Robustness on interdependent networks with a multiple-to-multiple dependent relationship

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ABSTRACT

Interdependent networks as an important structure of the real system not only include one-to-one dependency relationship but also include more realistic undirected multiple interdependent relationship. The study on interdependent networks plays an important role in designing more resilient real systems. Here, we mainly focus on the case of interdependent networks with a multiple-to-multiple correspondence of interdependent nodes by generalizing feedback and nonfeedback conditions. We develop a new mathematical framework and study numerically and analytically the percolation of interdependent networks with partial multiple-to-multiple dependency links by using percolation theory. By analyzing the process of cascading failure, the size of the giant component and the critical threshold are analytically obtained and testified by simulation results for couple Erdös-Rènyi and scale-free networks. The results imply that the system shows a discontinuous phase transition for different coupling strengths. We find that the system becomes more resilient and easy to defend by increasing the coupling strength and the connectivity density. Our model has the potential to shed light on designing more resilient real-world dependent systems.

The interdependent network as an important research field is attracting a lot of attention, but most of the previous works are limited to a one-to-one dependency relationship. The phenomenon of the dependent network with a multiple-to-multiple dependency relationship exists widely in the real world. The existing conclusions are not suitable to explain the resilience among more complicated multiple dependency relationships and also challenge the existing theoretical methods. To fill this gap, we develop a framework with two partially dependent networks with multiple-to-multiple dependency relationship to study the robustness of the system. We investigated the phase transition behaviors and testified the simulation results by using the percolation theory. Interestingly, the system shows a discontinuous phase transition behavior for strong coupling strength, which is a remarkable contrast to two interacting network models. The results help us to obtain a deeper understanding of the dependent system with multiple-to-multiple relationship, and the developed theoretical method could also be applied for designing more resilient real systems.

I. INTRODUCTION

A complex network as a momentous research field is attracting together researchers from different scientific communities working on these areas. Recently, interdependent networks with different types of dependent links have drawn a lot of attention.[1–11] The initial investigations of network properties are mainly on single networks, which are viewed as independent and do not exchange any information with other networks.[12–21] However, the real systems are always coupled or are interdependent on each other to function. More recently, some analytical frameworks for exploring the percolation properties of two interdependent networks or multilayered interdependent networks have been put forward because many real systems interact with each other virtually.[22–27] Previous studies have shown two strongly interdependent networks consisting of two networks with one-to-one correspondence, where distinct phase transition and discontinuous phase transition occur, from a single network, which shows a continuous transition.[22] Buldyrev *et al.* developed a framework for understanding the robustness of two dependent networks with a one-to-one correspondence of dependent nodes subject to such cascading failures.[27] Gao *et al.* introduced a more general case of percolation of interdependent networks, Network of Networks, by considering undirected one-to-one correspondence with feedback and nonfeedback conditions between networks, respectively.[28] Shao *et al.* presented a model of cascading failure on interdependent networks by considering directed support-dependence relations between two coupled network systems.[3] Dong *et al.* introduced a coupling network model with similar types of connectivity-link within and between communities' structure. The results highlight that the coupling community structure can significantly affect the resilience of the system in that it removes the phase transition present in a single module, and the network remains resilient at this transition.[29] However, a node within the network not only depends on one node in another network but most often mutually depends on multiple nodes in the real system. Examples of this case include global trade coupled networks, power grid networks, and communication networks. Due to a shortage of resources in various countries, there exist trade dependencies between countries. Since each national demand is different and necessary, multiple-to-multiple dependent relationship can be described between countries. In addition, based on economic considerations, one power station often provides power to several communication stations, and one communication station functions by depending on multiple power stations. The undirected one here means that there is a demand for each system in real systems. This real undirected multiple dependent relationship is different from directed multiple support-dependence relations, where active nodes have at least one directed support node in the system. For an interdependent network with two networks A and B, a single node in network A depends on multiple nodes in network B and functions since at least one of the dependent nodes of another network is functional. Similarly, a node in network B also depends on several nodes in network A. In an undirected multiple dependence relationship, node failures in one network will cause the failure of interdependent nodes in the other network and vice versa. Furthermore, this recursive process leads to a cascading failure until a stable state is reached, in which no node fails. Based on this motivation, we develop a theoretical framework for understanding the robustness of the interdependent networks with a multiple-to-multiple correspondence of interdependent nodes. Furthermore, the system threshold value, which plays an important role in determining the system robustness, is also numerically predicted by using the percolation theory.

