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How Fear of Future Outcomes Affects Social Dynamics

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Abstract Mutualistic relationships among the different species are ubiquitous in nature. To prevent mutualism from slipping into antagonism, a host often invokes a "carrot and stick" approach towards symbionts with a stabilizing effect on their symbiosis. In open human societies, a mutualistic relationship arises when a native insider population attracts outsiders with benevolent incentives in hope that the additional labor will improve the standard of all. A lingering question, however, is the extent to which insiders are willing to tolerate outsiders before mutualism slips into antagonism. To test the assertion by Karl Popper that unlimited tolerance leads to the demise of tolerance, we model a society under a growing incursion from the outside. Guided by their traditions of maintaining the social fabric and prizing tolerance, the insiders reduce their benevolence toward the growing subpopulation of outsiders but do not invoke punishment. This reduction of benevolence intensifies as less tolerant insiders (e.g., "radicals") openly renounce benevolence. Although more tolerant insiders maintain some level of benevolence, they may also tacitly support radicals out of fear for the future. If radicals and their tacit supporters achieve a critical majority, herd behavior ensues and the relation between the insider and outsider subpopulations turns antagonistic. To control the risk of unwanted social dynamics, we map the parameter space within which the tolerance of insiders is in balance with the assimilation of outsiders, the tolerant insiders maintain a

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sustainable majority, and any reduction in benevolence occurs smoothly. We also identify the circumstances that cause the relations between insiders and outsiders to collapse or that lead to the dominance of the outsiders.

Keywords Game theory · Complex networks · Social thermodynamics · Open systems · Tolerance · Herd behavior

1 Introduction

Karl Popper famously stated that unlimited tolerance leads to the demise of tolerance [1]. The tag-based [2] quantitative model of Riolo et al. [3] indicates that a combination of kin selection and mutation causes times of high tolerance to be replaced by times of low tolerance towards those who are different. These tides of (in)tolerance [4] seemingly dismiss the role of human reasoning in the selection process as unable to stave off periods during which undesirable states of affairs prevail. In contrast, the basic negative result of evolutionary game theory that unconditional cooperators are vulnerable to rare occurrences of unconditional defectors [5] can be avoided by appending an indirect reciprocity mechanism to the selection process, i.e., a concern for such abstract realities as reputation, and the ability to use some form of language to spread information. It would thus seem that humans *are* able to draw on such collective mechanisms as democracy [6] to adjust the course of selection in a preferable direction—and sometimes they are not [7].

Humans are conditional cooperators [2,8,9], yet on occasion may benevolently help even those who can never repay in kind. Although this cooperativeness is seen as the result of evolutionary selection [2,3,6,8–13], it is less clear why benevolence would permeate human societies. Popper's statement, however, helps us identify how certain mechanisms may sustain benevolence. When a benevolent population is attracting an inflow from the outside, such that a society undergoes the transient dynamics, understanding the relationship between the inflow rate and human behavior is key. If the rate of inflow is low the original population may feel safe, but if the inflow is high it may be perceived as aggression and provoke—in a sort of Popperian twist—a violent response. The very idea of these two limits suggests that benevolence is a relative category and is dependent on the inflow from the outside and the resulting state [14]. Little is known about these dependencies of benevolence.

2 Model

To quantify the dynamics of an open society with a benevolent population, we create a theoretical framework by combining the elements of biology (e.g., evolutionary games) and statistical physics (e.g., complex networks). It was argued almost a decade ago that the methods of statistical physics could contribute to social science [15]. This notion of the usefulness of statistical physics in researching social phenomena appears to have been appreciated in quantitative sociology, particularly the works on economic complexity [16,17]. In line with these works, we represent human relationships in an idealized manner by placing agents into a random network of friendships wherein each possible link occurs independently with precisely such a probability that the average degree is 50—a number consistent with Refs. [18,19]. Agents in the model are thus conditional cooperators in the sense that their interactions are restricted only to the nearest neighbors defined by the network. At each time t, a total

Table 1 Key symbols (parameters and state variables)

