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Intersection of two fractal objects: Useful method of estimating the fractal dimension

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We consider the "overlap pattern" formed by the intersection of two diffusion-limited aggregation (DLA) clusters. The fractal dimension of the disconnected set of points belonging to the intersection is given by $d_f^{\cap}=2d_f-d$, where d_f is the fractal dimension of the original DLA, and d is the space dimension. We measure $d_f^{\cap}$ from simulations in $d=2,3$ based on two DLA clusters, and then calculate the corresponding value of d_f. Also, we use the more general equation $d_f^{\cap}=d_f+d_f'-d$ to analyze overlap patterns obtained by slicing a $d=3$ DLA cluster with a $d_f'=2$ plane. We find good agreement with independent estimates of d_f for DLA.

Progress has been made recently in the investigation of the diffusion-limited aggregation (DLA) model.[1-7] The DLA model describes a wide range of growth phenomena.[8-11] Further, variants of DLA such as cluster-cluster aggregation[12] have many realizations in nature (such as chemical systems,[13] aerosol physics,[14] and polymer physics[15]).

Let us now consider another aspect of DLA, one related to the situation in which two DLA clusters are grown independently. When these two DLA clusters are overlapped, with their centers separated by a distance l, we obtain a disconnected fractal object which we call the "overlap pattern" or intersection. The fractal dimension $d_f^{\cap}$ of this fractal object is given by[16]

$$d_f^{\cap}=2d_f-d, \text{ for } 0<l<R_g . \quad (1)$$

Here d_f is the fractal dimension of a DLA, d the space dimension, and R_g the radius of gyration of DLA.

We will use this formula to obtain an independent estimate of d_f for DLA clusters of 5000 sites in dimension $d=2,3$. We deliberately use clusters of modest size that can be quickly calculated on almost any machine. Figure 1 shows the dependence on l of the number of sites in the overlap pattern, averaged over only ten samples. We omit large values of l. We find $d_f^{\cap}=1.46\pm0.05(d=2)$ (see Fig. 2) and $d_f^{\cap}=2.0\pm0.10(d=3)$ (see Fig. 3). Equation (1) then predicts that $d_f=1.73\pm0.02(d=2)$ and $d_f=2.5\pm0.05(d=3)$, in excellent accord with independent measurements.[1,2] Note that the error bars of 3%

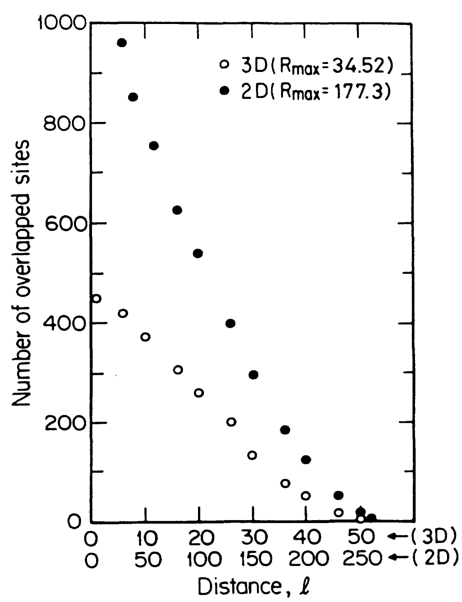

FIG. 1. Dependence on l, the distance between the seeds of two DLA clusters, of the number of sites in the overlap pattern.

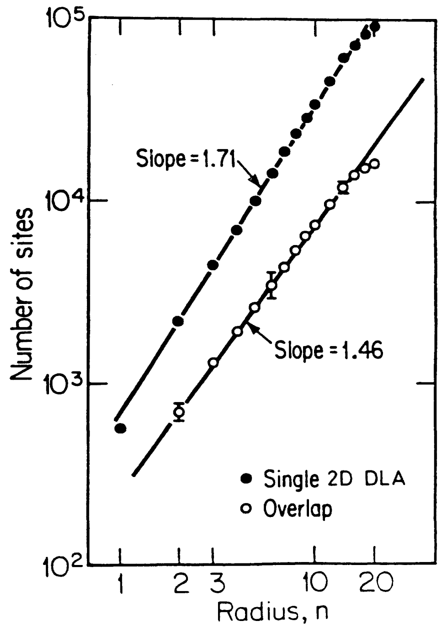

FIG. 2. The number of sites inside a circle of radius n, where n denotes the distance of $10n$ lattice spaces. ● is for one DLA and ○ is for the overlap pattern (obtained by overlapping two DLA clusters). Here $d=2$.

