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Predicting oil recovery using percolation

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Abstract

One particular practical problem in oil recovery is to predict the time to breakthrough of a fluid injected in one well and the subsequent decay in the production rate of oil at another well. Because we only have a stochastic view of the distribution of rock properties we need to predict the uncertainty in the breakthrough time and post-breakthrough behaviour in order to calculate the economic risk. In this paper we use percolation theory to predict (i) the distribution of the chemical path (shortest path) between two points (representing well pairs) at a given Euclidean separation and present a scaling hypothesis for this distribution which is confirmed by numerical simulation, (ii) the distribution of breakthrough times which can be calculated algebraically rather than by very time consuming direct numerical simulation of large numbers of realisations.

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1. Introduction

The most common method of oil recovery is by displacement. Either water or a miscible gas (carbon dioxide or methane) is injected in a well (or wells) to displace the oil to other wells. Ultimately the injected fluid will breakthrough into a production well where it has to be separated from the oil, which is a very costly process. Once the injected fluid has broken through the rate of production declines as more injected fluid is produced. For economic purposes it is important to know when the injected fluid will break through and what the rate of decline of the oil will be so that the economic limit of production can be determined.

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The porous rock through which the fluids flow is highly heterogeneous due to the sedimentary processes that deposited it. In many cases the rock can be separated into two types "good" – high permeability and "bad" – low or zero permeability and for all practical purposes it can be taken that the flow only takes place in the good rock. The spatial distribution of the rock types is often random, in which case this is a classical percolation problem. The place of the occupancy probability p is taken by the volume fraction of the good rock, called the net-to-gross ratio in the petroleum literature.

We have very little direct knowledge about the spatial distribution of rock properties in the reservoir. Direct measurements are limited to samples that represent around 10^{-13} of the total reservoir volume. These samples are taken from the well locations. Elsewhere the properties have to be inferred from knowledge of the general geological environment and by analogy with other reservoirs or surface outcrops. Hence, there is a great deal of uncertainty in our prediction of these rock properties. This leads to an uncertainty in our ability to predict the flow performance, principally the breakthrough time and the production decline rate. We need to estimate the uncertainty accurately so that economic risk evaluations can be made.

The conventional approach to this problem is to build a detailed (numerical) model of the rock properties. These models will honour the one and two point statistics observed from the wells and analogue outcrops. The models must also agree with the observed data values at wells. These models are statistical in nature and conventionally one samples realisations from the models and performs numerical flow calculations on the realisations to give a Monte Carlo prediction of breakthrough and production decline. Unfortunately this process is so time consuming as to be impractical in many cases. Typically the flow simulation can take some hours on reasonable workstations. When hundreds of realisations are sampled to get good statistics, the total computing time becomes unwieldy to use in practice. Thus there is a strong need to make this more efficient so as to come up with very quick, but accurate, predictions of recovery and the uncertainty due to the lack of knowledge of the underlying rock properties. The purpose of this study is to do just this using methods derived from percolation theory. It is based on two key assumptions. One is that for many cases the permeability disorder can be approximated by either permeable or impermeable rock. There are many cases where this is a very good approximation. For example the reservoir may have been deposited by meandering river belts in which case the good sand occurs as " packages" in an insulating background. The other assumption is that the flow paths are strongly controlled by the permeability disorder and not strongly modified by the flow dynamics themselves. Again there are many cases when this is reasonable. In particular, if the viscosity ratio between the injected and displaced fluids is not too large or when the system is highly disordered.

Under these assumptions we can consider the underlying heterogeneity to be that of a percolating system (not necessarily at threshold), the occupancy probability p corresponds to the volume fraction of permeable rock (known as the net-to-gross ratio in the oil industry). This has previously been done [1] to study the fraction of sand connected to well pairs. We then look at the dynamic displacement on this cluster

where the flow is controlled by Darcy's law (analogous to Ohm's law in electrical current flow). We assume that the injected fluid can be treated as a passive tracer that is convected along these flow paths (by passive tracer we mean one that is not absorbed by the rock). Then the breakthrough time is the same as the first passage time and is strongly controlled by the shortest path length between two points (the wells). This is the chemical path in percolation parlance. Then the post breakthrough production decline is controlled by the longer paths.

