A THEORY OF LIMITED LIQUIDITY AND LARGE INVESTORS CAUSING SPIKES IN STOCK MARKET VOLATILITY AND TRADING VOLUME

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Abstract

We survey a theory of the economic underpinnings of the fat-tailed distributions of a number of financial variables, such as returns and trading volumes. Our theory posits that they have a common origin in the strategic trading behavior of very large financial institutions in a relatively illiquid market. We show how the fat-tailed distribution of fund sizes can indeed generate extreme returns and volumes, even in the absence of fundamental news. Moreover, we are able to replicate the individually different empirical values of the power law exponents for each distribution. Large investors moderate their trades to reduce their price impact; coupled with a concave price impact function, this leads to volumes being more fat-tailed than returns but less fat-tailed than fund sizes. The trades of large institutions offer a unified explanation for apparently disconnected empirical regularities that are otherwise a challenge for economic theory. (JEL: G12, G14, G23)

1. Introduction

This paper, written by three physicists and an economist, is part of "econophysics," a broad movement utilizing concepts and methods from physics to study economic issues. Econophysics is similar in spirit to behavioral economics in that it postulates simple plausible rules of agent behavior and explores their implications. However, it differs by putting less emphasis on the psychological microfoundations and more on the results of the interactions among agents. Paradigms are the Pareto law and Zipf's law, both of which stem from a variety of broadly related random growth processes (Simon 1955;

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Gabaix and Ioannides 2004). Econophysics is a broad movement comprising several hundred researchers. One goal of this paper is to bridge the gap between the microfoundations of economics and the regularities uncovered in econophysics.

In a series of investigations (Gopikrishnan et al. 2000; Plerou et al. 2000, 2002; Gabaix et al. 2003), we established stylized facts on the extreme movements of behavior of stock market activity. This coalesced into an economic model published recently in Gabaix et al. (2006). Though most econophysics literature is published in physics journals, some of it is now being published in economics journals (Santa-Clara and Sornette 2001; Fu et al. 2005; Wyart and Bouchaud 2007).

In this paper, we motivate our results and present a simplified version of a model that may shed light on the puzzle of excess volatility in stock market prices. Indeed, even after the fact, it is hard to explain changes in the stock market using only observable news (Cutler, Poterba, and Summers 1989). In our theory, spikes in trading volume and returns are created by a combination of news and trades by investors in relatively illiquid markets. Suppose news or proprietary analysis induces a large investor to trade a particular stock. Because his desired trading volume is then a significant proportion of daily turnover, he will moderate his actual trading volume to avoid paying too much in price impact. The optimal volume will nonetheless remain large enough to induce a significant price change.

Traditional measures, such as variances and correlations, are of limited use in analyzing spikes in market activity. A better measure is the power law exponent of the distributions, since most distributions can be well approximated by power law distributions. The accumulated evidence on tail behavior is useful to guide and constrain any theory of the impact of large investors. Specifically, our theory unifies the following stylized facts:

(i) the power law distribution of returns, with exponent $\zeta_r \simeq 3$;
(ii) the power law distribution of trading volume, with exponent $\zeta_q \simeq 1.5$;
(iii) the power law of price impact;
(iv) the power law distribution of the size of large investors, with exponent $\zeta_S \simeq 1$.

Existing models have difficulty explaining facts (i)–(iv) together—not only the power law behavior in general but also the specific exponents. For example, efficient markets theories rely on news to move stock prices and thus can explain the empirical finding only if news is power law distributed with exponent $\zeta_r \simeq 3$. However, there is nothing a priori in the efficient markets hypothesis that justifies this assumption. Similarly, GARCH models generate power laws, but they must be fine-tuned to replicate the exponent of 3.

2. The Empirical Findings That Motivate Our Theory

The Power Law Distribution of Price Fluctuations: $\zeta_r \simeq 3$. The tail distribution of returns has been analyzed in a series of studies that use an ever-increasing number of data points (Gopikrishnan et al. 1999). Let r_t denote the logarithmic return over a time interval Δt. The distribution function of returns for the 1,000 largest U.S. stocks and several major international indices has been found to be

$$P(|r| > x) \sim \frac{1}{x^{\zeta_r}} \quad \text{with} \quad \zeta_r \simeq 3. \tag{1}$$

Here, $\sim$ denotes asymptotic equality up to numerical constants. This relationship holds for positive and negative returns separately and is best illustrated in Figure 1. It is not automatic that this graph should be a straight line or that the slope should be -3: In a Gaussian world it would be a concave parabola. In the sequel, we shall refer to equation (1) as *the cubic law of returns*.

