network, centrality strongly impacts an employee's role, status, and ability to communicate and that high status generates high performance and high network centrality. Friendship networks among employees benefit their mutual communication and their willingness to help each other. Employees in a friendship network trust each other and voluntarily take on extra work, and degree centrality is the key determinant of structural position [16].

Traditional studies base their analysis of the relationship between network structure and individual performance on a single type of social relationship. This structure-based approach considers the effect of network structure to be core [14,17] and tends to disregard the varying impacts of the different types of network relationships [18,19]. In real-world social systems, interpersonal interactions often comprise a superposition of several types of relationship rather than a single type. Individuals in real-world social systems have different categories of social relationships, and these we can describe using a multiplex network in which each layer represents a single type of social relationship [20,21]. For example, interpersonal communication patterns within an organization are either formal or informal. Formal patterns are based on a formal organization composed of leaders, managers, and staff and take a bureaucratic form to promote and coordinate the organization's formal activities. Informal patterns are spontaneous and are the primary mode of knowledge exchange [22,23]. Previous studies indicate that a multilayer network structure strongly influences dynamic processes [24,25]. If we do not comprehensively understand network types and take into full consideration the differing network relationships, we will be unable to adequately describe the interpersonal interactions in the network or understand how structural differences in the network impact performance.

Here we use empirical data come from three Chinese firms to construct formal and informal networks to characterize the relationships among employees. In each firm we collect anonymous employee work-related interactions and four categories of social connections. We then build multiplex networks by integrating these five different types of relations to describe the complex social structure of the employees in the firms. We introduce the superimposed multiplex network (SMN) and the unfolded multiplex network (UMN) to evaluate the structural centrality of nodes. Using the SMN and UMN, we take more structural features into consideration and find correlations between network topology and individual performance. The results indicate that employee centrality in a weighted UMN predicts performance, and this expands our understanding of the role of social structures in a network.

Section 2 presents the statistics of the dataset, a mathematical description of multiplex networks, and structural measures for multiplex networks. Section 3 explains the significance of regression in single layer networks and multiplex networks. Section 4 lists our conclusions.

2. Materials and methods

2.1. Data description

The datasets used in this paper were collected using a sampling survey carried out in the Xi'an hi-tech industrial development zone in Shaanxi Province, China. We selected three small and medium-sized firms (SMFs) designated YZ, BD,

Table 1
Basic information of sampling survey.

Firms name	Total questionnaires	Reclaiming questionnaires	Valid questionnaires	Reclaiming rate(%)	Valid rate (%)
YZ	147	125	119	85.0	81.0
BD	72	64	63	88.9	87.5
SL	113	58	52	51.3	46.1
Total	332	247	234	74.4	70.5

Table 2
Social network measures.

Networks	Description of questions
Formal networks	Question 1. With whom do you have working relations?
Informal networks	Question 2. With whom do you have informal social interaction (such as dinner together, drinking together, and shopping together)?
	Question 3. With whom do you have discussed about your important business?
	Question 4. Who will you turn to if you need borrow money from colleagues?
	Question 5. With whom will you discuss if you want to resign?

and SL (not the actual names) for study. And these three firms are Limited Liability Company, Foreign-funded Enterprises and Corporation respectively. Using cluster sampling, we selected all employees in three firms and sent them a questionnaire. Employees were asked to report current status of their networks in firm, and then the employee performance was obtained from finance department. A total of 247 questionnaires were returned, of which 234 were valid. The overall return rate of the questionnaire was 74.4 percent, and the return rate of the valid questionnaires was 70.5 percent with a validity rate of 94.7 percent. Table 1 shows the data of the valid responses to the questionnaire.

Note that the sampling frames for all the employees in the three firms were set in advance, and that the cluster sampling is strictly executed according to the sample frames. This makes our data acquisition valid and usable in our research. Note also that some employees in all three firms were away on a business trip. This removed them from our sample and possibly introduced some bias. Thus our research is only on the social networks and individual performances of employees who were actually in residence at the firms.

The social networks we examine here are both formal and informal. Because the formal interaction of employees in SMFs is primarily work-related, we consider work-related interactions to be an institutional formal relationship and non-work interactions to be non-institutional informal relationships. In particular, formal networks are formed among employees in work settings and ensure the routine operation of the firms, and informal networks are formed among employees in the non-work settings of daily life. Because of the differing levels of trust among employees, the formation rate of informal networks can vary. For example, a person wanting to change jobs might discuss the subject with close friends privately but not publicly with everyone in the firm. In our survey we numbered all employees, and each respondent in Table 1 answered with employee numbers. Thus the data collection is limited within the boundary of the firm network. Based on Ref. [26], Question 2,3,5 are selected to measure informal networks. Besides that, Question 1 is designed to measure formal network and Question 4 is designed to measure tangible support network, which is another typical informal network [27,28]. Using the questions in Table 2, formal networks, i.e., the working relationship networks (WRN, Question 1) were established. Informal networks were also constructed, and these included the informal-communication network (ICN, Question 2), the important business network (IBN, Question 3), the borrowing network (BN, Question 4), and the resignation discussion network (RDN, Question 5). To make up for any lack of information caused by self-reporting, we applied the max-symmetrization method to the network data, which made all five networks symmetric.