2. Chemical path

The first issue of importance is the shortest path between a pair of wells. That is given a pair of wells separated by a Euclidean distance r, what is the shortest path length on the connected cluster? This path determines the first break through of the injected fluid. To be more specific we need the conditional probability $P'(\ell|r)$ that given the two wells are connected at that separation r, the chemical path length is ℓ. This will be a function of the net-to-gross ratio p and the finite size of the system L. For the present we shall confine ourselves to consideration of a two-dimensional (areal) system where the wells can be considered as points (remember that many reservoirs are tens of metres thick but tens of kilometres in lateral extent) and that the sand bodies can be thought of as independently located and isotropically distributed. The issues of anisotropy, spatial correlation and three dimensions will be considered in future work.

It has been shown previously [2,3] that the conditional probability $P'(r|\ell)$ (at $p=p_c$ and for $L\to\infty$) obeys the scaling ansatz [4]

$$P'(\ell|r)=A_r\left(\frac{\ell}{r^{d_{\min}}}\right)^{-g'_\ell} f_1\left(\frac{\ell}{r^{d_{\min}}}\right), \tag{1}$$

where $A_r\sim 1/r^{d_{\min}}$, $d_{\min}=1.13$, $g'_\ell=2.14$ [5] and the scaling function $f_1(x)=\exp(-ax^{-\phi_1})$ with $\phi_1=(d_{\min}-1)^{-1}$. The relation $g'_\ell=2+(2-d_f)/d_{\min}$ was proposed by Havlin [6], who used arguments similar to those of de Gennes [7] for SAWs. Further in Refs. [8,9] we used this to demonstrate the plausibility of the scaling form for $P'(\ell|r)$

$$P'(\ell|r)\sim\frac{1}{r^{d_{\min}}}\left(\frac{\ell}{r^{d_{\min}}}\right)^{-g'_\ell} f_1\left(\frac{\ell}{r^{d_{\min}}}\right) f_2\left(\frac{\ell}{L^{d_{\min}}}\right) f_3\left(\frac{\ell}{\xi^{d_{\min}}}\right), \tag{2}$$

where the scaling functions are $f_2(x)=\exp(-bx^{\phi_2})$ and $f_3(x)=\exp(-cx)$. Here ξ is a characteristic length of the pair connectedness function and has a power-law dependence of occupancy p as $\xi\sim|p-p_c|^{-\nu}$. The validity of this assumption is shown in Fig. 1. It shows that below a "typical" chemical path that scales as $\ell^*\sim r^{d_{\min}}$ the distribution falls off very rapidly towards r. Above this there is a long power-law tail, $P'(\ell)\sim\ell^{-2.14}$, indicating that there can be highly tortuous paths that are the shortest way between the two wells. Away from p_c and in finite systems these tortuous paths are truncated either by the finite size of the system or by the finite correlation length of the clusters, leading to the terms f_2 and f_3 in Eq. (2). The importance of Eq. (2) for practical

