ONSET OF HELICAL ORDER

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Renormalization group techniques are used to treat the onset of helical order at higher order critical points for a large class of helical states First order perturbation results are given for all the critical point exponents for critical points of order θ and arbitrary anisotropic propagators. The critical point exponent η is calculated to second order for arbitrary isotropic propagators.

Recently several authors have used renormalization group techniques to discuss the onset of helical order in magnetic systems.[1-3] In particular, the existence of new types of critical behavior has been postulated for the "Lifshitz" point where the transition from a uniformly ordered to helically ordered state occurs[1].

Here we consider a class of systems which can exhibit phases of very general helical character, and report the renormalization group calculation of critical exponents for such systems. These critical points will be termed "generalized Lifshitz points". These systems are characterized by critical or fixed point propagators which differ from the usual k^2 dependence.[4]

By "helical" phases we mean magnetically ordered states with a periodic spatial structure whose periodicity need not be related to that of the lattice. Included thereby are spiral structures of various types, as well as states in which the magnetic moments are aligned uniformly in direction, but with sinusoidally varying magnitudes. The presence of these (and even more complicated phases) is well established, largely through neutron diffraction studies. Typical examples include the screw spiral structure of MnO_2, cone-spiral order in spinel-type compounds such as $MnCr_2O_4$, and sinusoidal phases in certain rare earth metals (Er, Nd and others).

We use continuum spin Wilson models with explicit wave-vector dependent terms in the Hamiltonian density which are coupled to the thermodynamic fields[6]. For motivational purposes we consider a free energy functional F of an isotropic single component magnetization M(x):

$$F=\int d^d_x \{a_1(\nabla M(\vec{x}))^2 + a_2(\nabla^2 M(\vec{x}))^2 + \dots + A_1 M^2(\vec{x}) + A_2 M^4(\vec{x}) + A_3 M^6(\vec{x}) + \dots\} \quad (1)$$

Here the coefficients a_i and A_i are functions of the thermodynamic fields (temperature, pressure, magnetic field,...). At an ordinary critical point we have $A_1=0$, with $A_2>0$ and $a_1>0$; variations in the field variables which preserve these conditions sweep out a surface of second order ($\theta=2$) critical points. Similarly, $A_1=A_2=0$, $A_3>0$, $a_1>0$ characterizes a points of three phase criticality ($\theta=3$). For $A_1=A_2=\dots=A_{\theta-1}=0$, $A_\theta>0$, we have a critical point of order θ.

For $a_1>0$, all of the competing phases are spatially uniform. However, if $a_1<0$, the free energy F will be minimized for some particular non-uniform phase. This is most easily seen by considering the Fourier transform representation of (1), where we include derivative terms up to order 2L and magnetization terms up to order 2θ:

$$F(\{M(\vec{k})\})=\int d^d k\Big(\sum_{j=1}^{L} a_j k^{2j}\Big) M^2(\vec{k}) + \sum_{i=1}^{\theta} A_i \Big(\prod_{m=1}^{2i} \int d^d k_m M(\vec{k}_m)\Big)\delta\Big(\sum_{=1}^{2i} \vec{k}\Big). \quad (2)$$

The minimum of F is obtained for a sinusoidally varying magnetization $M(\vec{k})$, where $\vec{k}$ is determined by minimizing the $\vec{k}$-dependent parts of (2). For L=2 and $a_1>0$, the system displays ordinary criticality between spatially uniform phases; for $a_1<0$ criticality is achieved between helical states of equal and opposite $\vec{k}$. As the Lifshitz point $(a_1=0)$[1] is approached, $\vec{k}$ approaches zero as $k\sim(-a_1)^{\beta_k}$ ($\beta_k=1/2$ in mean field theory). If both a_1 and a_2 vanish, it is necessary to include in (2) the k^6 term. In mean field theory, the conditions $A_1=\dots=A_{\theta-1}=0$, $A_\theta>0$, and $a_1=a_2=\dots=a_{L-1}=0$ specify a critical point with order θ and "Lifshitz character L". In the vicinity of such a point, there are L different values of the helicity wave-vector, each of which is associated with θ different values of $M(\vec{k})$. At the generalized Lifshitz point, all of these phases are simultaneously critical.

We will use as a model Hamiltonian for our renormalization group calculations $H=F(\{s(\vec{k})\})$, where $s(\vec{k})$ is the spin fluctuation variable. The critical propagator (the term in H proportional to $s^2(k)$) has leading dependence k^{2L}. It is also possible to have non-integral propagator exponents. By the introduction of a long range force[7] with interaction strength decaying like $r^{-(d+\sigma)}$, we can add a term proportional to $|k|^{\sigma}s^2(\vec{k})$ to H. If both terms are present in the critical region, then only $\tilde{\sigma}\equiv\min(\sigma,2L)$ is important; the critical propagator is proportional to $|k|^{\tilde{\sigma}}$.