Table 3 summarizes the basic topological features of the formal network and the four informal networks in three firms. Networks formed by different kinds of relationship differ in different firms. For example, the centralization in WRN is higher than in any of the informal networks in the YZ firm. In contrast in the BD and SL firms, IBN and BN are more centralized, respectively, than in the other networks. Similarly, $\langle k \rangle$ in Table 3 indicates that employees in the YZ firm have more frequent work-related contacts, while employees in BD and SL firms have more informal-communication and resignation discussions, respectively, than other types of relationships. When $k_{min} = 0$ there is at least one isolated network employee, and when $k_{max} = N - 1$ there is at least one employee that has a relationship with every other employee in the firm. The networks with a high average degree have high clustering coefficients, small average shortest path lengths, and modularity, i.e., they are so dense that all the nodes are grouped together [29]. Most of the networks (except for IBN, BN, and RDN in YZ firm and BN in BD firm) are highly disassortative, indicating that highly-connected employees tend to not establish relations with each other and are more inclined to establish relations with individuals with fewer connections. In contrast, an assortative network is a "rich club" with close interactions among the highly-connected employees, the so-called "core nodes". The corresponding social networks are layered, and the status of these core node employees is enhanced in the social stratification process [30].

Table 3
The basic topological features of the five networks. YZ_WRN represents the working relationship network of YZ firm, YZ_ICN represents the informal-communication network of YZ firm, and so on. N is the network size, C is the centralization of the network [31], k_{min} is the minimum degree in the network, k_{max} is the maximum degree in the network. $\langle k\rangle$ is the average degree, $\langle d\rangle$ is the average shortest path length. R is the assortative coefficient [32], Q is the modularity of a network calculated by the Louvain method [33], and CC is the clustering coefficient [34].

Networks	N	C	k_{min}	k_{max}	$\langle k\rangle$	$\langle d\rangle$	R	Q	CC
YZ_WRN	119	59.58%	20	118	48.874	1.586	−0.151	0.217	0.728
YZ_ICN	119	57.09%	0	78	11.765	2.740	−0.097	0.313	0.579
YZ_IBN	119	14.91%	0	23	5.697	3.440	0.031	0.379	0.405
YZ_BN	119	23.42%	1	33	5.832	4.017	0.002	0.382	0.511
YZ_RDN	119	10.25%	0	15	3.109	5.038	0.198	0.374	0.301
BD_WRN	63	79.80%	4	62	14.095	1.773	−0.539	0.095	0.796
BD_ICN	63	42.73%	10	62	36.349	1.414	−0.245	0.081	0.745
BD_IBN	63	93.76%	1	62	5.714	1.908	−0.322	0.199	0.675
BD_BN	63	22.71%	0	19	5.365	2.647	0.014	0.284	0.424
BD_RDN	63	46.88%	0	33	4.857	2.413	−0.300	0.224	0.391
SL_WRN	52	68.00%	4	51	17.654	1.654	−0.443	0.114	0.767
SL_ICN	52	55.14%	1	32	4.962	2.537	−0.258	0.252	0.501
SL_IBN	52	18.35%	0	13	4.000	3.340	−0.142	0.408	0.400
SL_BN	52	96.00%	1	51	3.923	1.923	−0.422	0.252	0.761
SL_RDN	52	55.45%	8	51	23.808	1.533	−0.472	0.089	0.805

Employee performance (denoted by Z) is measured by calculating the Z-score of employee average monthly income including wages and bonuses obtained directly from the firm's finance department. The Z-score can be expressed

$$Z = (Y - \mu)/\sigma, \tag{1}$$

where Y is the actual income of employees, μ is the average income for all employees, and σ is the standard deviation of income. Since the Z-score is a normalization of Y, the mean value of Z is 0, and standard deviation of Z is 1. When $Z = 0$ the employee earns an average wage and bonus, and when $Z = 1$ the employee earns a wage and bonus one standard deviation higher than average. In YZ, BD and SL firms, the range of Z are (−1.058, 3.824), (−0.815, 5.318), and (−1.509, 2.353) respectively.

2.2. Mathematical description of multiplex network

We introduce multiplex networks to describe the formal and informal relationships in the above three firms [35–37]. A generalized multilayer network can be defined as a pair $\mathcal{M} = (\mathcal{G}, \mathcal{C})$, where $\mathcal{G} = \{G_\alpha; \alpha \in \{1, 2, \ldots, M\}\}$ consists of a family of graphs $G_\alpha = (V_\alpha, E_\alpha)$ and $\mathcal{C} = \left\{E_{\alpha\beta} \subseteq V_\alpha \times V_\beta; \alpha, \beta \in \{1, 2, \ldots, M\}, \alpha \neq \beta\right\}$ is the set of interconnections between nodes in different graphs G_α and G_β with $\alpha \neq \beta$.