The Power Law Distribution of Trading Volume: $\zeta_q \simeq 3/2$. To better constrain a theory of large returns, it is helpful to understand the structure of large trading volumes. Gopikrishnan et al. (2000) find that trading volumes for the 1,000 largest U.S. stocks are also power law distributed:

$$P(q > x) \sim \frac{1}{x^{\zeta_q}} \quad \text{with} \quad \zeta_q \simeq \frac{3}{2}. \tag{2}$$

Figure 2 illustrates equation (2). The density satisfies $p(q) \sim q^{-2.5}$. We find $\zeta_q = 1.5 \pm 0.1$ for each of the three stock markets. The exponent appears essentially identical in the three stock markets, which is suggestive of "universality". Despite the institutional differences between markets, some simple economic force might be at work that generates similar outcomes between stock markets. We shall refer to equation (2) as the *half-cubic law of trading volume.*

It is intriguing that the exponent of returns is (approximately) 3 and the exponent of trading volumes is (approximately) 3/2. To see if there is an economic connection between those values, we turn to the relation between return and volume.

The Power Law of Price Impact: $r \sim V^{\gamma}$. The following parameterization is often used to represent the price impact r of a trade of size V for large V: $r \sim kV^{\gamma}$ with $k > 0$ and $0 \leq \gamma \leq 1$, which yields a concave price impact function (Hasbrouck 1991; Plerou et al. 2002). The parameterization $\gamma = 1/2$ is often used. The relation $r \sim kV^{\gamma}$ implies $\zeta_r = \zeta_V/\gamma$ by general rules on Paretian variables. Hence, given $\zeta_r = 3$ and $\zeta_V = 3/2$, the value $\gamma = 1/2$ is a particularly plausible null hypothesis. From this relationship we see a natural