The first order results in a perturbation expansion can be obtained (both in the isotropic propagator considered here and for the anisotropic case considered below) by utilizing the techniques developed for the simple L=1 case[8-11]. For a critical point of order θ of an isotropically interacting n-component spin system, the borderline dimension (above which mean field behavior holds) is given by $d_b=\tilde{\sigma}\,\theta/(\theta-1)$. Below this dimension, we obtain a correction to the pth eigenvalue (corresponding to s^{2p}) in terms of the unperturbed or Gaussian eigenvalues $\{\lambda_j\}$:

$$\lambda'_p = \lambda_p - 2\lambda_\theta \langle\theta,p;p\rangle_n / \langle\theta,\theta;\theta\rangle_n \quad (3a)$$

where[11]

$$\langle\theta,p;p\rangle_n \equiv \sum_{j=0}^{\theta/2} \binom{p}{j}\binom{p+\frac{n}{2}-1}{j}\binom{2p-2j}{\theta-2j} \quad (3b)$$

For isotropic propagators, $\lambda_p = d+p(\tilde{\sigma}-d)$. The expansion parameter $\varepsilon_\theta(\tilde{\sigma}) \equiv \lambda_\theta$ is thus

$$\varepsilon_\theta(\tilde{\sigma}) = d+\theta(\tilde{\sigma}-d) = (\theta-1)(d_b-d). \quad (4)$$

Thus, for the Ising model ($n=1$), we have

$$\lambda'_p = d+p(\tilde{\sigma}-d)-2\varepsilon_\theta(\tilde{\sigma})\binom{2p}{\theta}\Big/\binom{2\theta}{\theta} \quad (5a)$$

while for $n=\infty$

$$\lambda'_p = d+p(\tilde{\sigma}-d)-\frac{2\varepsilon_\theta(\tilde{\sigma})\binom{p}{[1/2(\theta+1)]}}{\binom{\theta}{[1/2(\theta+1)]}} \quad (5b)$$

This defines all the eigenvalues to first order. We now define the critical point exponent η by the shift in the exponent of the critical two-point correlation function

$$\Gamma_2(\vec{k}) \propto |\vec{k}|^{\tilde{\sigma}-\eta} \quad (6)$$

For η, we find that if $\tilde{\sigma}\neq 2L$, there is no shift, i.e. that $\eta=0$ to $O(\varepsilon_\theta(\tilde{\sigma})^2)$. For $\tilde{\sigma}=2L$ we find:[9]

$$\eta_\theta(2L) = \frac{(-1)^{L+1}\,4\varepsilon_\theta^2(2L)}{L\binom{2\theta}{\theta}^3}\;\frac{\theta\,\Gamma^2\left(\frac{d_b}{2}\right)C_n}{\Gamma\left(\frac{1}{2}(d_b-2L)\right)\Gamma\left(\frac{1}{2}(d_b+2L)\right)} \quad (7a)$$

$$\text{with } C_n \equiv \left[\frac{\langle\theta,\theta;\theta\rangle_n}{\langle\theta,\theta;\theta\rangle_{n=1}}\right]^2 \prod_{j=1}^{\theta-1}\frac{2j+n}{2j+1} \quad (7b)$$

Equation (7) agrees with the result of Ref. 1 for the special case $\theta=L=2$ and has also been verified for $\theta=2$ and all L by a differential renormalization group method[10]. Note that the combinatorial factor $C_n=1$ for $n=1$, and that for large n

$$C_n \to n^{-1}\binom{2\theta}{\theta}^3\Big/\binom{\theta}{\theta/2}^3 + O(n^{-2}),\ \theta \text{ even}, \quad (8a)$$

$$C_n \to \binom{2\theta}{\theta}^3\Big/\left\{(\theta+1)\binom{\theta}{1/2[\theta+1]}\right\}^3 + O(n^{-1}),\ \theta \text{ odd}. \quad (8b)$$

Other wave-vector associated exponents also have second order corrections; e.g., $\beta_k = 1/(2(L-1)) + O(\varepsilon^2(2L))$.[10]

<u>Anisotropic Propagators</u>

In the above we used a propagator <u>isotropic</u> in $\vec{k}$-space. However, the lattice structure of a real material can induce a preferred direction for the periodic behavior. Moreover, since catastrophic infrared divergences[10-11] set in at dimensions less than $d_{min}=2L$, the theory as developed above is unlikely to produce realistic predictions at $d=3$ if $L \geq 2$. These infrared divergences are, of course, intimately related to the appearance of infinitely many relevant Gaussian eigenvalues below d_{min}. For these reasons, we now consider <u>anisotropic</u> propagators.