Each graph G_α is a layer of $\mathcal{M}$ that can be used to represent a relationship among individuals. The nodes in the layer G_α are denoted by $V_\alpha = \left\{v_1^\alpha, v_2^\alpha, \ldots, v_{N_\alpha}^\alpha\right\}$, and the edges in the layer G_α by $E_\alpha = \left\{\left(v_i^\alpha, v_j^\alpha\right)\right\}$. The unweighted adjacency matrix of each layer G_α is $A^{[\alpha]} = (a_{ij}^\alpha) \in \mathbb{R}^{N_\alpha \times N_\alpha}$, the element of $A^{[\alpha]}$ is

$$a_{ij}^\alpha = \begin{cases} 1, & \text{if } \left(v_i^\alpha, v_j^\alpha\right) \in E_\alpha \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

where $1 \le i, j \le N_\alpha$, and $1 \le \alpha \le M$. The unweighted adjacency matrix describing the interlayer connections $E_{\alpha\beta} (\alpha \neq \beta)$ is $A^{[\alpha,\beta]} = (a_{ij}^{\alpha\beta}) \in \mathbb{R}^{N_\alpha \times N_\beta}$, the element of $A^{[\alpha,\beta]}$ is

$$a_{ij}^{\alpha\beta} = \begin{cases} 1, & \text{if } \left(v_i^\alpha, v_j^\beta\right) \in E_{\alpha\beta} \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

where $1 \le i \le N_\alpha$, $1 \le j \le N_\beta$, $1 \le \alpha, \beta \le M$, and $\alpha \neq \beta$.

Social systems are made up of individuals within a certain proximity who engage in a variety of social relations. The overall system of interactions is a superposition of a number of social networks in which nodes are individuals and edges are social connections. Employee relationships within a firm are both formal in response to job requirement and informal. Informal relationships are of four different types. Here each type of social relation is represented by a layer in the multilayer network, and the same group of individuals (nodes) are represented in all layers, i.e., nodes in a given layer have counterpart nodes in other layers. This type of multilayer network is a multiplex network in which the interlayer edges are connections between a given node and its counterpart nodes in other layers. We define a multiplex network to be [37]

$$\begin{cases} \mathcal{M} \\ s.t. V_\alpha = V, \quad 1 \le \alpha \le M \\ E_{\alpha\beta} = \{(v, v); v \in V\}, \quad 1 \le \alpha, \beta \le M \quad \text{and} \quad \alpha \neq \beta \end{cases}$$

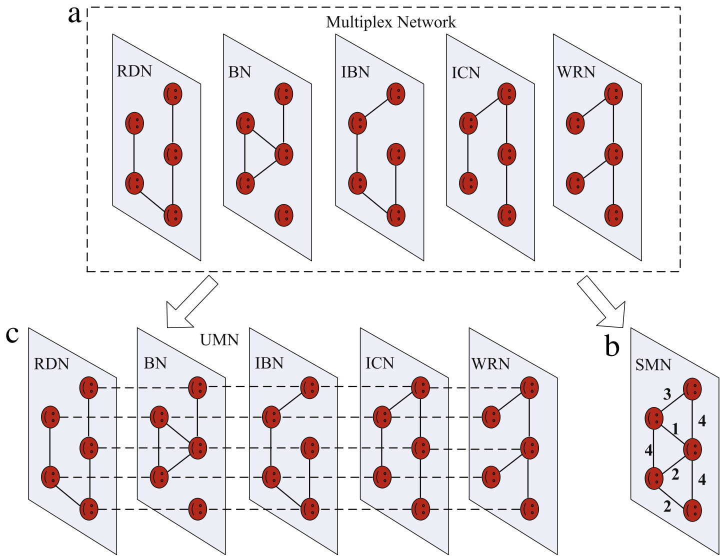

Fig. 1. Multiplex network, superimposed multiplex network (SMN) and unfolded multiplex network (UMN). Each layer reflects one kind of relationship among individuals in multiplex network, including RND, BN, IBN, ICN and WRN. Solid line and dotted line between nodes indicate the interconnections and interlayer connections respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

We introduce two ways of analyzing the structural centrality of nodes in a multiplex network (see Fig. 1). One uses a superimposed multiplex network (SMN) and the other an unfolded multiplex network (UMN).

SMN is denoted by $\widetilde{\mathcal{M}}_{\mathrm{s}} = \left(\widetilde{V}_{\mathrm{s}}, \widetilde{E}_{\mathrm{s}}, W_{\mathrm{s}}\right)$, where $\widetilde{V}_{\mathrm{s}} = V$, $\left|\widetilde{V}_{\mathrm{s}}\right| = |V| = N$ and $\widetilde{E}_{\mathrm{s}} = \left\{\left(v_i, v_j\right); \left(v_i^{\alpha}, v_j^{\alpha}\right) \in E_{\alpha}, 1 \le i, j \le N_{\alpha}, 1 \le \alpha \le M\right\}$. For each $\left(v_i, v_j\right) \in \widetilde{E}_{\mathrm{s}}$ there is a non-negative real number $w_{\mathrm{s}} \in W_{\mathrm{s}}$ that quantifies its weight. Fig. 1(b) shows that SMN is a superposition of all monolayers, and does not take into consideration the interlayer connections $E_{\alpha\beta}(\alpha \neq \beta)$. The strength of the relationship between two individuals in SMN depends on the total number and the type of their relationships.

