Persistence and uncertainty in the academic career

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Understanding how institutional changes within academia may affect the overall potential of science requires a better quantitative representation of how careers evolve over time. Because knowledge spillovers, cumulative advantage, competition, and collaboration are distinctive features of the academic profession, both the employment relationship and the procedures for assigning recognition and allocating funding should be designed to account for these factors. We study the annual production $n_i(t)$ of a given scientist i by analyzing longitudinal career data for 200 leading scientists and 100 assistant professors from the physics community. Our empirical analysis of individual productivity dynamics shows that (*i*) there are increasing returns for the top individuals within the competitive cohort, and that (*ii*) the distribution of production growth is a leptokurtic "tent-shaped" distribution that is remarkably symmetric. Our methodology is general, and we speculate that similar features appear in other disciplines where academic publication is essential and collaboration is a key feature. We introduce a model of proportional growth which reproduces these two observations, and additionally accounts for the significantly right-skewed distributions of career longevity and achievement in science. Using this theoretical model, we show that short-term contracts can amplify the effects of competition and uncertainty making careers more vulnerable to early termination, not necessarily due to lack of individual talent and persistence, but because of random negative production shocks. We show that fluctuations in scientific production are quantitatively related to a scientist's collaboration radius and team efficiency.

career trajectory | labor market | science of science | tenure | computational sociology

Institutional change could alter the relationship between science and scientists as well as the longstanding patronage system in academia (1, 2). Some recent shifts in academia include the changing business structure of research universities (3), shifts in the labor supply demand balance (4), a bottleneck in the number of tenure track positions (5), and a related policy shift away from long-term contracts (3, 6). Along these lines, significant factors for consideration are the increasing range in research team size (7), the economic organization required to fund and review collaborative research projects, and the evolving definition of the role of the academic research professor (3).

The role of individual performance metrics in career appraisal, in domains as diverse as sports (8, 9), finance (10, 11), and academia, is increasing in this data rich age. In the case of academia, as the typical size of scientific collaborations increases (7), the allocation of funding and the association of recognition at the varying scales of science [individual $\leftrightarrows$ group $\leftrightarrows$ institution (12)] has become more complex. Indeed, scientific achievement is becoming increasingly linked to online visibility in a considerable reputation tournament (13).

Here we seek to identify (*i*) quantitative patterns in the scientific career trajectory towards a better understanding of career dynamics and achievement (14–20), and (*ii*) how scientific production responds to policies concerning contract length. Using rich productivity data available at the level of single individuals, we analyze longitudinal career data keeping in mind the roles of spillovers, group size, and career sustainability. Although our empirical analysis is limited to careers in physics, our approach is general. We speculate that similar features describe other disciplines where academic publication is a primary indicator and collaboration is a key feature.

Specifically, we analyze production data for 300 physicists $i = 1 \ldots 300$ who are distributed into 3 groups: (*i*) Group A corresponds to the 100 most cited physicists with average h-index $\langle h \rangle = 61 \pm 21$, (*ii*) Group B corresponds to 100 additional highly cited physicists with $\langle h \rangle = 44 \pm 15$, and (*iii*) Group C corresponds to 100 assistant professors in 50 US physics departments with $\langle h \rangle = 15 \pm 7$. We define the annual production $n_i(t)$ as the number of papers published by scientist i in year t of his/her career. We focus on academic careers from the physics community to approximately control for significant cross-disciplinary production variations. Using the same set of scientists, a companion study has analyzed the rank-ordered citation distribution of each scientist with a focus on the statistical regularities underlying publication impact (17). We provide further description of the data and present a parallel analysis of 21,156 sports careers in *SI Appendix*.

We begin this paper with empirical analysis of longitudinal career data. Our empirical evidences serve as statistical benchmarks used in the final section where we develop a stochastic proportional growth model. In particular, our model shows that a short-term appraisal system can result in a significant number of "sudden" early deaths due to unavoidable negative production shocks. This result is consistent with a Matthew Effect model (16) and recent academic career survival analysis (21), which demonstrate how young careers can be stymied by the difficulty in overcoming early achievement barriers. Altogether, our results indicate that short-term contracts may increase the strength of the "rich-get-richer" mechanism in science (22, 23) and may hinder the upward mobility of young scientists.

Results

Scientific Production and the Career Trajectory. The academic career depends on many factors, such as cumulative advantage (16, 19, 22, 23), the "sacred spark," (24, 25), and other complex aspects of knowledge transfer manifest in our techno-social world (26). To exemplify this complexity, a recent case study on the impact trajectories of Nobel prize winners shows that "scientific career shocks" marked by the publication of an individual's "magnum opus" work(s) can trigger future recognition and reward, resembling the cascading dynamics of earthquakes (27).

We model the career trajectory as a sequence of scientific outputs which arrive at the variable rate $n_i(t)$. Because the reputation of a scientist is typically a cumulative representation of his/her