Symbol	Definition	Value
c/b	Cost-benefit ratio	10 %
Δ_b/b	Relative benefit differential	25 %
p_1	Assimilation rate	–
$T_r^{\min}$	Minimum tolerance of insiders	0.05
σ	Tolerance range (individuality)	–
I_c	Critical threshold for herd behavior	50 %
f_g	Fraction of outsiders in the total population	
R	Fitness ratio measuring benevolence of insiders	
$\langle T_r \rangle$	Average population tolerance	

of m donor-recipient pairs are randomly chosen [11] among neighboring agents, whereupon a donor pays cost c for the recipient to receive benefit b. To this trivial scenario we add an asymmetry in which insiders incur a higher cost of cooperation and provide more benefit to outsiders than they receive in return. In this way, benefit differential Δb emerges between insider and outsider subpopulations, creating an incentive for outsiders to immigrate. The overall purpose is to form a mutualistic relationship in which everyone experiences a higher standard due to the extra labor provided by outsiders. When the selected m donor-recipient pairs finish interacting, we calculate the fitness of both insiders and outsiders—denoted Φ_1 and Φ_2, respectively—as the average per-capita benefit net of the cost of cooperation. The details on the mathematical representation of the described setup are found in Supporting Information (SI text). Here, we just note that the quantities of interest are the cost-benefit ratio, c/b, and the relative benefit differential, Δ_b/b. A list of key symbols with some default parameter values is given in Table 1.

We envision a society that is an open dynamic system in the sense that its size changes over time. To that end we adopt replicator-type equations. Specifically, if N_1 and N_2 denote the population sizes of insiders and outsiders, respectively, then the time-change of these two subpopulations is given by $N_i(t + 1) = \Phi_i N_i(t)$, $i = 1, 2$. The fraction of outsiders in the system is defined as $f_g = N_2/(N_1 + N_2)$, yielding

$$f_g(t+1) = \frac{\Phi_2(t)N_2(t)}{\Phi_1(t)N_1(t) + \Phi_2(t)N_2(t)} = R(t)f_g(t), \tag{1}$$

where R is the ratio of the fitness of outsiders to the average fitness of the whole population, i.e., $R = \Phi_2/\left[\Phi_1(1 - f_g) + \Phi_2 f_g\right]$. Here the time-change is both biological and sociological, i.e., successful individuals can attract migrants of similar origin from the outside.

We assume that outsiders can "mutate" into insiders (i.e., they can assimilate the cultural patterns of the insiders) and vice versa (e.g., due to education or intermarriage). Assimilation, often almost synonymously referred to as integration, has been receiving considerable attention in the literature based on statistical physics [20–23]. Herein, we represent the results of the assimilation process by a net rate p_1. The net rate is either positive or negative, where positive values indicate that the absolute assimilation rate is higher for outsiders than insiders. Our analysis, however, revolves only around the case $p_1 > 0$ because, as long as the asymmetry in the system persists, and thus $R > 1$, outsiders grow at a faster rate than insiders. In this sense, fitness ratio R is a global measure of benevolence exhibited by insiders towards outsiders. For the two subpopulations to equilibrate, the necessary and sufficient condition is $p_1 = R - 1$, which indicates that the more benevolent insiders are, the higher the net

assimilation rate needs to be for insiders to avoid being overrun by outsiders. If we allowed $p_1 < 0$, the two subpopulations could equilibrate only at some $R < 1$, which is impossible unless insiders start to punish outsiders.

How do insiders decide whether to cooperate with outsiders? Relying on the network of friendships as another layer of social fabric, insiders face three options: (i) remain benevolent, (ii) behave benevolently, but question the wisdom of doing so, or (iii) no longer cooperate with outsiders. Insiders who are surrounded by an overwhelming number of outsiders can begin to resent providing benefit without receiving the same in return. Thus when the local fraction of outsiders, f_ℓ^i, in the neighborhood of insider i exceeds the tolerance threshold [24–26], T_r^i, the insider defects by severing all connections with outsiders. Insiders who terminate all their friendships with outsiders we label "radical agents." As a proxy for T_r^i we take a uniform random variable on the interval from $T_r^{\min}$ to $T_r^{\min} + \sigma$, i.e., $T_r^i \sim \mathcal{U}(T_r^{\min}, T_r^{\min} + \sigma)$. The higher the value of $T_r^{\min}$, the greater the willingness of insiders to tolerate outsiders. Parameter σ is a measure of the degree of individuality, while its inverse $1/\sigma$ measures societal responsiveness—the lower its value, the greater the extent to which insiders are willing to tolerate a large benefit differential. When $\sigma = 0$, individuality among the insiders disappears and threshold distribution $\mathcal{U}$ degenerates into a Dirac delta distribution. However, $\sigma \neq 0$ is more realistic because tolerance is likely to vary among people. It is important that the complex network of friendships, depending on the choice of tolerance parameters $T_r^{\min}$ and σ relative to assimilation parameter p_1, may generate herd behavior [27] such that most connections between insiders and outsiders get abruptly broken [26].