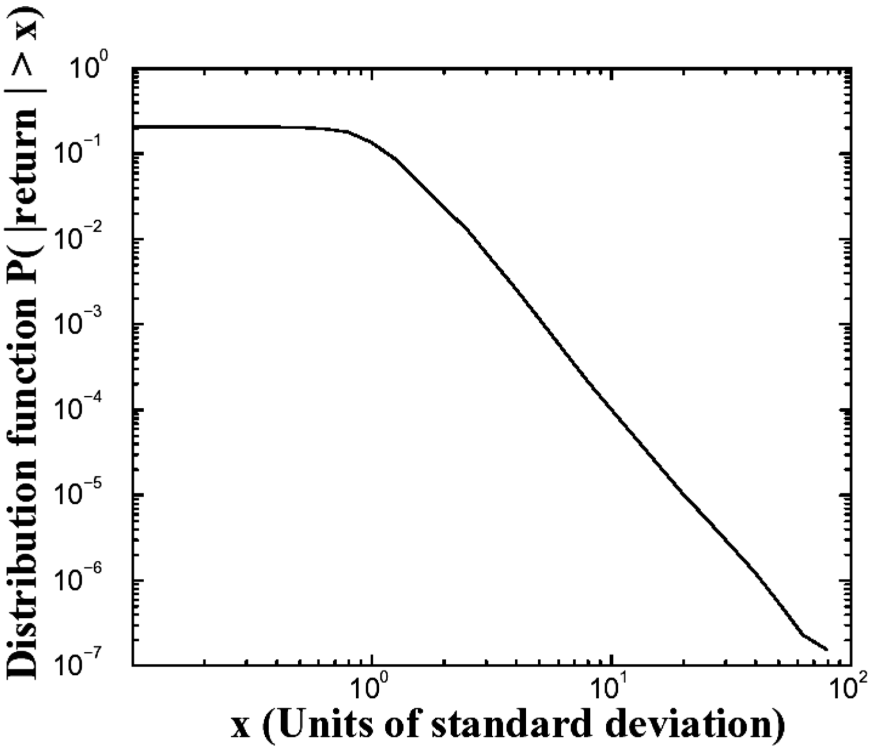

FIGURE 1. Empirical cumulative distribution of the absolute values of the normalized 15-minute returns of the 1,000 largest companies in the Trades and Quotes database for the 2-year period 1994–1995 (12 million observations). We normalize the returns of each stock so that the normalized returns have a mean of 0 and a standard deviation of 1. For instance, for a stock i, we consider the returns $r'_{it} = (r_{it} - r_i)/\sigma_{r,i}$, where r_i is the mean of the r_{it} and $\sigma_{r,i}$ is their standard deviation. In the region $2 \leq x \leq 80$ we find an ordinary least squares fit $\ln P(|r| > x) = -\zeta_r \ln x + b$ with $\zeta_r = 3.1 \pm 0.1$. This means that returns are distributed with a power law $P(|r| > x) \sim x^{-\zeta_r}$ for large x between 2 and 80 standard deviations of returns.

Source: Gabaix et al. (2003).

connection between the power laws of returns and volumes. Gabaix et al. (2006) survey the current evidence, which is often consistent with $\gamma = 1/2$.

The Power Law Distribution of the Size of Large Investors: $\zeta_S \simeq 1$. It is highly probable that substantial trades are generated by very large investors. This motivates us to investigate the size distribution of market participants. A power law formulation $P(S > x) \sim 1/x^{\zeta_S}$ often yields a good fit. The exponent $\zeta_S \simeq 1$, often called Zipf's law, is particularly common. This relation is true for cities (Gabaix 1999; Gabaix and Ioannides 2004) as well as for firm size by employees (Axtell 2001) and market value (Gabaix and Landier 2006). In Gabaix et al. (2003) we report evidence that is consistent with $\zeta_S \simeq 1$, a Zipf's law for the size distribution of mutual funds.

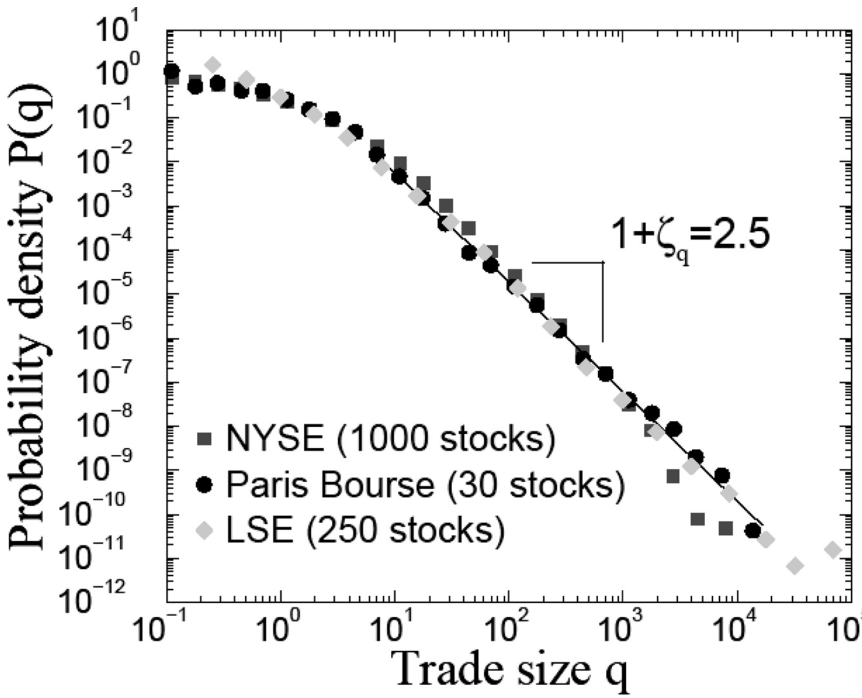

FIGURE 2. Probability density of normalized individual transaction sizes q for three stock markets: (i) NYSE for 1994–1995; (ii) the London Stock Exchange for 2001; and (iii) the Paris Bourse for 1995–1999. An OLS fit yields $\ln p(x) = -(1 + \zeta_q) \ln x +$ constant for $\zeta_q = 1.5 \pm 0.1$. This means a probability density function $p(x) \sim x^{-(1+\zeta_q)}$ and a countercumulative distribution function $P(q > x) \sim x^{-\zeta_q}$. The three stock markets appear to have a common distribution of volume, with a power law exponent of 1.5 ± 0.1. The horizontal axis shows individual volumes that are up to 10^4 times larger than the absolute deviation, $E\left[|q - \bar{q}|\right]$.

Source: Gabaix et al. (2006).