II. MODEL DESCRIPTION

We simply assume that two independent networks A and B with the same number of nodes N, which have the given degree

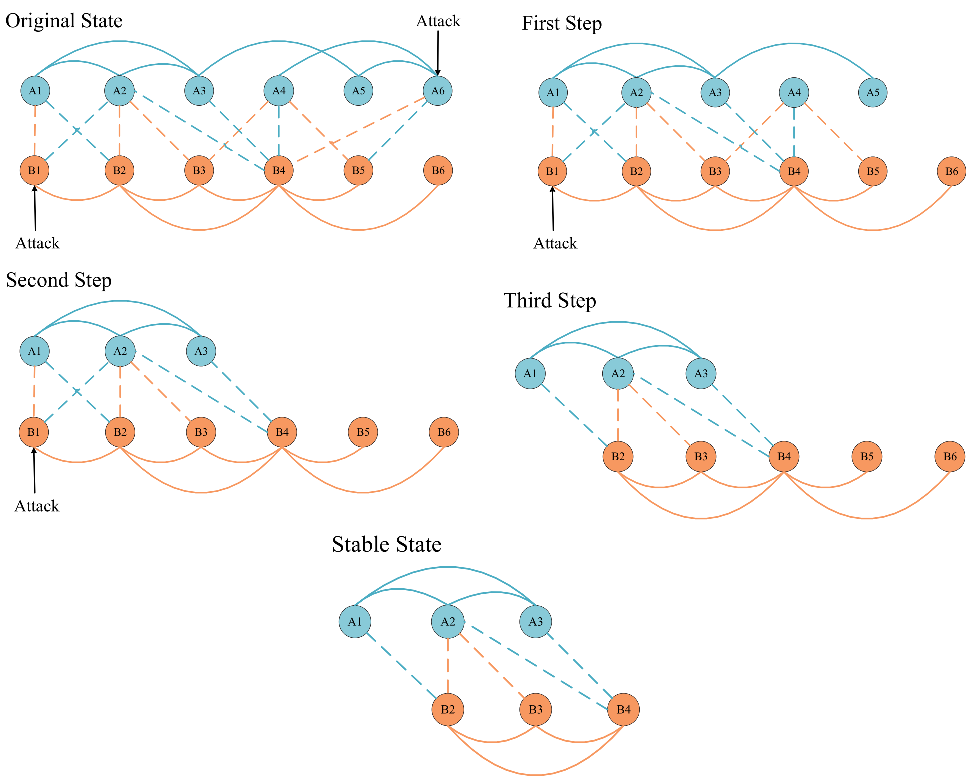

FIG. 1. Demonstration of the process of cascading failures of dependent networks A and B with undirected multiple-to-multiple dependency links. Curves represent connectivity-links within networks and dashed lines represent different multiple dependency links between networks A (blue) and B (orange).

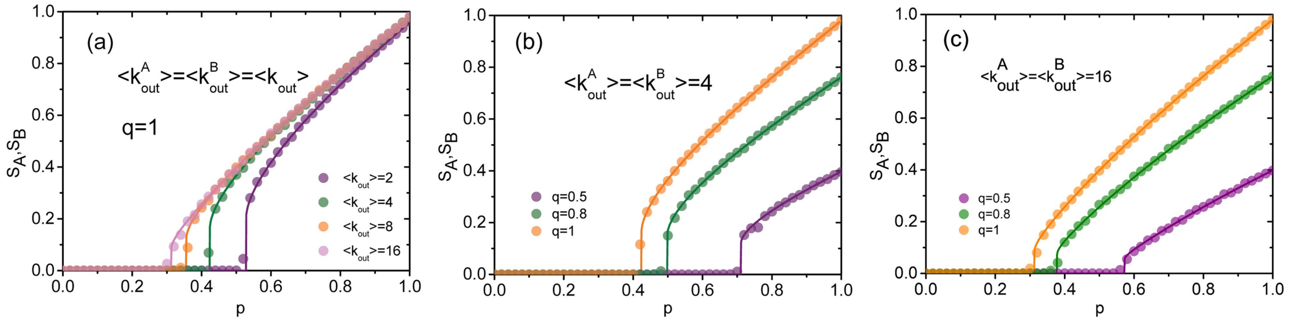

FIG. 2. S_A and S_B as functions of p with parameters $\langle k_{in}^A\rangle = \langle k_{in}^B\rangle = 4$. Analytic predictions in comparison with simulation results. Symbols and lines represent simulation results and theoretical results. (a) $\langle k_{out}^A\rangle = \langle k_{out}^B\rangle = k_{out} = 2, 4, 8, 16$, and $q = 1$. (b) and (c) Different coupling strength $q = 0.5, 0.8, 1$ with $k_{out} = 4$ and 8, respectively. Simulation results are averaged over 100 realizations with the size $N = 10^5$.