($d=2$) and 5% ($d=3$) in $d_f^{\cap}$ correspond to error bars of only 1% ($d=2$) and 2% ($d=3$) in d_f, corresponding to the fact that Eq. (1) expresses d_f as the difference of two numbers, one of which is a perfect integer.

We can also simulate the intersection of a DLA cluster with any other object, fractal or nonfractal, of dimension $d_f'^{\cap}$. Equation (1) is replaced by[16]

$$d_f^{\cap} = d_f + d_f' - d \ . \qquad (2)$$

We studied the patterns obtained by slicing the three-dimensional DLA by a plane ($d_f'=2$). Our simulation gives $d_f^{\cap} \sim 1.7$. Equation (2) then predicts $d_f \sim 2.7$, in rough accord with the estimate above.

In summary, we have used the two formulas, (1) and (2), for the fractal dimension of the overlap pattern, to calculate independent estimates of d_f for DLA in dimensions two and three. The agreement with previous estimates is good, considering relatively small clusters were used, because the basic equations (1) and (2) express d_f as the difference of two numbers, one of which is a perfect integer. Of course, (1) and (2) hold for any fractal object; thus, e.g., the fractal dimension in percolation can be estimated by studying the overlap between two large percolation clusters. The percolation case is physically appealing: Measuring $d_f^{\cap}$ is the same as measuring γ/ν, since[17]

$$\frac{\gamma}{\nu} = d_f^{\cap} \ . \qquad (3)$$

Here γ and ν are the critical exponents describing the divergence as $p \rightarrow p_c$ of the mean cluster size $\chi(p)$ [the first moment of the cluster size distribution $P(s)$] and the connectedness length $\xi(p)$, respectively.

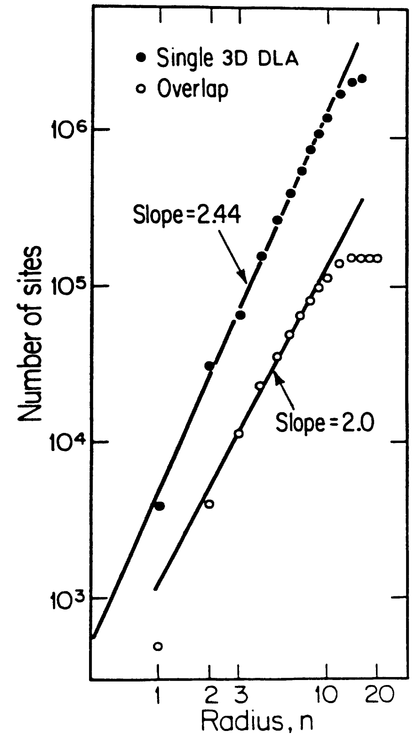

FIG. 3. The number of sites inside spheres of radius n, where n denotes the distance of $5n$ lattice spaces. ● is for one DLA and ○ is for the pattern obtained by overlapping two DLA clusters. Here $d=3$.

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[16]One way of understanding (1) and (2) is as follows. Suppose we consider the overlap pattern formed by two fractals A and A' with fractal dimensions d_f and d_f'. Consider a $L\times L$ box centered on a site belonging to the overlap pattern. The density of sites belonging to the overlap pattern decreases with increasing L with exponent $d_f^{\cap}-d$. Now the density of A sites decreases with L with exponent d_f-d, while the density of A' sites has exponent $d_{f'}-d$. The probability that a site belongs to both the A and A' fractals is the product of the individual probabilities. Hence $d_f^{\cap}-d=(d_f-d)+(d_f'-d)$, from which (2) follows immediately. Equation (1) is a special case of (2) in which $d_f'=d_f$.

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