3. The Model

We consider a large fund in a relatively illiquid market. We first describe a rudimentary model for the price impact of its trades. Next, we link the various power law exponents; this represents the theory's core contribution. One could employ different microfoundations for price impact without changing our conclusions.

3.1. A Simple Model to Generate a Power Law Price Impact

Before presenting the core of the model in Section 3.2, we first sketch a simple microfoundation for the square-root price impact. The model used in this section is a formalized version of a useful heuristic argument, sometimes called the “Barra model" of Torre and Ferrari (Barra 1997). A large trader wants to buy a large block V, and she buys it from a "liquidity provider." After selling the block of size V, the liquidity provider needs to buy it again on the market in order to make his inventory return to the baseline. Under some simple conditions, it will take an amount of time $T = V/\bar{V}$ to buy back the V shares. During that time, the price diffuses at a rate σ. Hence the liquidity provider faces a price uncertainty with standard deviation $\sigma\sqrt{T}$, which is proportional to $\sigma\sqrt{V}$. This is the standard deviation of the risk he faces per dollar of block sold.

To see how much he requires to be compensated, we assume that the liquidity provider has the following mean variance utility function on the total amount W of money earned during the trade: $U = E[W] - \lambda[\text{var}(W)]^{\delta/2}$, with $\lambda > 0$ and $\delta > 0$. The liquidity supplier requires compensation equal to $\lambda\sigma^{\delta}$ in order to bear a risk of standard deviation σ (i.e., he has "δth-order risk aversion"), with standard mean-variance preferences, $\delta = 2$. In many cases, a better description of behavior is first-order risk aversion, which corresponds to $\delta = 1$. One justification for first-order risk aversion comes from psychology: Prospect theory presents psychological evidence for this behavior.

In equilibrium, one obtains the following price impact function (the details are in Gabaix et al. 2006):

$$\tau(V) = HV^{\gamma} \text{ with } H = \lambda\sigma^{\delta}/(3\bar{V})^{\delta/2}, \tag{3}$$

and $\gamma = 3\delta/2 - 1$. In particular, with first-order risk aversion ($\delta = 1$) we have $\gamma = 1/2$ so that $\tau \sim \sqrt{V}$. We obtain a square-root price impact of trades. In other words, if the liquidity provider is first-order risk averse, then the price concession τ is proportional to the standard deviation and hence $\tau \sim \sigma\sqrt{V}$, that is, $\gamma = 1/2$.

In practice, $\bar{V}$ is likely to be proportional to the daily trading volume. Hence the scaling predictions of equation (3) can be almost directly examined. In the spirit of econophysics, we use simple arguments—particularly about how quantities scale with size—and make testable implications about the scaling of variables.

3.2. The Core Model: Behavior of a Large Fund

We now lay out the core of our model. The fund periodically receives signals about trading opportunities, which indicate that the excess risk-adjusted return on the asset is $s_t M_t \tilde{C}$; here s_t,M_t and $\tilde{C}$ are independent and $s_t = \pm 1$ is the sign of the mispricing. The expected absolute value of the mispricing, M_t, is drawn from a distribution $f(M)$ that we assume to be not too fat-tailed, $E[M^{1+1/\gamma}] < \infty$.

The model misspecification risk $\tilde{C}$ captures uncertainty over whether the perceived mispricing is in fact real. For example, the fund's predictive regressions may result from data mining, or the mispricing may have since been arbitraged away. Observe that $\tilde{C}$ can take two values, 0 and C^*. If $\tilde{C} = 0$, then the signals the fund perceives are pure noise, and the true average return on the perceived mispricings is 0. If $\tilde{C} = C^*$, the mispricings are real. We specify $E[\tilde{C}] = 1$, so that M represents the expected value of the mispricing.

The fund has S dollars in assets. If it buys a volume V_t of the asset and pays a price concession $R(V_t)$ then the total return of its portfolio is

$$r_t = \frac{V_t(\tilde{C}M_t - R(V_t) + u_t)}{S}, \tag{4}$$

where u_t is mean zero noise. Hence, if the model is wrong, expected returns are $E[r_t \mid \tilde{C} = 0] = -V_t R(V_t)/S$.

We assume that the manager has a concern for robustness. She does not want her expected return to be below some value $-\Lambda$ percent if her trading model is wrong. Formally, this may be written as $E[r_t \mid \tilde{C} = 0] \geq -\Lambda$.