distributions $P(k_{in}^A)$ and $P(k_{in}^B)$. Here, k_{in}^A and k_{in}^B are the degrees of a node in single network A and B, respectively. The topological structure of undirected multiple-to-multiple dependency links between networks A and B is described by degree distributions $P(k_{out}^A)$ and $P(k_{out}^B)$, which means that a node in network A has k_{out}^A dependency links from network B and a node in network B has k_{out}^B support links from network A. The process of the cascading of failures for small networks is described in Fig. 1. Without loss of generality, we assume that the attack on network A occurs before network B. At the initial state, let every node in the interdependent system be active. At every stage, the nodes without any dependency link from the other network or separated from the giant component within the network are considered to be inactive and removed from the interdependent system. This process will be continuous until there is no further node removed in the system. We first consider the process of failure in network A. Node A_6 becomes inactive because of the random attack, and nodes A_5 and A_4 are removed due to the absence of one interdependency link and not belonging to the set of the giant component, respectively. Since there are failures of nodes in network A, some nodes in network B are triggered to fail. Correspondingly, nodes B_5 and B_6 become inactive due to loss of all interdependent links after B_1 fails in the interdependent system. After the cascading failure, the system achieves a stable state and no more nodes fail.

III. ANALYTICAL FRAMEWORK

Here, we develop an analytical framework to study the system robustness by applying the method of generating functions[19,30] for partially interdependent networks. Let x_A and x_B be the possibility that a randomly chosen connectivity-link in network A is connected to the giant component of networks A and B, respectively. The symbol x_{BA} denotes the probability of a node in network A with in-degree k_{in}^A and connecting by k_{out}^A dependency nodes within the giant component of the network B. Analogously, we define probability x_{AB} for the dependency networks A and B. For partially interdependent networks A and B, x_A satisfies

$$x_A = pq\left[1 - G_{A,1}^{in}(1 - x_A)\right]\left[1 - G_{A,0}^{out}(1 - x_{BA})G_{B,0}^{out}(1 - x_{BA})\right], \quad (1)$$

where we assumed that only a fraction q of nodes within networks A and B depends on each other. $G_{i,0}^{in}(x) = \sum_{k_{in}^i=0}^{\infty} P(k_{in}^i)x^{k_{in}^i}$ and $G_{i,1}^{in}(x) = \sum_{k_{in}^i=1}^{\infty} \frac{P(k_{in}^i)x^{k_{in}^i-1}}{\langle k_{in}^i\rangle}$ are the generation function and branch

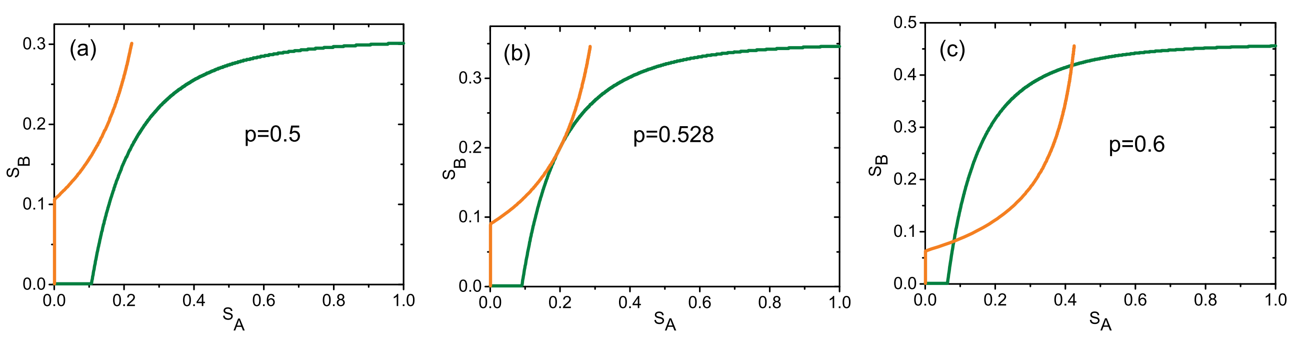

FIG. 3. Graphic solutions of S_A and S_B with parameters $q = 1$ and $\langle k_{in}^A\rangle = \langle k_{in}^B\rangle = 4$ for different p [(a) $p = 0.5$, (b) $p = 0.528$, (c) $p = 0.6$] from Eq. (8) and the simulation results are averaged over 100 realizations with size $N = 10^5$.