Envisioning herd behavior in a different yet related context [27], as the fraction of outsiders (f_g) increases, the fraction of radical agents increases, and the probability of violent incidents occurring also increases. As violence increases in a society, tolerant insiders ($T_r^i \gg f_g$) may consider open animosity towards outsider friends overly radical, yet adopt a conservative position [28] that, in effect, supports the radical faction—option (ii) above. There emerges a disparity between how some insiders act and think [29]. If g_t is the fraction of tacit supporters of radicals in a large population of N_1 insiders, then the probability of event $Y = \{N_1 g_t \geq I_c\}$ is approximated by a cumulative Poisson distribution, i.e., $\Pr(Y) = \exp(-\lambda) \sum_{k \geq I_c} \lambda^k / k!$, where λ is a size-dependent variable that increases with the global fraction of outsiders f_g. Specifically, we set $\lambda = N_1(1 - g_r)f_g$, where g_r is the fraction of insider radical agents. The term in parentheses indicates that radicals cannot be tacit supporters at the same time. We further assume that the increase in tacit supporters causes tension that, at some critical I_c, becomes a violent reaction. Herd behavior ensues, resulting in the abrupt and simultaneous severing of all connections between insiders and outsiders. An example of a critical I_c in a democracy would be a value that gives a majority to radicals and their supporters during an election. Figure 1 shows the basic aspects of the implied social dynamic, i.e., benevolence as a function of the fraction of outsiders in the system.

Mean-field theory provides a deeper understanding of the social dynamics implied by the model. For example, we derive the probability that randomly picked insider i is radical, i.e., the probability of event $X = \{f_\ell^i \geq T_r^i\}$, where f_ℓ^i is the local fraction of outsider neighbors surrounding agent i and T_r^i is i's tolerance. In the mean field approximation, a substitute for f_ℓ^i is its global counterpart, f_g, which can further be multiplied with the average degree of the network, $\langle k \rangle$, to estimate the number of outsider neighbors of an insider. The insider can tolerate any number of outsiders between $k_{\min} = \lceil T_r^{\min} \langle k \rangle \rceil$ and $k_{\max} = \lceil (T_r^{\min} + \sigma) \langle k \rangle \rceil$ with equal probability, where $\lceil \cdot \rceil$ is the ceiling function. Consequently we obtain
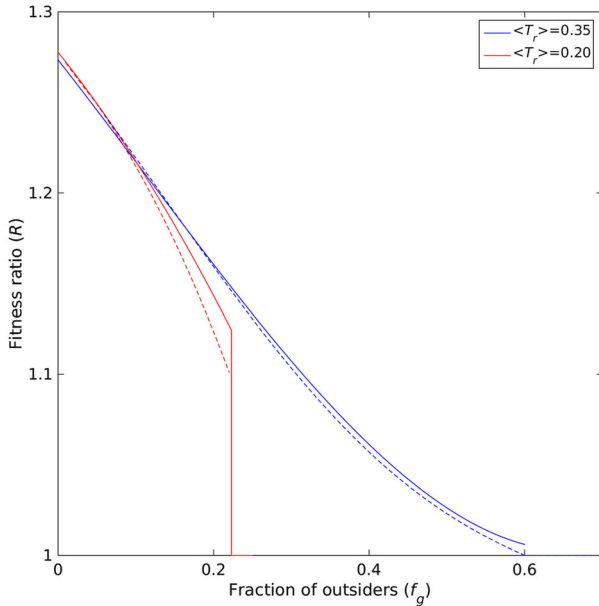
Fig. 1 Benevolence is dependent on the ubiquity of outsiders. Benevolence, as reflected by fitness ratio R, is a decreasing function of the fraction of outsiders, f_g, although no explicit assumptions were made to that effect. If the average tolerance of insiders is low enough (*red curve*), herd behavior causes a discontinuity of function $R = R(f_g)$. Otherwise (*blue curve*), the functional dependence is continuous. *Dashed curves* are analytical approximations (SI text). The assimilation rate is $p_1 = 0.01$

$$\Pr(X) = \frac{1}{k_{\max} - k_{\min} + 1} \sum_{l=k_{\min}}^{k_{\max}} \sum_{k=l}^{\langle k \rangle} \binom{\langle k \rangle}{k} f_g^k (1 - f_g)^{\langle k \rangle - k}. \tag{2}$$

Figure 2a compares the results of Eq. (2) with the corresponding numerical simulations. Note that a population of insiders with a higher than average tolerance contains fewer radicals for a given fraction of outsiders and allows outsiders to reach a higher saturation level. The agreement between the analytical results and the simulations is favorable.