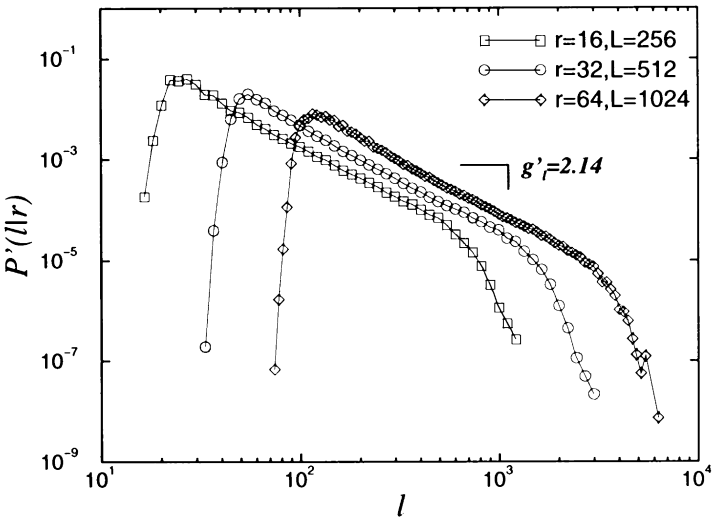

Fig. 1. Log–log plot of minimal path distribution $P'(\ell|r)$ at the percolation threshold, $p=p_c=0.593$ and for different sets of parameters, (r,L). The straight line regime has slope $g'_\ell=2.14$. The most probable lengths scale $\ell^* \sim r^{d_{\min}}$ and $d_{\min}=1.13$.

purposes is that it enables us to estimate the distribution of shortest path lengths at once for any given reservoir from just the statistics of the geological heterogeneities (that is the net-to-gross ratio and the size of the reservoir as a multiple of the sand body dimensions). Once the coefficients a, b and c in the scaling functions have been found from simulations then the whole distribution can be determined algebraically. Clearly this is very much faster than the traditional route of generating a large number of realisations and then calculating the shortest path lengths.

3. Breakthrough times

In fact we are more interested in the time it takes for the injected fluid to breakthrough rather than the length of the path it takes. The transition to the distribution of breakthrough times can be made easily if we assume that injection takes place at a constant volumetric rate (which it usually does). Then for a given dimension d_B of the backbone the mass of the backbone is $M(r)\sim r^{d_B}$. Constant injection rate is equivalent to $dt\sim dM\sim r^{d_B-1}\,dr$. Given that ℓ scales like $\ell\sim r^{d_{\min}}$ and therefore $d\ell\sim r^{d_{\min}-1}\,dr$ we find that the first passage time scales with the chemical path like $t\sim\ell^z$ where $z=d_B/d_{\min}=1.625/1.13\approx 1.44$. If we substitute $\ell\sim t^{1/z}$ into Eq. (2) we get the scaling form for the first passage time when injecting at a constant rate.

In order to investigate the decline in oil production after breakthrough we first start by investigating the homogeneous case ($p=1$). To do this we use bipolar coordinates ($x=a\sinh u/(\cosh u-\cos v)$, $y=a\sin v/(\cosh u-\cos v)$). In this coordinate system the