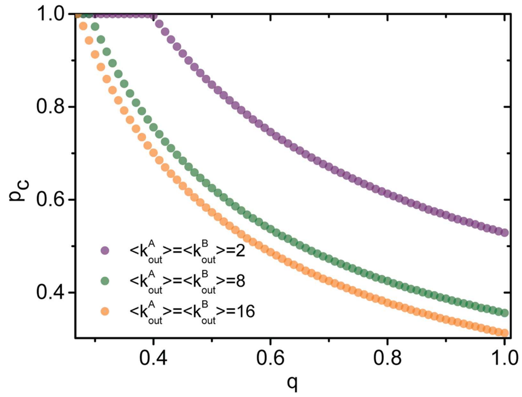

FIG. 4. p_c as a function of q for different $\langle k^A_{out}\rangle = \langle k^B_{out}\rangle = 2, 8, 16$ with $\langle k^A_{in}\rangle = \langle k^B_{in}\rangle = 4$.

generation function of network i $(i = A, B)$, respectively. Similarly, we can get the expression of $G^{out}_{i,0}(x)$ and $G^{out}_{i,1}(x)$ $(i = A, B)$. $G^{in}_{A,1}(1 - x_A)$ represents the probability of a randomly selected link attaching a node with in-degree k^A_{in} in network A excluded in the largest component of network A. $G^{out}_{A,0}(1 - x_{BA})$ represents the probability of the node with in-degree k^A_{in} and has k^A_{out} interdependent nodes in network B, which does not belong to the giant component of network B. $G^{out}_{B,0}(1 - x_{BA})$ can be explained similarly as $G^{out}_{A,0}(1 - x_{BA})$. Similarly, we obtain the expression of x_B with the same method as follows:

$$x_B = pq\left[1 - G^{in}_{B,1}(1 - x_B)\right]\left[1 - G^{out}_{B,0}(1 - x_{AB})G^{out}_{A,0}(1 - x_{AB})\right]. \quad (2)$$

The probability x_{BA} of a node in network A with in-degree k^A_{in} connecting via an interdependency link to a node, which included in the giant component from network B, using the following formula can be obtained:

$$x_{BA} = p\left[1 - G^{in}_{B,0}(1 - x_B)\right]. \quad (3)$$

The probability x_{AB} can be similarly written as

$$x_{AB} = p\left[1 - G^{in}_{A,0}(1 - x_A)\right]. \quad (4)$$

Furthermore, S_A and S_B, which describe the probability that a chosen node belongs to the giant component of the steady networks A and B at the stable state, can be obtained as

$$\begin{aligned} S_A &= pq\left[1 - G^{in}_{A,0}(1 - x_A)\right]\left[1 - G^{out}_{A,0}(1 - x_{BA})G^{out}_{B,0}(1 - x_{BA}\right], \\ S_B &= pq\left[1 - G^{in}_{B,0}(1 - x_B)\right]\left[1 - G^{out}_{B,0}(1 - x_{AB})G^{out}_{A,0}(1 - x_{AB})\right]. \end{aligned} \quad (5)$$

IV. COMPARING THEORETICAL RESULTS WITH SIMULATION RESULTS

In this section, we study two interdependent ER networks, whose degree distribution follow the Poisson distribution $P(k^i_{in}) = \frac{e^{-k^i_{in}}\langle k^i_{in}\rangle^{k^i_{in}}}{k^i_{in}!}$ $(i = A, B)$ with the equal average degree $\langle k^A_{in}\rangle = \langle k^B_{in}\rangle$. Equations (1)–(5) become