Using Eq. (2) we can characterize the dependence of the probability of herd behavior on the fraction of outsiders. In accordance with our definitions, herd behavior occurs when radicals and their tacit supporters reach a critical threshold I_c. Because the number of radicals in the system is given by $N_1 g_r$, we only need to consider the probability of event $Y = \{N_1 g_t \geq I_c - N_1 g_r\}$, which—as described above—is given by the cumulative Poisson distribution. In general, the population of insiders, N_1, is large and parameter $\lambda \propto N_1$ of the Poisson distribution is also large, and this allows us to use the normal approximation for probability $\Pr(Y)$. Accordingly, $\Pr(Y) \approx 1 - F_{\mathrm{norm}}(I_c/N_1 - g_r; \lambda/N_1, \sqrt{\lambda}/N_1)$, where F_{norm} is the cumulative distribution function of a normal random variable with mean $\lambda/N_1 = (1 - g_r) f_g$ and standard deviation $\sqrt{\lambda}/N_1 = \sqrt{(1 - g_r) f_g / N_1}$. During the calculations g_r is replaced with $\Pr(X)$ from Eq. (2). Figure 2b shows the favorable comparison between this approximation and the simulation results and indicates that herd behavior becomes possible only when the fraction of outsiders is already high, i.e., $f_g \approx 0.4$. As f_g continues to increase, the possibility of herd behavior becomes increasingly remote because it is now difficult for radicals and their tacit supporters to acquire an electoral majority.
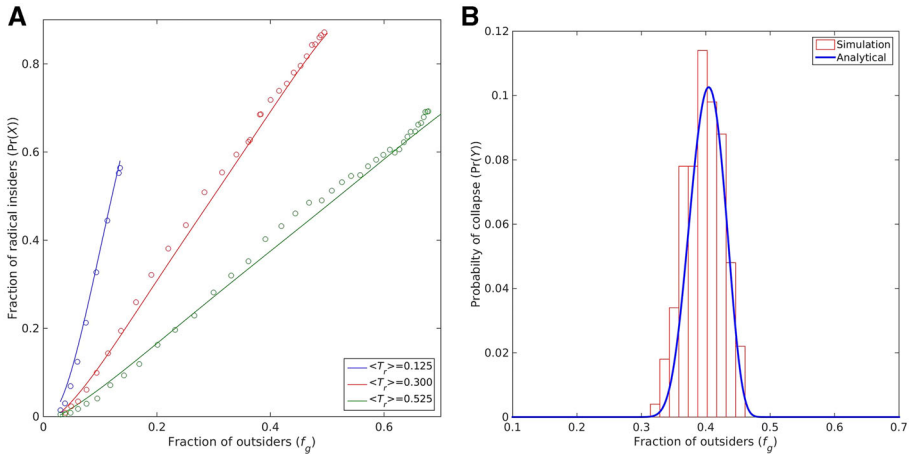
Fig. 2 Mean-field approximation of the implied social dynamics. **a** The fraction of radical insiders is a monotonically increasing function of the fraction of outsiders until the latter reach a saturation level. *Circles* indicate the simulation results. **b** The probability of herd behavior during which cooperation between insiders and outsiders collapses need not always increase with the fraction of outsiders because a high level of outsiders may preclude radicals and their tacit supporters from reaching the necessary majority. Here we study the case in which I_c is set to 50 % of the total population. The assimilation rate is $p_1 = 0.01$

Although analytical results help us understand the dynamics of the model, comprehensive mapping of the parameter space is often impossible without resorting to numerical simulations. Before detailing the results of these simulations, we emphasize how Fig. 1 shows that benevolence measured by fitness ratio R is a decreasing function of the fraction of outsiders f_g. A diminishing benevolence in the presence of a benefit differential is hardly surprising given that benevolent populations disappear by virtue of evolutionary dynamics. The question, therefore, is whether and under what conditions adjusting benevolence is sufficient to accommodate an incursion of outsiders without causing societal turmoil. Accordingly, we recognize three different types of outcomes in numerical simulations (Fig. 3):

> *Mutualism* is a set of equilibrium states reached by a smooth reduction of benevolence to a level at which insiders maintain a sustainable majority (continuous blue curve in Fig. 1).
> *Outsider dominance* is a set of equilibrium states wherein the outsiders form a majority.
> *Antagonism* is a set of non-equilibrium, absorbing states due to a complete breakdown of cooperation between the two subpopulations (discontinuous red curve in Fig. 1)

We find that over a considerable portion of the parameter space the tolerance of insiders is in balance with the assimilation of outsiders, and there is a smooth reduction of benevolence to a level at which insiders maintain a sustainable majority (see the *Mutualism* region in Fig. 3).