UMN is denoted by $\widetilde{\mathcal{M}}_{\mathrm{u}} = \left(\widetilde{V}_{\mathrm{u}}, \widetilde{E}_{\mathrm{u}}, W_{\mathrm{u}}\right)$, where

$$\widetilde{V}_{\mathrm{u}} = \bigcup_{1 \le \alpha \le m} V_{\alpha} = \left\{v^{\alpha}; v \in V_{\alpha}\right\}, \qquad \left|\widetilde{V}_{\mathrm{u}}\right| = |V| \times M = N \times M,$$

where $\widetilde{E}_{\mathrm{u}}$ is given by

$$\widetilde{E}_{\mathrm{u}} = \left(\bigcup_{1 \le \alpha \le m} \left\{\left(v_i^{\alpha}, v_j^{\alpha}\right); \left(v_i^{\alpha}, v_j^{\alpha}\right) \in E_{\alpha}\right\}\right) \bigcup \left(\bigcup_{1 \le \alpha, \beta \le M, \alpha \neq \beta} \left\{\left(v_i^{\alpha}, v_j^{\beta}\right); v_i \in V\right\}\right),$$

and $w_{\mathrm{u}} \in W_{\mathrm{u}}$ is the weight value of each element $\left(v_i, v_j\right) \in \widetilde{E}_{\mathrm{u}}$. Fig. 1(c) shows that UMN considers the interconnections in each layer separately and treats the relationships between an individual and its mirrors as interlayer connections $E_{\alpha\beta}(\alpha \neq \beta)$.

2.3. Centrality of nodes in multiplex network

Here we introduce node centrality metrics based on SMN and UMN that can predict employee performance. Centrality measures the structural relevance of each node in a network and reflects its importance [38,39] in the objective social structure. By analyzing whether an individual is in the center of its social network and measuring the quantity and quality of their network resources, we can quantify how relationships impact individuals. Node centrality can be evaluated both locally and globally. When a node has a high local centrality it has many direct contacts. When a node has a high global centrality it occupies a strategic position in entire whole network [40]. Degree centrality and eigenvector centrality are typical local and global centrality indicators, respectively.

Degree centrality is the simplest index of centrality measure. The degree of a node refers to the number of nodes to which it is directly connected (and not to those indirectly connected), and it quantifies the local centrality of the node [40]. The degree centrality of node i in layer G_{α} of a monolayer network is

$$k_i^{\alpha} = \sum_{j=1}^{N_{\alpha}} a_{ij}^{\alpha}. \tag{4}$$

The degree centrality indicates the direct influence of an individual in a network and measures their ability to connect with neighboring individuals. The higher the degree centrality of an individual, the higher the number of direct contacts and the higher the probability that they will be in the center of the network and wield higher power. Individuals with high degree centralities can also impede or distort the dissemination of network information [41].

The degree of node $i \in X$ in a multiplex network $\mathcal{M}$ is based on the definition of degree centrality in a monolayer network, which is the vector [42]

$$K_i = \left(k_i^{[1]}, k_i^{[2]}, \ldots, k_i^{[M]}\right) \in \mathbb{R}^M, \tag{5}$$

where $k_i^{[\alpha]}$ is the degree centrality of node i in layer α.

Measuring degree centrality does not take into account the role of indirect connections in the network. When a node is connected to other high centrality nodes its own centrality increases. Thus node centrality measures not only the number of connected nodes but also their centrality value [43]. Employing the spectral properties of the adjacency matrix, the eigenvector centrality of node i in layer G_α becomes

$$c_i^\alpha = \frac{1}{\lambda_{\max}\left(A^{[\alpha]}\right)} \cdot \sum_{j=1}^{N_\alpha} \left(a_{ij} \cdot c_j^\alpha\right). \tag{6}$$

If the centralities of all nodes are $\boldsymbol{c}^\alpha = \left(c_1^\alpha, c_2^\alpha, \ldots, c_{N_\alpha}^\alpha\right)$, Eq. (6) can be written $\lambda \boldsymbol{c} = \boldsymbol{Ac}$, which can be solved by computing the eigenvalues and eigenvectors of the adjacency matrix $\boldsymbol{A}$. If $\boldsymbol{A}$ both an adjacency matrix of an undirected graph and nonnegative, the eigenvector corresponding to the largest eigenvalue will be the eigenvector centrality according to the Perron–Frobenius theorem [43,44]. The eigenvector centrality of a node is the linear superposition of the centralities of its neighbor nodes. The eigenvector centrality measurement can be extended to multiplex networks in the form of degree centrality, which is given by

$$\boldsymbol{C}_i = \left(c_i^{[1]}, c_i^{[2]}, \ldots, c_i^{[M]}\right) \in \mathbb{R}^M, \tag{7}$$

where $c_i^{[\alpha]}$ is the eigenvector centrality of node i in layer α.