We write the wave vector $\vec{k}$ as

$$\vec{k} = \vec{k}_1 + \vec{k}_2 + \ldots + \vec{k}_J \quad (9)$$

where each $\vec{k}_i$ is a d_i-dimensional vector, so that $\sum d_i = d$. We consider a critical propagator G^{-1} of the form

$$G^{-1} = \sum_{i=1}^{J} |\vec{k}_i|^{\sigma_i} \quad (10)$$

with $\sigma_1 < \sigma_2 < \ldots < \sigma_J \equiv \sigma_>$. For such systems, d_b for a critical point of order is determined by

$$\sum_{i=1}^{J} d_i/\sigma_i = \theta/(\theta-1) \quad (11a)$$

and d_{min} by the condition

$$\sum_{i=1}^{J} d_i/\sigma_i = 1. \quad (11b)$$

The introduction of anisotropy in the propagator lowers both d_b and d_{min}. For example, if only one component of $\vec{k}$ enters G^{-1} as k^{2L} and the remaining components have k^2 dependence, then (11) gives $d_b=(3\theta-1)/(\theta-1)-1/L$ and $d_{min}=3-1/L$. Thus, we have $d_b \geq 3 > d_{min}$ for all $\theta<2L+1$.

For anisotropic systems, the critical point exponents $\{\eta_i\}$ are defined by examing the behavior of the two-point function for a wave-vector lying entirely in one of the d_i dimensional subspaces:

$$\Gamma_2(\vec{k}_i) \propto |\vec{k}_i|^{\sigma_i-\eta_i} \quad (12)$$

There will also be difference values of the correlation length exponent ν_i in each of the subspaces. The following relationships between the exponents hold generally:

$$2-\alpha = \sum_{i=1}^{J} d_i\,\nu_i\ ;\ \gamma=(\sigma_i-\eta_i)\nu_i\ ;$$

$$\delta = \left\{\sum_{i=1}^{J} d_i/(\sigma_i-\eta_i) + 1\right\}\Big/\left\{\sum_{i=1}^{J} d_i/(\sigma_i-\eta_i)-1\right\}. \quad (13)$$

The results (13) can be derived by constructing an exact differential[10-12] renormalization group equation suitable for the propagator (10). A similar approximate generator for a Hamiltonian $H(\vec{s},\ell)$ is[13]

$$\partial H/\partial\ell = \sum_{i=1}^{J} d_i\,\sigma_>/\sigma_i\}H + \frac{1}{2}\,(\sigma_> - \sum_{i=1}^{J} d_i\,\sigma_>/\sigma_i)\ \vec{s}.\nabla H \quad (14)$$
$$+\ \nabla^2 H - \vec{\nabla} H\cdot\vec{\nabla} H$$

where ℓ is the renormalization parameter[11]. The first order eigenvalue corrections are again given by (3); only the value of the unperturbed eigenvalues are changed. From (14), we see that the Gaussian eigenvalues are now:

$$\lambda_p = \sum_{i=1}^{J} d_i\sigma_>/\sigma_i + p(\sigma_> - \sum_{i=1}^{J} d_i\sigma_>/\sigma_i) \quad (15)$$

The expansion parameter for $d<d_b$ is again $\varepsilon_\theta(d_i,\sigma_i)\equiv\lambda_\theta$. Thus, the corrected eigenvalues for the general anisotropic case are

$$\lambda'_p = \sum_{i=1}^{J} d_i\, \sigma_> /\sigma_i + p\left(\sigma_> - \sum_{i=1}^{J} d_i \sigma_>/\sigma_i\right)$$

$$-2\varepsilon_\theta(d_i,\sigma_i)\ \langle\theta',p;p\rangle_n/\langle\theta,\theta;\theta'\rangle_n \quad . \qquad (16)$$

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13. The approximate generator is derived as in Ref. 11 from an exact generator similar to the Wilson incomplete integration generator (Ref. 12), except that we renormalize in such a way as to hold each of the propagator exponents separately fixed. Wave-vector dependent terms enter the calculation only at second order in perturbation expansions; by neglecting these terms we obtain (14). Note that to this order the wave-vector dependent portions of the Hamiltonian are invariant and therefore we need not write them explicitly. A similar exact generator can be obtained in Wegner-Houghton form, cf. Ref. 10-11.

ORDER-DISORDER AND α-γ TRANSITIONS IN FeCo*

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Electrical resistivity (ρ) measurements in an annealed FeCo alloy (with 46.29 at.% Co) in the temperature range of 500-1350^o K are reported. Special attention is given to the temperature regions of 900-1100^o K and 1160-1300^o K. A change in the slope of ρ vs T curve, associated with the order-disorder (OD) transition, yields a λ-type anomaly in the computed $\partial\rho/\partial T$ with maximum at $T_c \simeq 1006^o$ K. To our knowledge this is the first observation of the OD transition in FeCo using temperature-dependent electrical resistivity measurements. Similar to the observations in β brass [1], $\partial\rho/\partial T$ in the critical region is found to be proportional to the specific heat associated with the OD transition [2]. Near 1235^o K, a discontinuous change in ρ of about 20% with hysteresis of about 12^o K, is observed. This first order transition is associated with the α-γ (bcc-fcc) transition. However, the nature of the anomaly (a jump in ρ with increasing T) in FeCo is quite opposite to that observed in Fe. A detailed account of this work will be published elsewhere.

*Supported in part by the National Science Foundation.
⁺A. P. Sloan Research Fellow.

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