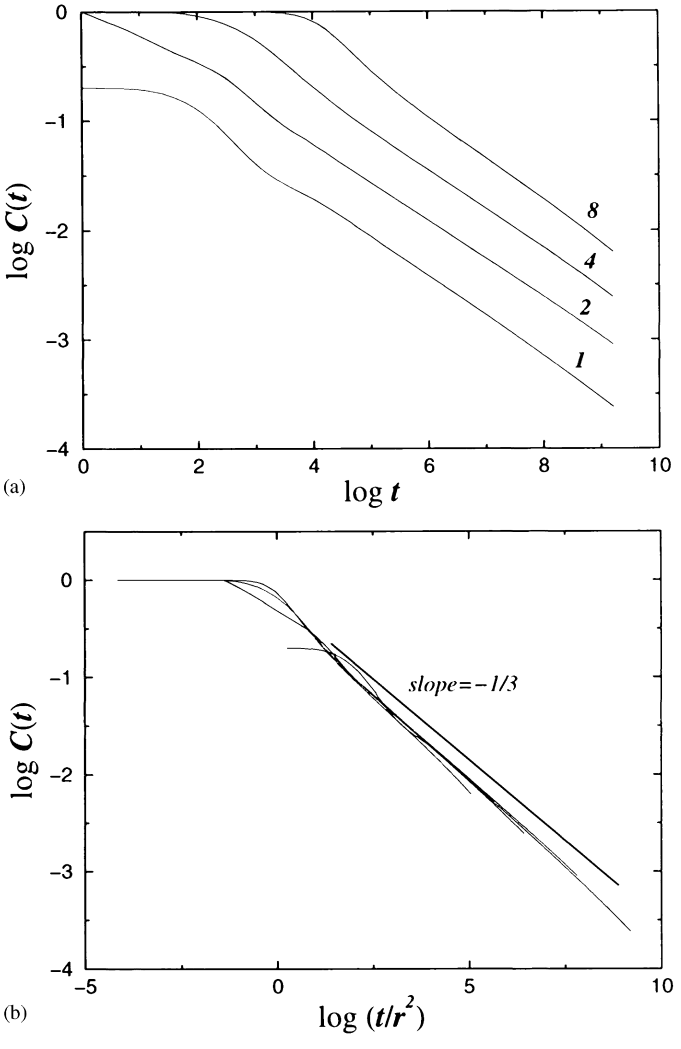

Fig. 2. (a) Log–Log plot of production curve $C(t)$ for homogeneous media ($p=1.0$), and for different values of the interwell separation $r=1,2,4,8$. (b) The scaling collapse using breakthrough time $t_{br}\sim r^2$. The slope of the curve gives exponent $g_c=\frac{1}{3}$, consistent with the theoretical value.

point $(a,0)$ is a unit source and $(-a,0)$ is a unit sink. The curves of constant u are isobars and curves of constant v are streamlines (both of these curves are arcs of coaxal circles). Elementary analysis shows that the length of streamline v is $2a(\pi-v)/\sin v$ and the time taken to traverse each streamline is