Once all the centralities have been calculated in each layer $1 \le \alpha \le M$ separately, the degree centrality and eigenvector centrality of multiplex network $\mathcal{M}$ can be denoted by matrices $K = (K_1|K_2|\cdots|K_N) \in \mathbb{R}^{N\times M}$ and $C = (C_1|C_2|\cdots|C_N) \in \mathbb{R}^{N\times M}$ respectively. However matrix degree centrality and eigenvector centrality measurements do not produce an ordered list of the nodes in a multiplex network. To rank the nodes in a multiplex network, we introduce two centrality measurement methods based on the SMN and UMN respectively.

Because previous research has shown that the strength of a relationship between two individuals is positively related to the number of different types of relationship they have [9], the node centralities in SMN can be computed using the weighted matrix given by

$$\widetilde{A}_s = \sum_{\alpha=1}^{M} w_s^\alpha A^{[\alpha]} \in \mathbb{R}^{N\times N}, \tag{8}$$

where w_s^α is the degree of importance (or influence) of layer α in multiplex network $\mathcal{M}$.

SMN measurements are limited in that they do not fully consider the multilevel interactions between layers and their effect on the centralities of each node. UMN measurements both consider the influence of connections among nodes in same layer and analyze the impact of interlayer connections. The adjacency matrix of UMN is a block matrix

$$\widetilde{A}_{\mathrm{u}} = \begin{pmatrix} w_u^{11}A^{[1]} & w_u^{12}I_N & \cdots & w_u^{1M}I_N \\ w_u^{21}I_N & w_u^{22}A^{[2]} & \ldots & w_u^{2M}I_N \\ \vdots & \vdots & \ddots & \vdots \\ w_u^{M1}I_N & w_u^{M2}I_N & \cdots & w_u^{MM}A^{[M]} \end{pmatrix} \in \mathbb{R}^{(NM)\times(NM)}, \tag{9}$$

where I_N is the N-dimensional identity matrix, and $W_{\mathrm{u}} = w_u^{\alpha\beta} \in \mathbb{R}^{M\times M}$, $(1 \le \alpha, \beta \le M)$ is a non-negative matrix $W_{\mathrm{u}} \ge 0$ such that $w_u^{\alpha\beta}$ measures the degree of importance of different kinds of connections. The degree centrality and eigenvector centrality computed by $\widetilde{A}_{\mathrm{u}}$ is $\widetilde{K}_{\mathrm{u}} = \left(\widetilde{k_1}, \widetilde{k_2} \ldots, \widetilde{k_{NM}}\right) \in \mathbb{R}^{NM}$ and $\widetilde{C}_{\mathrm{u}} = (\widetilde{c_1}, \widetilde{c_2}, \ldots, \widetilde{c_{NM}}) \in \mathbb{R}^{NM}$, respectively. Then the degree centrality of node $i \in V$ can be aggregated using $\widetilde{k_i}$ and its mirrors, denoted by

$$k_i = \sum_{\lambda=0}^{M-1} \widetilde{k_{i+\lambda N}}. \tag{10}$$

Analogously, the eigenvector centrality of node $i \in V$ is

$$c_i = \sum_{\lambda=0}^{M-1} \widetilde{c_{i+\lambda N}}. \tag{11}$$

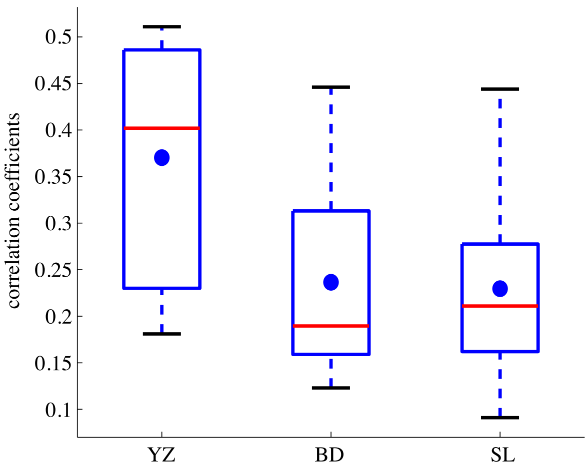

Fig. 2. Boxplot graph of QAP analysis of three firms. The abscissa stands for firms, and the ordinate stands for correlation coefficients obtained in QAP analysis. The blue dots represent mean values of correlation coefficients of networks in each firm. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

3. Results

Here we test the effectiveness of two centrality measurements based on SMN and UMN using datasets from three firms. Different types of social relationships among the same group of employees tend to mutually embed [45]. Since dyadic relations are not independent of each another, we use a quadratic assignment procedure (QAP) that is robust against autocorrelation to test correlations between social networks [46]. We use the QAP to calculate the correlation coefficient between two matrices by comparing each element in these two matrices. We also perform a nonparametric test on the coefficients and find that the probability that the correlation coefficient after random permutations will be larger than that prior [47]. Fig. 2 shows a boxplot of the statistically significant correlation coefficients of networks obtained in QAP analysis.