$$\begin{aligned} x_A &= pq\Big[1 - e^{-k^A_{in}x_A}\Big]\Big[1 - e^{-k^A_{out}p(1-e^{-k^B_{in}x_{BA}})}e^{-k^B_{out}p(1-e^{-k^B_{in}x_{BA}})}\Big], \\ x_B &= pq\Big[1 - e^{-k^B_{in}x_B}\Big]\Big[1 - e^{-k^B_{out}p(1-e^{-k^A_{in}x_{AB}})}e^{-k^A_{out}p(1-e^{-k^A_{in}x_{AB}})}\Big], \end{aligned} \quad (6)$$

$$\begin{aligned} x_{BA} &= p\Big[1 - e^{-k^B_{in}x_B}\Big], \\ x_{AB} &= p\Big[1 - e^{-k^A_{in}x_A}\Big], \end{aligned} \quad (7)$$

$$\begin{aligned} S_A &= pq\Big[1 - e^{-k^A_{in}x_A}\Big]\Big[1 - e^{-k^A_{out}p(1-e^{-k^B_{in}x_{BA}})}e^{-k^B_{out}p(1-e^{-k^B_{in}x_{BA}})}\Big], \\ S_B &= pq\Big[1 - e^{-k^B_{in}x_B}\Big]\Big[1 - e^{-k^B_{out}p(1-e^{-k^A_{in}x_{AB}})}e^{-k^A_{out}p(1-e^{-k^A_{in}x_{AB}})}\Big]. \end{aligned} \quad (8)$$

From Fig. 2, one can observe that the simulation results agree well with the theoretical predictions and the system shows a discontinuous phase transition. As p increases, S_A and S_B gradually decrease to zero at the critical threshold p_c. As shown in Fig. 2(a), the system becomes more and more vulnerable and easy to defend as the average degree increases. Also, for strong coupling strength $q = 1$, the system becomes more robust, but for weak coupling strength $q = 0.5$, the system becomes more vulnerable. This follows the fact that $1 - q$ fraction of nodes will fail due to the absence of dependency links for weak coupling strength, whereas the network connectivity mainly depends on the nodes with interdependency links in the system. Now, we will give an analysis of how to obtain the critical value p_c. Figure 3 gives graphic solutions of S_A and S_B by using Eq. (8). From Fig. 3(a), we find that there exists a trivial solution $S_A = S_B = 0$ at $p = 0.5$.

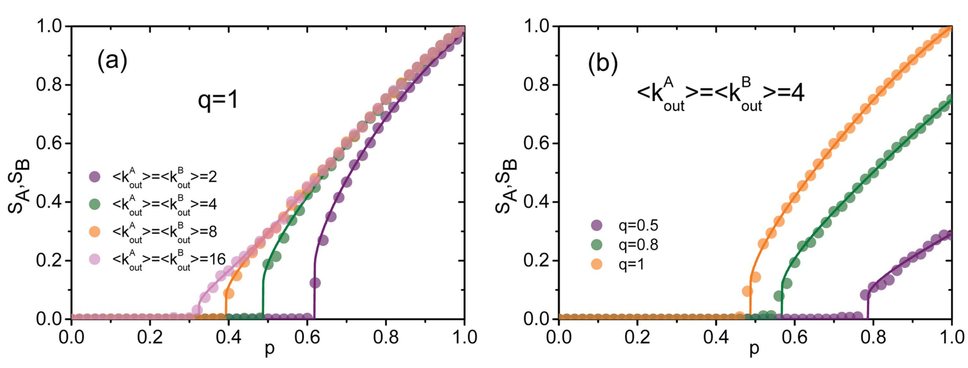

FIG. 5. Analytic predictions (line) in comparison with simulation results (symbol) with different parameters. (a) The parameters $q = 1$, $\lambda = 3.2$ with different $\langle k^A_{out}\rangle = \langle k^B_{out}\rangle$. (b) The parameters $\langle k^A_{out}\rangle = \langle k^B_{out}\rangle = 4$, $\lambda = 3.2$ with different q. Simulation results are averaged over 100 realizations with size $N = 10^5$.