$$t=\frac{2a^2}{K}[1+(\pi-v)\cot v]/\sin^2 v\,, \tag{3}$$

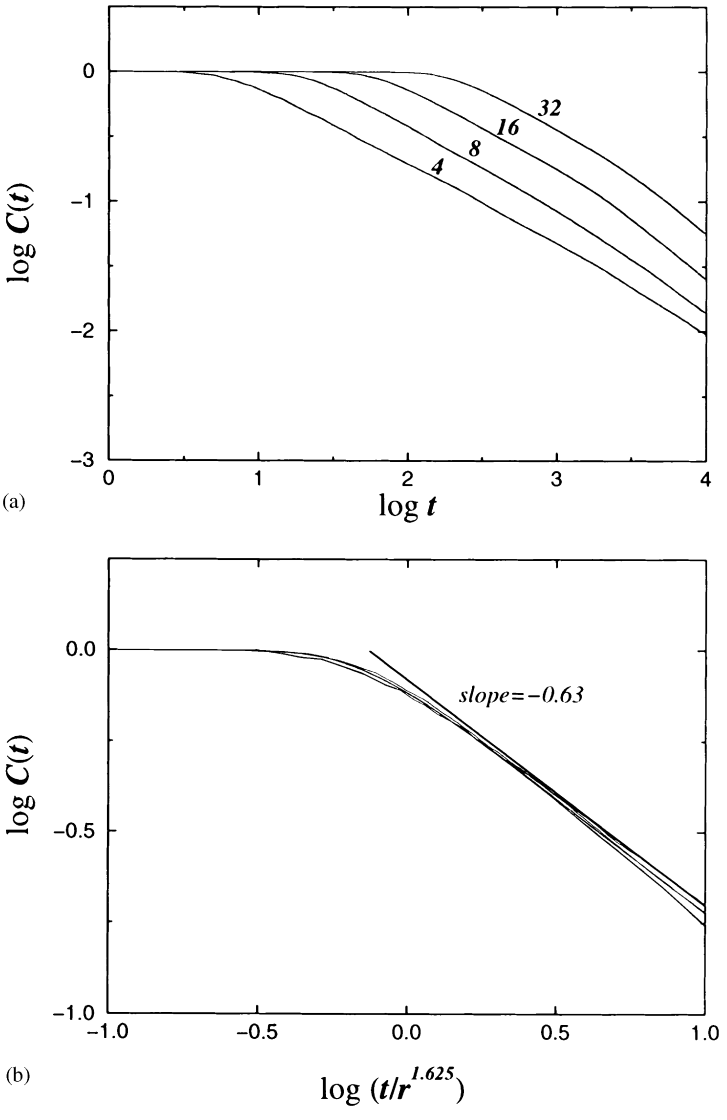

Fig. 3. (a) Log–Log plot of production curve $C(t)$ for percolating clusters ($p = p_c$), and for different values of interwell separation $r = 4, 8, 16, 32$. (b) The scaling collapse of the data using the breakthrough time $t_{br} \sim r^{1.625}$. The slope of the curve gives the exponent $g_c = 0.63 \pm 0.05$.

where K is a pumping rate. From this is can be seen that the earliest breakthrough is on $v = \pi$ and that the breakthrough time is $t_{br} = 2a^2/3K$. Conversely the long breakthrough times correspond to those streamlines around $v = 0$ for which $t_{br} \approx 2\pi a^2/Kv^3$, which implies that $v \approx (2\pi a^2/Kt)^{1/3}$ for long times. Note that at time t the oil comes out of the production well along the streamlines of which angle is greater than $2v$. Hence the

concentration at the production well at time t is of the oil at time t is $C(t) = v(t)/\pi$ where $v(t)$ should be found in Eq. (3).

So we have two results for the homogeneous case which we can express as scaling laws as a function of the well spacing $r = 2a$. First we expect the breakthrough time to scale as

$$t_{br} \sim r^2 \tag{4}$$

and the rate of production of oil at long times to decline like

$$C(t) \sim (r^2/t)^{1/3} . \tag{5}$$

We now wish to extend these ideas to the percolating system. So far we have only investigated the case $p = p_c$. In this case we would expect the breakthrough time to depend on the backbone of the percolating cluster in which case we would anticipate that $t_{br} \sim r^{d_B}$. Next we expect the decline in production to be a power but we have no a priori notion of what this might be, so we write

$$C(t) \sim (r^{d_B}/t)^{g_c} . \tag{6}$$

To test these hypotheses we carried out some numerical simulations on a square bond lattice. We find (Figs. 2 and 3) that they are very well confirmed and we can establish that the value of exponent $g_c = 0.63 \pm 0.05$. We do not yet have a hypothesis to explain this value.

As we move above the percolation threshold we would expect a crossover to the homogeneous case behaviour with exponent $g_c = 1/3$. We would also expect this to be the case for a finite size system. So we anticipate that the distribution of breakthrough times curves have a similar scaling form to Eq. (2) with similar exponential cutoffs for finite size and $p = p_c$. This is currently under investigation.

4. Conclusions

We have demonstrated that percolation concepts can be useful in predicting oil recovery and uncertainty in the recovery characteristics. In particular, we have shown that various results can be calculated algebraically and therefore computationally extremely quickly compared with the more traditional Monte Carlo approach. This may have commercial benefits in terms of very rapid risk assessment.

Specifically we have extended earlier work on the scaling of the chemical distance on percolating systems to propose a scaling conjecture for the conditional distribution for the chemical path between two points given that they are connected. This result has been extended to include finite size effects and the behaviour away from the threshold. Numerical simulations have been carried out to validate this conjecture.

Further, we have investigated the scaling behaviour of the breakthrough time for a passive tracer convected between the injector and the producer as well as the decline in production post-breakthrough. This has been done by first considering the homogeneous case and then using plausible extensions to the percolation case at threshold. We are

still investigating the way this behaviour is modified away from threshold and for finite size systems.

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