Fig. 2 shows that there are significant data correlations in employee social networks. Exceptions to this are in WRN–ICN, ICN–IBN in the BD firm and WRN–RDN, BN–RDN in the SL firm. This indicates that although mutual embeddings exist between formal and informal networks, and also between informal networks, this correlation is not strong. The maximum values of the correlation coefficient between networks in these three companies are 0.511, 0.446, and 0.444, and the mean values are 0.370, 0.237, and 0.230, respectively.

Table 4 provides the details of this QAP analysis of the three firms. The significant correlations between WRN and the other informal relationships suggest that the establishment of informal relationships among employees tend to emerge from a formal working relationship, and ultimately result in mutual embedding. Note that the data overlap between formal and informal networks is not serious. Maximal correlation coefficients of the three companies are 0.298, 0.187, and 0.328, respectively, i.e., only a small number of employees in the firm become members of the informal network.

The characteristics of the correlations between networks differ among the three firms. In the YZ firm there are significant correlations among all networks, indicating that the various social relations in the firm are mutually embedded, and that the employees are closely connected. In the BD and SL firms the formal network unrelated to any informal networks, indicating that employees leave the formal organizational structure and spontaneously form informal groups. The average degree in Table 3 shows that the SL firm in particular has a range of contacts discussing resignation issues that is larger than that discussing work issues. When having resignation discussions, employees tend to form groups that do not overlap with formal groups, and this can result in organizational conflict [48].

To calculate centralities in multiplex networks, we use two strategies for determining the parameters in the adjacent matrix with respect to SMN and UMN. In the first we consider every layer G_α in multiplex network $\mathcal{M}$ to have the same level of importance. Thus the influence on employee performance is the same in all types of relationship, and thus only structural characteristics are incorporated into our network analysis. We thus define the unweighted superimposed multiplex network (uSMN) by setting $w_s^\alpha=1$ for $\forall\alpha \in \{1, 2, \ldots, M\}$ in Eq. (8) and the unweighted unfolded multiplex network (uUMN) by setting $w_u^{\alpha\beta}=1$ for $\forall\alpha, \beta \in \{1, 2, \ldots, M\}$ in Eq. (9).

The second and more complex strategy is to allow different degrees of importance in different network layers. To reflect the different effects of different types of relationship on individual performance, we define r^α to be the value of R^2 of regression analysis in which the centrality of layer G_α is an independent variable and the individual performance a dependent variable. Subsequently, the weighted superimposed multiplex network (wSMN) is defined

$$w_s^\alpha = \frac{r^\alpha}{\sum_{\theta=1}^{M} r^\theta} \tag{12}$$

Table 4
QAP test for social network data. Unstandardized correlation coefficients are displayed. Number of Permutations is 5000.

(a)Correlations between employees social networks in YZ firm					
Networks	YZ_WRN	YZ_ICN	YZ_IBN	YZ_BN	YZ_RDN
YZ_WRN	1.000***	0.298***	0.230***	0.210***	0.181***
YZ_ICN	0.298***	1.000***	0.440***	0.486***	0.364***
YZ_IBN	0.230***	0.440***	1.000***	0.479***	0.511***
YZ_BN	0.210***	0.486***	0.479***	1.000***	0.504***
YZ_RDN	0.181***	0.364***	0.511***	0.504***	1.000***
(b) Correlations between employees social networks in BD firm					
Networks	BD_WRN	BD_ICN	BD_IBN	BD_BN	BD_RDN
BD_WRN	1.000***	0.044	0.123+	0.150***	0.187**
BD_ICN	0.044	1.000***	0.059	0.192***	0.168***
BD_IBN	0.123+	0.059	1.000***	0.324***	0.302***
BD_BN	0.150***	0.192***	0.324***	1.000***	0.446***
BD_RDN	0.187**	0.168***	0.302***	0.446***	1.000***
(c) Correlations between employees social networks in SL firm					
Networks	SL_WRN	SL_ICN	SL_IBN	SL_BN	SL_RDN
SL_WRN	1.000***	0.328***	0.195***	0.129+	−0.014
SL_ICN	0.328***	1.000***	0.444***	0.211*	0.091+
SL_IBN	0.195***	0.444***	1.000***	0.211***	0.227***
SL_BN	0.129+	0.211*	0.211***	1.000***	0.104
SL_RDN	−0.014	0.091+	0.227***	0.104	1.000***

*** Significant at 0.001 level.
** Significant at 0.01 level.
* Significant at 0.05 level.
+ Significant at 0.1 level.

for $\forall\alpha \in \{1, 2, \ldots, M\}$ in Eq. (8), and the weighted unfolded multiplex network (wUMN) is defined

$$w_u^{\alpha\beta} = \frac{r^\alpha r^\beta}{\sum_{\theta=1}^{M}(r^\theta)^2} \tag{13}$$

for $\forall\alpha, \beta \in \{1, 2, \ldots, M\}$ in Eq. (9).