Outsiders dominate when there is an imbalance between their assimilation and the tolerance of insiders, i.e., the insiders maintain their benevolence at a very high level or for a very long time (see the *Outsider dominance* region in Fig. 3). If insider tolerance is insufficient relative to the assimilation of outsiders, the society becomes polarized, and cooperation between the two subpopulations breaks down completely [30,31] (see the *Antagonism* region in Fig. 3). From the perspective of the native insider population, the two latter outcomes are unsatisfactory. On the one hand, outsider dominance may lead to the disappearance of the
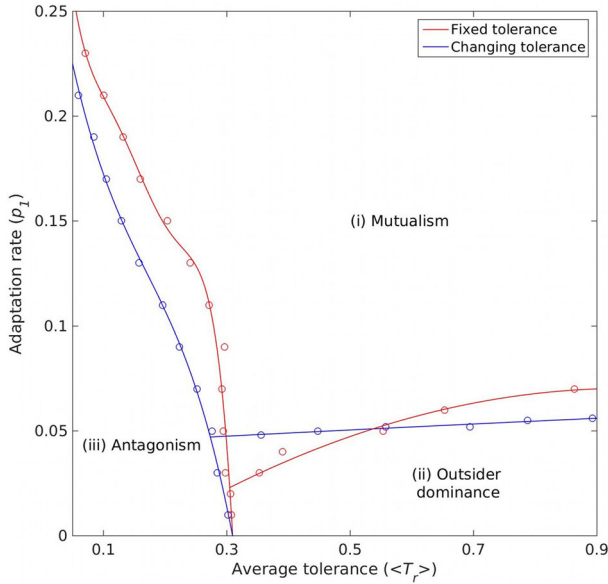
Fig. 3 Outcomes of the social dynamics. A benevolent society faces three possible outcomes of the social dynamics when dealing with an inflow of individuals from the outside: (i) benevolence is adjusted to a sustainable level for insiders to hold a majority in a balanced interplay between their tolerance and the assimilation of outsiders, (ii) benevolence is kept too high for too long and insiders turn to a minority owing to their excessive tolerance relative to the assimilation of outsiders, and (iii) the society is polarized due to a failure to adjust the benevolence of insufficiently tolerant insiders relative to the assimilation of outsiders. We studied cases in which the tolerance of insiders is unaffected (*red*) and affected (*blue*) by assimilated outsiders. Curves of best fit added as a visual aid

cultural patterns of insiders. On the other hand, antagonism leads to social turmoil, thus defeating the purpose of the initial incentive to attract outsiders.

The simplified structure of the present model makes it possible to find a necessary condition for the state of antagonism in an analytical form. We begin by noting that when the fraction of outsiders equals the average tolerance, i.e., when $f_g = \langle T_r \rangle$, approximately one-half the insiders become radicals [32,33]. This makes the fraction of radicals in the total population $f_r = 0.5(1 - f_g)$ and the fraction of tacit supporters among the remaining insiders $f_t = 0.5(1 - f_g)f_g$. Thus $f_r + f_t = 0.5(1 - f_g^2) < 50\%$ for any $0 < f_g \leq 1$, indicating that radicals and their tacit supporters achieve the majority needed to engage in herd behavior when $f_g > \langle T_r \rangle$. The condition for herd behavior, $f_r + f_t \geq 50\%$, is thus supplemented by $f_g = (1 + \epsilon)\langle T_r \rangle$ to yield $f_r = (0.5 + \epsilon) \times (1 - f_g)$, $f_t = (0.5 - \epsilon) \times (1 - f_g)f_g$, and $\epsilon \geq 0.5f_g^2/(1 - f_g)^2$. We find that herd behavior occurs only when the dynamics permit $\langle T_r \rangle \leq f_g(1 - f_g)^2/(1 - 2f_g + 1.5f_g^2)$ while still $f_g < 50\%$. When large p_1 values stop the fraction of outsiders from reaching values higher than $f_g \approx 0.2$, the condition for herd behavior simplifies to $\langle T_r \rangle \leq f_g$.