As $p = 0.528$, $S_B(S_A)$ forms a tangent point, at which S starts to appear at a finite value. It means that p_c can be obtained from the condition $\frac{dS_A(S_B)}{dS_B} \cdot \frac{dS_B(S_A)}{dS_A} = 1$, as shown in Fig. 3(b). Furthermore, Fig. 3(c) shows that $S_B(S_A)$ has two intersection points for $p = 0.6 > 0.528$. For this case, the bigger value is chosen to $S_A = S_B \approx 0.4223$, since it denotes the size of the biggest component of the system. In order to explore the influence of coupling strength on system robustness, we show the critical value p_c as a function of q in Fig. 4. From Fig. 4, we notice p_c as a function of q with different parameters $\langle k^A_{out}\rangle = \langle k^B_{out}\rangle = \langle k_{out}\rangle$ for $\langle k^A_{in}\rangle = \langle k^B_{in}\rangle = \langle k_{in}\rangle = 4$. The results imply that the system becomes more robust as q increases, as shown in Fig. 4. As the network density increases, the system becomes more resilient and easy to defend. This will prompt a better understanding and further realization in designing more resilient dependent systems.

Next, we will compare our theoretical results with the numerical results in partially interdependent networks with multiple-to-multiple relationship, whose degree distributions follow power law distribution $p(k) \sim k^{-\lambda}$[20] and dependency links follow Poisson degree distribution. Thus, Eqs. (1)–(5) become

$$
\begin{aligned}
x_A &= pq\left[1-\sum_{k^A_{in}=1}^{\infty}\left(\left(\frac{k_{min}}{k^A_{in}}\right)^{\lambda-1}-\left(\frac{k_{min}}{k^A_{in}+1}\right)^{\lambda-1}\right)\frac{x_A^{k^A_{in}-1}}{\langle k^A_{in}\rangle}\right]\\
&\quad\times\left[1-\sum_{k^A_{out}=0}^{\infty}\left(\left(\frac{k_{out}}{k^A_{out}}\right)^{\lambda-1}-\left(\frac{k_{out}}{k^A_{out}+1}\right)^{\lambda-1}\right)x_{BA}^{k^A_{out}}\sum_{k^B_{out}=0}^{\infty}\left(\left(\frac{k_{out}}{k^B_{out}}\right)^{\lambda-1}-\left(\frac{k_{out}}{k^B_{out}+1}\right)^{\lambda-1}\right)x_{BA}^{k^B_{out}}\right],\\
x_B &= pq\left[1-\sum_{k^B_{in}=1}^{\infty}\left(\left(\frac{k_{min}}{k^B_{in}}\right)^{\lambda-1}-\left(\frac{k_{min}}{k^B_{in}+1}\right)^{\lambda-1}\right)\frac{x_B^{k^B_{in}-1}}{\langle k^B_{in}\rangle}\right]\\
&\quad\times\left[1-\sum_{k^B_{out}=0}^{\infty}\left(\left(\frac{k_{out}}{k^B_{out}}\right)^{\lambda-1}-\left(\frac{k_{out}}{k^B_{out}+1}\right)^{\lambda-1}\right)x_{AB}^{k^B_{out}}\sum_{k^A_{out}=0}^{\infty}\left(\left(\frac{k_{out}}{k^A_{out}}\right)^{\lambda-1}-\left(\frac{k_{out}}{k^A_{out}+1}\right)^{\lambda-1}\right)x_{AB}^{k^A_{out}}\right],
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
x_{BA} &= p\left[1-\sum_{k^B_{in}=0}^{\infty}\left(\left(\frac{k_{min}}{k^B_{in}}\right)^{\lambda-1}-\left(\frac{k_{min}}{k^B_{in}+1}\right)^{\lambda-1}\right)x_B^{k^B_{in}}\right],\\
x_{AB} &= p\left[1-\sum_{k^A_{in}=1}^{\infty}\left(\left(\frac{k_{min}}{k^A_{in}}\right)^{\lambda-1}-\left(\frac{k_{min}}{k^A_{in}+1}\right)^{\lambda-1}\right)x_A^{k^A_{in}}\right],
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
S_A &= pq\left[1-\sum_{k^A_{in}=1}^{\infty}\left(\left(\frac{k_{min}}{k^A_{in}}\right)^{\lambda-1}-\left(\frac{k_{min}}{k^A_{in}+1}\right)^{\lambda-1}\right)x_A^{k^A_{in}}\right]\\
&\quad\times\left[1-\sum_{k^A_{out}=0}^{\infty}\left(\left(\frac{k_{out}}{k^A_{out}}\right)^{\lambda-1}-\left(\frac{k_{out}}{k^A_{out}+1}\right)^{\lambda-1}\right)x_{BA}^{k^A_{out}}\sum_{k^B_{out}=0}^{\infty}\left(\left(\frac{k_{out}}{k^B_{out}}\right)^{\lambda-1}-\left(\frac{k_{out}}{k^B_{out}+1}\right)^{\lambda-1}\right)x_{BA}^{k^B_{out}}\right],\\
S_B &= pq\left[1-\sum_{k^B_{in}=0}^{\infty}\left(\left(\frac{k_{min}}{k^B_{in}}\right)^{\lambda-1}-\left(\frac{k_{min}}{k^B_{in}+1}\right)^{\lambda-1}\right)x_B^{k^B_{in}}\right]\\
&\quad\times\left[1-\sum_{k^B_{out}=0}^{\infty}\left(\left(\frac{k_{out}}{k^B_{out}}\right)^{\lambda-1}-\left(\frac{k_{out}}{k^B_{out}+1}\right)^{\lambda-1}\right)x_{AB}^{k^B_{out}}\sum_{k^A_{out}=0}^{\infty}\left(\left(\frac{k_{out}}{k^A_{out}}\right)^{\lambda-1}-\left(\frac{k_{out}}{k^A_{out}+1}\right)^{\lambda-1}\right)x_{AB}^{k^A_{out}}\right].
\end{aligned}
\tag{11}
$$