To test how employee networks affect employee performance we use the linear polynomial regression model $Z = p_1c+p_2$ in which c is degree/eigenvector of employee centrality. Figs. 3 and 4 show the results of the regression analysis. The results on single layer networks indicate that both degree centrality and eigenvector centrality have a statistically significant positive impact on employee performance, indicating that employees in a central network position will have a higher performance. Note that because a single employee can occupy positions of differing centralities in different categories of network, to comprehensively analyze the effect of employee networks on employee performance we need a multiplex network model.

Although uSMN and uUMN aggregate the information from all single-layer networks, the regression analysis results in Fig. 4 show that their ability to accurately predict employee performance is even worse than that of some single-layer networks. In contrast, the wSMN and wUMN, which assimilate the characteristics of the relationship categories, can use social networks to explain improvements in employee performance. Measuring the degree and eigenvector centralities they can predict the employee performance range from high to low, i.e., $E_{\mathrm{wUMN}} > E_{\mathrm{wSMN}} > E_{\mathrm{uUMN}} = E_{\mathrm{uSMN}}$ and $E_{\mathrm{wUMN}} > E_{\mathrm{wSMN}} > E_{\mathrm{uSMN}} > E_{\mathrm{uUMN}}$, respectively, where E_α is the effectiveness of network α.

Why the regression results of degree centrality in uUMN is the same as that in uSMN can be determined using mathematical logic. The degree centrality of node $i \in V$ in uUMN is

$$\begin{aligned} k_i^{\mathrm{uUMN}} &= \sum_{\lambda=0}^{M-1} \widetilde{k_{i+\lambda N}} \\ &= \sum_{\alpha=1}^{M} \left(k_i^{[\alpha]} + M - 1\right) \\ &= \sum_{\alpha=1}^{M} k_i^{[\alpha]} + M\,(M-1) \\ &= k_i^{\mathrm{uSMN}} + \varphi, \end{aligned} \tag{14}$$

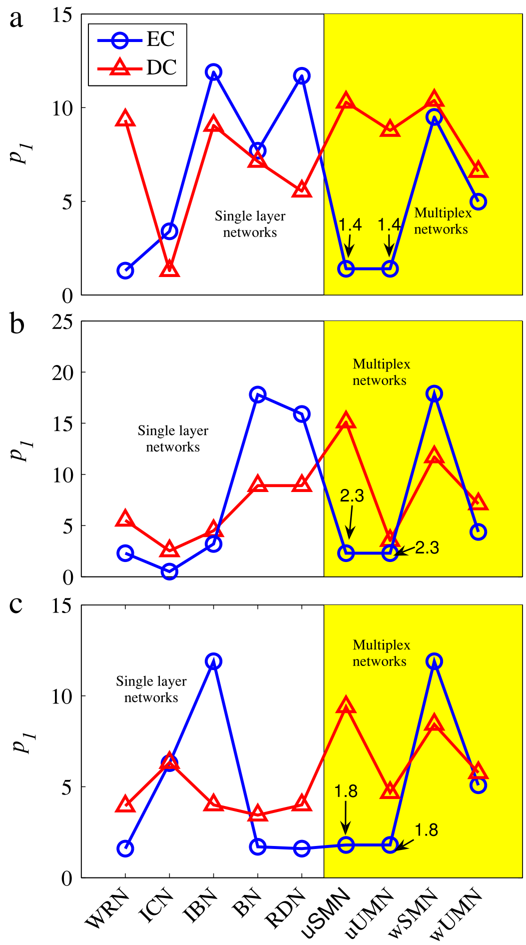

Fig. 3. Coefficients of centralities in linear regression models for predicting individual performance by employee networks in YZ, BD and SL firms. The subfigure (a), (b) and (c) show the results in YZ, BD and SL firms respectively. The abscissa stands for networks. The results from five different types of single layer network are presented in the left area delimited by dashed line, while the results from multiplex networks are showed in the right area with yellow background. The red line and the blue line indicate the results of regression analysis where degree centrality (abbreviate to DC) and eigenvector centrality (abbreviate to EC) are brought into as the independent variables respectively. The ordinate stands for coefficients of centralities (with 95% confidence bounds) in regression analysis. In order to show the coefficients with appropriate scale in the graph, the coefficient of degree centrality in wUMN is multiplied by 10 and coefficients of degree centrality in other networks are multiplied by 100. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

where ${k_i}^{\mathrm{uSMN}}$ is the degree centrality of node $i \in V$ in uSMN and $\varphi = M(M-1)$ is a constant. Consequently only the intercept value p_2 is changed when ${k_i}^{\mathrm{uUMN}}$ and ${k_i}^{\mathrm{uSMN}}$ are used as independent variables in linear regression model $Z = p_1 c + p_2$, respectively.