Although the tolerance of the original insider population is preserved in this case—i.e., outsiders who assimilate insider cultural patterns become a part of the same distribution of tolerance—this is not always the outcome. Assimilated outsiders may either be more tolerant towards "their own" than the average insider, or they may want to radically separate from their own past. Lacking reliable data, we study a scenario in which the tolerance of an assimilated outsider is determined using a uniform distribution between zero and unity

(see the blue curves in Fig. 3). Thus tolerance turns into a dynamic variable that, depending on the initial average tolerance, may increase or decrease the tolerance of the society. Using the simplified condition for herd behavior, $\langle T_r \rangle \leq f_g$, we focus on the "race" between the tolerance level and the fraction of outsiders to determine whether a society splits into two non-interacting subpopulations or ends up being dominated by one of them. We can understand this qualitatively by assuming that both $\langle T_r \rangle$ and f_g grow over time linearly. Thus $\langle T_r \rangle(t) = \langle T_r \rangle(0) + v_T t$ and $f_g(t) = f_g(0) + v_g t$, where $\langle T_r \rangle(0)$ and $f_g(0)$ denote the initial values of $\langle T_r \rangle$ and f_g, respectively, and v_T and v_g are the corresponding growth rates. Herd behavior and a subsequent breakdown ensue when

$$f_g(t) \geq \frac{\langle T_r \rangle(0) - f_g(0)}{1 - \frac{v_T}{v_g}} + f_g(0). \tag{3}$$

In other words, a small initial average tolerance value and a rapid outsider growth rate increases the probability that there will be a breakdown.

Another scenario deserving attention is that the assimilation process may require a minimum amount of time. Accordingly, only outsiders that entered the system at a given moment in the past could be assimilated into insiders. We analyzed the delay in the assimilation process by means of numerical simulations and described the results in SI text.

3 Data

We are using the terms "insider" and "outsider" metaphorically, but the model can be applied to any situation with a built-in asymmetry, e.g., if a majority provides support to a minority. When the European Parliament recently discussed the migration of refugees into Europe, they concluded that this current global trend is not just short-term or temporary. When the question of whether to increase the number of immigrants from the Middle East and sub-Saharan Africa was presented, some EU countries voted against any additional immigration [34]. If we assume that these votes are proxies for the level of a country's tolerance towards minorities and use our model to interpret the outcome of the voting, we find that (i) different countries have different levels of tolerance (parameters $T_r^{\min}$ and σ in the model) and (ii) some countries have already reached a critical point in which anti-immigrant radicals and tacit supporters are in the majority.

We extend this analogy and use the Migrant Integration Policy Index (MIPEX) [35] as a proxy for the assimilation rate (parameter p_1) and plot the data in Fig. 4 using the same procedure as in Fig. 3. In this manner, x-axes (y-axes) of both figures point in the direction of increasing tolerance (assimilation), controlled by parameter σ (p_1) in Fig. 3 and represented by the fraction of pro-immigrant votes (MIPEX) in Fig. 4. We find that openly anti-immigrant countries end up in the left lower corner, as expected from the results of the model, but that we can obtain the EU country borderlines that delineate the different dynamical regimes only when information on the distribution of tolerance—presently approximated by a uniform distribution in the model—is available.

The lack of any clear correlation in Fig. 4 is indicative of the complexity of the real world in that at least one additional, and possibly more, explanatory factor(s) affect the relationship between assimilation and tolerance. However, despite being a very rough approximation of reality, the scatter pattern in Fig. 4 is rather informative when interpreted in conjunction with the model results.
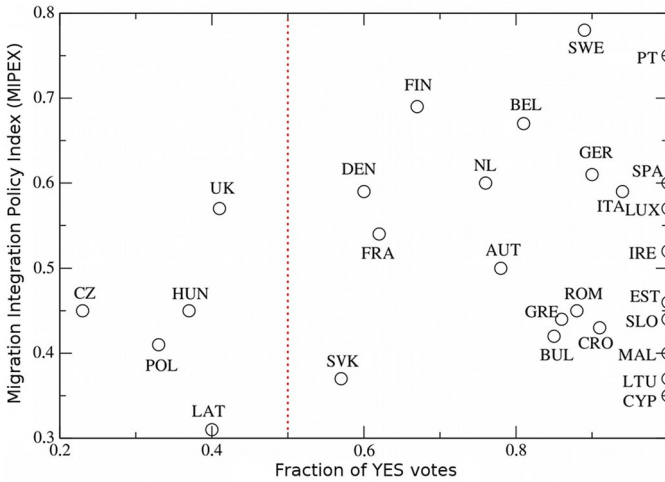
Fig. 4 Proxies for assimilation and tolerance in the real world: the case of the EU countries. The net assimilation rate is quantified by MIPEX index, whereas the average tolerance is quantified by the fraction of pro-immigration votes. A considerable scatter emphasizes that countries differ in preparedness to accept immigrants (parameter p_1) and their levels of tolerance (parameters $T_r^{\min}$ and σ). Left of the *red dashed line* are the countries that are openly anti-immigrant