To simplify the algebra, we assume that, subject to the robustness constraint, the manager wants to maximize the expected value of her excess returns $E[r]$. The fund's optimal policy is a function $V(M, S)$ that specifies the quantity of shares V traded when the fund perceives a mispricing of size M. It maximizes the expected returns $E[r_t]$ subject to the robustness constraint:

$$\max_{V(M,S)} E[r_t] \quad \text{subject to:} \quad E[r_t \mid \tilde{C} = 0] \geq -\Lambda;$$

that is,

$$\max_{V(M,S)} \quad \frac{1}{S}\int_0^\infty V(M, S)(M - R(V(M, S)))f(M)dM,$$

subject to

$$\frac{-1}{S}\int_0^\infty V(M, S)R(V(M, S))f(M)dM \geq -\Lambda.$$

One solves the problem by simply deriving the Lagragian. This yields the following statement.

PROPOSITION 1. *The optimal policy for a fund of size S that perceives a mispricing of size M is to trade a volume*

$$V(M, S) = vM^{1/\gamma}S^{1/(1+\gamma)}. \tag{5}$$

The price change after the trade is

$$R(M, S) = hv^\gamma MS^{\gamma/(1+\gamma)} \tag{6}$$

for a positive constant v, which is increasing in Λ and decreasing in h.

It would be greatly desirable to test Proposition 1 directly, for instance by looking at trade-by-trade data in a data set comprising the identity and size of the funds trading. Such a data set is not currently available but might be in the future. In the meantime, we can already extract some predictions from Proposition 1. Taking the power law exponents of the last two equations shows us that if the news is fat-tailed enough (i.e., if it has an exponent greater than $[1+\gamma]\zeta_S$), then the following proposition holds.

PROPOSITION 2. *Volumes and returns are power-law distributed, with respective exponents ζ_V and ζ_R, such that*

$$\zeta_V = (1+\gamma)\zeta_S, \tag{7}$$

$$\zeta_R = (1+\frac{1}{\gamma})\zeta_S. \tag{8}$$

For instance, equation (6) yields

$$\zeta_R = \min(\zeta_M, \zeta_{S^{\gamma/(1+\gamma)}}) = \zeta_{S^{\gamma/(1+\gamma)}} = ((1+\gamma)/\gamma)\zeta_S.$$

Proposition 2 relates the distribution of volumes and returns to the distributions of the size of financial institutions and to the curvature of price impact. With the empirical and theoretical baseline case of a square-root price impact ($\gamma = 1/2$) and Zipf's law for financial institutions ($\zeta_S = 1$), volumes and returns follow power law distributions with respective exponents of 3/2 and 3:

$$\zeta_V = 3/2, \tag{9}$$

$$\zeta_R = 3. \tag{10}$$

These are the empirically found exponents of the distribution of volume and returns.

The foregoing captures our explanation of the origins of the cubic law of returns and the half-cubic law of volumes. Random growth of mutual funds leads to Zipf's law of financial institutions, $\zeta_S = 1$, via the known mechanisms that lead to Zipf's law (Gabaix and Ioannides 2004). The model of Section 3.1 leads to a power law price impact with curvature $\gamma = 1/2$. Because large funds wish to lessen their price impacts, their trading volumes are less than proportional to their size. This generates a power law distribution of the size of trades that is less fat-tailed than the size distribution of mutual funds. The resulting exponent is $\zeta_V = 3/2$, which is the empirical value. Trades of large funds create large returns—indeed, the power law distribution of returns with exponent $\zeta_r = 3$.

On the other hand, equations (7)–(8) predict that in some circumstances the power laws of volumes and returns will differ from 3/2 and 3. In future research, it would be interesting to evaluate predictions (7)–(8) across different stocks or different stock markets. In any case, the fat-tailed distribution of stock market returns, which at the time of Mandelbrot (1963) seemed rather just an empirical fact without clear theoretical underpinning, now has a theoretical explanation that makes testable alternative predictions: in its relation to trading volume, the size of market participants, and the mechanisms of price impact. Because crashes do not seem to be outliers to the power law distributions of stock returns (Gabaix et al. 2005), it is even possible that the mechanism outlined here may explain stock market crashes, which start with one or a few very large institutions trading in a relatively illiquid market. Indeed, the Brady report repeatedly marvels at how concentrated trading was on Monday, October 19, 1987.

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