Employees with a high centrality in wUMN are more likely to perform well. Interpersonal relationships and interaction patterns strongly affect firm performance, and the different characteristics of the social network structure affect the formation process and the resulting performance [49]. The productivity, efficiency, and innovation capacity of an firm requires an effective pattern of knowledge interaction among employees [22,50], and the flow of information must be fair, open, and transparent in the network of working relationships. However in private social interactions, tacit knowledge leads to employee inequality. Thus the results in Fig. 3 show that occupying a central location in the formal network (WRN) has little effect on employee performance. Applying degree centrality and eigenvector centrality as independent variables in these three firms, we find the regression results of R^2 to be (0.078, 0.106, 0.04) and (0.107, 0.119, 0.058), respectively. Although informal relationships are not directly related to work in most cases, they bring a variety of low-cost employee benefits, including resources, information, and emotional support. This phenomenon is more evident in Asian countries [51]. Many companies are currently aware of this problem, and are adopting both written and unwritten rules to encourage informal relationships among employees [52].

Informal relationships are of various types and play differing roles in different settings. Because of this complexity, interpersonal relationships are difficult to describe using a single-dimensional network. Thus a multiplex network model is needed to integrate different social relations within bounded groups and more comprehensively reflect the structural status

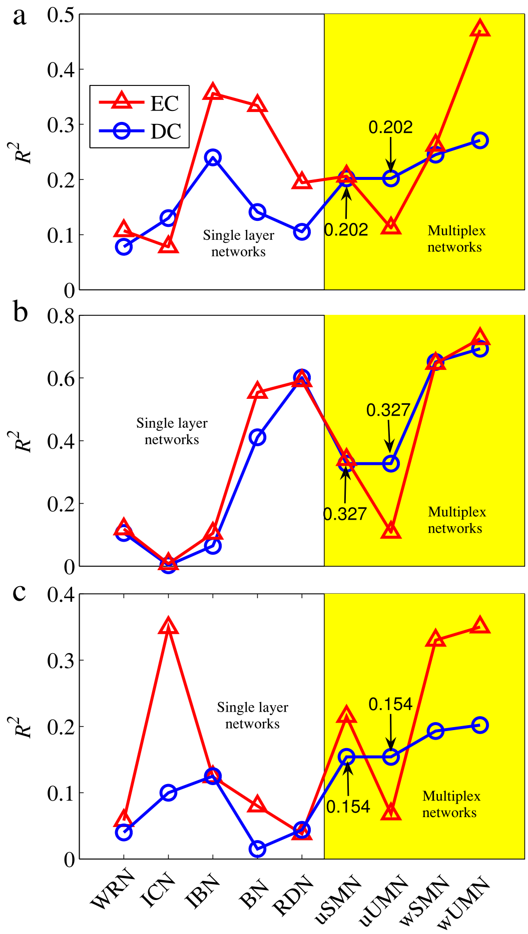

Fig. 4. R^2 of linear regression models for predicting individual performance by employees networks in YZ, BD and SL firms The ordinate stands for R^2 in regression analysis. Other captions are the same as Fig. 3. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

of individuals in an organization. UMN reflects the internal connections in each layer and reveals interactions between layers better than SMN, and wUMN reflects the importance of different layers better than uUMN. Thus the regression analysis results in Figs. 3 and 4 show that the highest performing employees are those that have an advantageous network position in wUMN.

4. Conclusion

We have used a multiplex network model to demonstrate how employee performance is affected by formal and informal employee relationship. We introduce degree centrality and eigenvector centrality in four multiplex network models (wUMN, wSMN, uUMN, and uSMN) to quantify the position of employees in their social structure. We test the effectiveness of our approach by conducting empirical research on three datasets of five kinds of relationship among employees in three firms. We found that a nuanced multiplex network model that integrates different types of social relations provides a richer explanation of employee performance than the single layer network model used in much prior research. The QAP analysis shows that there is some correlation among various social relations, but that this correlation is not significant. Thus different kinds of social relations result in differing network structures and have differing effects on employees, consistent with the results of a regression analysis of a single layer network. A regression analysis of a multiplex network predicts employee performance, from high to low, to be $E_{\mathrm{wUMN}} > E_{\mathrm{wSMN}} > E_{\mathrm{uUMN}} = E_{\mathrm{uSMN}}$ and $E_{\mathrm{wUMN}} > E_{\mathrm{wSMN}} > E_{\mathrm{uSMN}} > E_{\mathrm{uUMN}}$, respectively. This indicates that the wUMN more accurately reflects an individual's position in the complex social network, and that the highest performing employees are in the center position.

Our work contributes to the firm management literature by focusing on the pivotal role of social structures. Our findings provide a new and actionable understanding of human resources management and suggest that building a communication platform, enhancing employee interaction, and promoting the diffusion of information, whether work-related or non-work related, improves employee performance. Thus we suggest that human resources managers focus more on employees in the central position in multidimensional relationships because they are good organizers who can positively affect organizational development [28].

Despite the complexity of multidimensional relationships, we believe our work and the further quantitative analyses of multiplex network data will expand our understanding of firm networks and management practices. Although here we have used only degree and eigenvector centralities to measure an individual's position in positive symmetrical multiplex network, other centrality measurements such as closeness and betweenness and other connection types, such as non-reciprocal and negative relations, could be extended to multiplex networks and applied in empirical research [35]. Because organizational structures differ in different countries [53], further research on related issues in organizations with different cultures or in different domains would be a positive next step.

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