Three groups of countries are identifiable in Fig. 4. The countries historically exposed to large immigration inflows (e.g., the Netherlands, Portugal, Spain etc.) are both more tolerant and better equipped to receive immigrants. These countries appear in the upper-right corner of the plot, corresponding to the *Mutualism* domain in Fig. 3. By contrast, the countries in the lower-left corner (the Czech Republic, Hungary, Poland, and Latvia), corresponding to the *Antagonism* domain in Fig. 3, are former socialist states, ethnically and culturally homogeneous, and lacking substantial immigration. These countries therefore felt little pressure to enact effective immigration policies, which is reflected in their low proxy for assimilation. Furthermore, a low proxy for tolerance is less surprising if a country is unaccustomed to a large direct inflow of immigrants (e.g., Hungary) or, for that matter, to large inflows into neighboring countries (e.g., the Czech Republic and Poland in relation to Germany).

The third group of countries is found in the lower-right corner of Fig. 4, corresponding to the *Outsider dominance* domain in Fig. 3. Among the members of this group are countries that maintained ethnic and cultural homogeneity, and/or lacked substantial immigration, which is consistent with a low proxy for assimilation. As for the high tolerance, most of these countries (not Greece though) were largely unaffected by the immigration wave prior to the EU vote because of, among other reasons, geographic locations off the main migration pathways (e.g., Estonia, Lithuania, and Ireland, but also Romania, and at the time Croatia and Slovenia). Since the vote, however, some countries were struck by the crisis more seriously and at least one (Slovenia) reversed its previous policy. The overall conclusion is that the scatter pattern in Fig. 4 is reminiscent of the domains in Fig. 3.

When an incursion of outsiders decreases the level of benevolence in a benevolent society this decrease appears to be related to the radicalization process (see Fig. 5). For example, in the 1980s there was a dramatic increase in the number of refugees and illegal immigrants entering European countries and by the early 1990s this had provoked a wave of radical
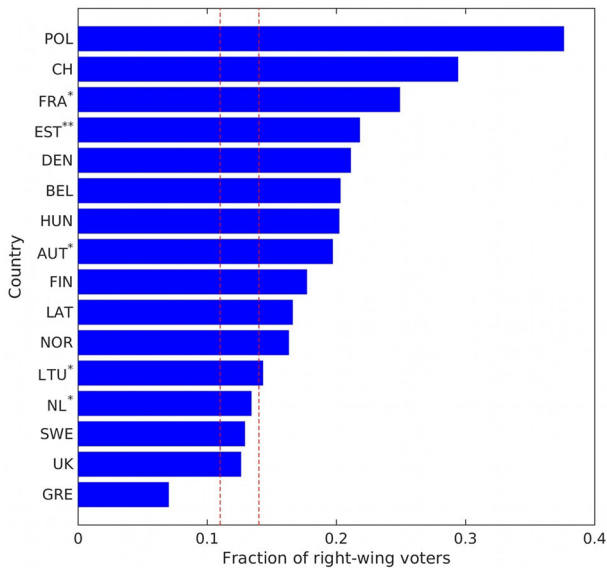
Fig. 5 Success of the right-wing nationalist parties during the recent elections across Europe. Listed are the European countries that contained more than 7 % of right-wing nationalist voters in nationwide elections held in 2014 or 2015. The fraction of right-wing nationalist voters in many countries exceeded the percentage of the population that "was troubled by the presence of people of other nationality, race, or religion" in Europe in the early 1990s [36] (range indicated by the *red dashed lines*). Only parties that openly oppose immigration were included. *For countries where national elections were held before 2014, we used the results of the last European parliament elections. **In Estonia, the results of the two major right-wing nationalist parties were summed

right-wing populism [36]. Between 11 and 14 % of the population of Europe found other nationalities, races, or religions unsettling, and this group became a major source of votes for right-wing nationalist parties [36]. The recent migration trend brought a new wave of success for right-wing nationalists, suggesting that the population of many European countries has been radicalized beyond that recorded in the early 1990s (see Fig. 5). The fraction of right-wing voters in a given country may not correlate with the fraction of immigrants because (i) the EU is a supra-national entity without internal borders between the member states, and (ii) EU countries are not equally tolerant, i.e., each country exhibits its own value of parameter σ. Note that supporters of right-wing nationalist parties seem to originate from two social groups [36], (i) those in competition with non-European immigrants for jobs and housing (identified as radicals in our model), and (ii) the "new middle class" consisting of highly educated, often self-employed, yet politically restless youth who are less likely to admit belonging to the political right (the tacit supporters in our model).