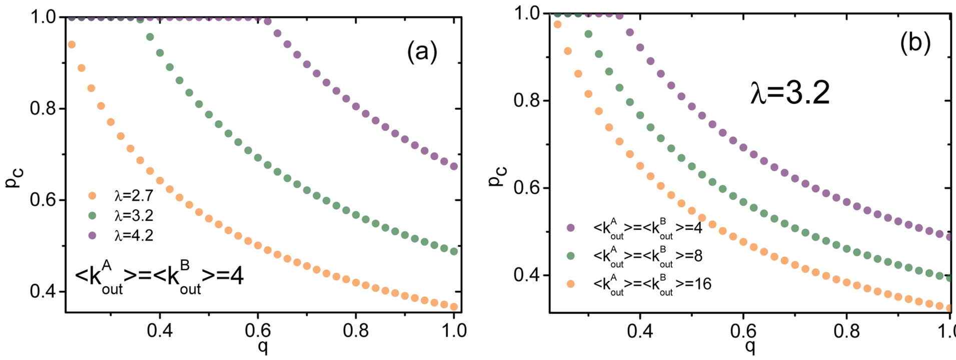

FIG. 6. p_c as a function of q. (a) For different $\lambda = 2.7, 3.2, 4.2$ with $\langle k^A_{out}\rangle = \langle k^B_{out}\rangle = 4$. (b) For different $k_{out} = 4, 6, 8$ with $\lambda = 3.2$.

By comparing analytical and simulation results, we find that the simulation results agree well with the analytical results, as shown in Fig. 5. Figure 5(a) shows that p_c becomes smaller if the value of $\langle k^A_{out}\rangle = \langle k^B_{out}\rangle$ becomes bigger, which means that one can get a robust system by increasing the dependency density. From Fig. 5(b), the results imply that the system becomes more robust as the fraction of dependency nodes increases for our model. For different power law exponents λ, p_c as a function of q is shown in Fig. 6(a) for $\langle k^A_{out}\rangle=\langle k^B_{out}\rangle = 4$. We find that the value of p_c is getting smaller with a smaller λ, which means that the interdependent system is more robust for smaller power law exponents. For fixed λ, p_c values get smaller as the average dependency degree increases, and the system here becomes more steady and robust, as shown in Fig. 6(b).

V. CONCLUSIONS

In this paper, the robustness of two multiple-to-multiple dependent networks is numerically and analytically studied by extending previous works.[28,31] By defining the mechanism of cascading failures of the system, we developed a framework and obtained the theoretical prediction for the size of the giant component at the stable state. The analytical results agree well with the simulation results for coupled networks. Additionally, the graphical solution of the critical threshold is numerically analyzed for the above models. The results suggest that the system behaves as a discontinuous phase transition for the above two systems. The robustness of a single network increases as the coupling strength q increases. We expect that the increased network density can also strengthen the robustness of the network for both Erdös-Rènyi and scale-free networks. Moreover, the studies imply that one can obtain a more robust system by increasing the power exponent index of the system. The case of robustness of the general case of n coupled networks with multiple-to-multiple dependent relationship has been studied very recently.

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