4 Discussion

We have devised and analyzed a social dynamics model in which the benevolence level of an insider population decreases in response to an incursion of outsiders. Because this decrease is a result of radicalization, it often generates an increase in violent incidents [28,37–39]. This increase can function as early-warning signal that the society is approaching a critical point

in which all relations between insiders and outsiders abruptly terminate. The heterogeneity in the tolerance among European countries can mean that the EU tolerance is as strong as its weakest link, and that if the cooperation in the least tolerant EU country abruptly terminates this breakdown can trigger a cascade of societal breakdowns through other EU countries. This cascade phenomenon is a general characteristic of complex systems, and thus cooperation as a counterforce is essential if societies are to continue functioning [30,31]. The EU is currently facing an increase in religious intolerance and a huge inflow of immigrants, and some societies are responding by raising walls or fences on their borders, a strong signal that these societies have already reached a critical point. It is thus extremely important that we understand how a society reacts to the increasing inflow of outsiders and identify the links that produce an increase in violent incidents. Understanding the underlying social dynamics may help policy makers undertake legislative actions that will prevent potential conflicts on a much larger scale. Nations need to strike a balance between the short-term benefit of accepting immigrants and any potential long-term violence.

One limitation of our approach is that we give the population topology a secondary role. We assume (i) that each member of each subpopulation (insiders and outsiders) is equally able to assimilate the cultural patterns of the other subpopulation and (ii) that new agents who enter the system as a part of population growth make friends randomly, irrespective of identity tags. Because of homophily [40], we envision adding a preferential attachment (PA) factor [41] which makes new outsiders more likely to connect to older outsiders and new insiders more likely to connect to older insiders. Using PA changes the topology of the population, and hubs of predominantly single subpopulation agents appear. In this structure will the assimilation rate be the same for, e.g., an outsider surrounded by outsiders and an outsider in a mixed neighborhood? A more realistic treatment of the assimilation issue under PA may be the formalism of Watts [25] or extension of Watts [42], i.e., an agent may be more likely to change their tag if they are befriended by an increasing number of agents carrying the opposite tag. These considerations again have important implications for a country's immigration policy. In France, for example, large immigrant communities exist in Greater Paris, Lyon, and Marseille, suggesting that immigration indeed tends to be a PA-type process. The same is true in much of Western Europe, suggesting that the assimilation rate is severely impeded by the topology of a mixed native and immigrant population. The result is two ethnically-split subpopulations, which, in light of the work of Esteban, Mayoral, and Ray [43], is a dangerous combination, i.e., two measures of ethnic division— fractionalization and polarization—jointly influence an increase in violent incidents and the possibility of intrastate wars. A potential measure against that outcome may be exemplified by Singapore, where tenants in government-built housing (which is 88% of all housing) must be of mixed ethnic origin.

Modulating the asymmetry of interactions to attain a preferred evolutionary result is found not only in human, but in animal "societies" as well. For instance, a benefit differential similar to our Δ_b arises in mutualistic relationships in nature if some symbionts provide less benefit to a host than others, and the host cannot discriminate between them [44]. The local stability of the cooperative state may be maintained by a non-equilibrium mechanism of density-dependent interference competition among symbionts [45]. However, the local stability means that mutualism between the host and symbionts may turn into antagonism for a period of time due to the outside (environmental) conditions. To stabilize mutualism, hosts generally punish less cooperative symbionts [44,46], thus pushing the system to a cooperative steady-state. Reminiscent of Popper's statement, the host cannot maintain unlimited tolerance towards less cooperative symbionts indefinitely.

Mathematical modeling helps us understand the relationships between the main forces shaping a phenomenon of interest. The society, however, is extremely complex in the sense that human interactions are inhomogeneous, processes occur at multiple time scales, and state variables—e.g., size, production, and wealth—are unconserved quantities. It is therefore inappropriate to treat the results of mathematical modeling as a substitute for an informed discussion between policymakers. At best, modeling results aid such discussions by making them more informed.

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