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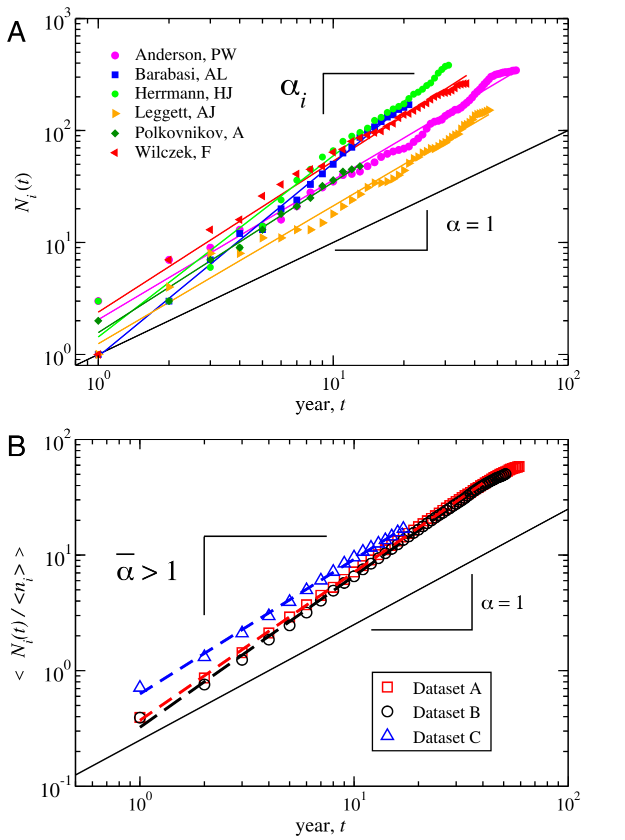

Fig. 1. Persistent accelerating career growth. (*A*) The career trajectory $N_i(t) \sim t^{\alpha_i}$ of six stellar careers from varying age cohorts. The α_i value characterizes the career persistence, where careers with $\alpha > 1$ are accelerating. α_i values calculated using OLS regression in alphabetical order are: $\alpha = 1.25 \pm 0.02$, $\alpha = 1.72 \pm 0.02$, $\alpha = 1.62 \pm 0.04$, $\alpha = 1.23 \pm 0.02$, $\alpha = 1.34 \pm 0.05$, $\alpha = 1.35 \pm 0.04$. (*B*) Defined in Eq. **1**, the average career trajectory $\langle N'(t) \rangle$ calculated from 100 individual $N_i(t)$ in each dataset demonstrates robust accelerating career growth within each cohort. We use the normalized career trajectory $N_i'(t)$ in order to aggregate $N_i(t)$ with varying publication rates $\langle n_i \rangle$. As a result, the aggregate scaling exponent $\bar{\alpha}$ quantifies the acceleration of the typical career over time, independent of $\langle n_i \rangle$. For the scientific careers, we calculate $\bar{\alpha}$ values: 1.28 ± 0.01 [A], 1.31 ± 0.01 [B], and 1.15 ± 0.02 [C]. These values are all significantly greater than unity, $\bar{\alpha} > 1$, indicating that cumulative advantage in science is closely related to knowledge and production spillovers. We calculate $\bar{\alpha}$ using OLS regression and plot the corresponding best-fit lines (dashed) for each dataset.

contributions, we consider the cumulative production $N_i(t) \equiv \sum_{t'=1}^{t} n_i(t')$ as a proxy for career achievement. Fig. 1*A* shows the cumulative production $N_i(t)$ of six notable careers which display a temporal scaling relation $N_i(t) \approx A_i t^{\alpha_i}$ where α_i is a scaling exponent that quantifies the career trajectory dynamics. The average and standard deviation of the α_i values calculated for each dataset are $\langle \alpha_i \rangle = 1.42 \pm 0.29$ [A], 1.44 ± 0.26 [B], and 1.30 ± 0.31 [C]. We justify this two-parameter model in the *SI Appendix* text using scaling methods and data collapse.

There are also numerous cases of $N_i(t)$ which do not exhibit such regularity (see *SI Appendix: Fig. S1*), but instead display marked nonstationarity and nonlinearity arising from significant exogenous career shocks. Positive shocks, possibly corresponding to just a single discovery, can spur significant productivity and reputation growth (24, 27). Negative shocks, such as in the case of scientific fraud, can end the career rather suddenly. We also acknowledge that the end of the career is a difficult phase to analyze, because such an event can occur quite abruptly, and so our analysis is mainly concerned with the growth phase and not the termination phase.

In order to analyze the average properties of $N_i(t)$ for all 300 scientists in our sample, we define the normalized trajectory $N_i'(t) \equiv N_i(t)/\langle n_i \rangle$. The quantity $\langle n_i \rangle$ is the average annual production of author i, with $N_i'(L_i) = L_i$ by construction (L_i corresponds to the career length of individual i). Fig. 1*B* shows the characteristic production trajectory obtained by averaging together the 100 $N_i'(t)$ belonging to each dataset,

$$\langle N'(t) \rangle \equiv \left\langle \frac{N_i(t)}{\langle n_i \rangle} \right\rangle \equiv \frac{1}{100} \sum_{i=1}^{100} \frac{N_i(t)}{\langle n_i \rangle}. \quad \textbf{[1]}$$

The standard deviation $\sigma(N'(t))$ shown in *SI Appendix: Fig. S2*B begins to decrease after roughly 20 y for dataset [A] and [B] scientists. Over this horizon, the stochastic arrival of career shocks can significantly alter the career trajectory (20, 24, 27, 28).

Each $N_i'(t)$ exhibits robust scaling corresponding to the scaling law $\langle N'(t) \rangle \sim t^{\bar{\alpha}}$. This regularity reflects the abundance of careers with $\alpha_i > 1$ corresponding to accelerated career growth. This acceleration is consistent with increasing returns arising from knowledge and production spillovers.

Fluctuations in Scientific Output over the Academic Career. Individuals are constantly entering and exiting the professional market, with birth and death rates depending on complex economic and institutional factors. Due to competition, decisions and performance at the early stages of the career can have long lasting consequences (16, 29). To better understand career uncertainty portrayed by the common saying "publish or perish" (30), we analyze the outcome fluctuation

$$r_i(t) \equiv n_i(t) - n_i(t - \Delta t) \quad \textbf{[2]}$$

of career i in year t over the time interval $\Delta t = 1$ y. Fig. 2 *A* and *B* show the unconditional probability density function (pdf) of r values which are leptokurtic but remarkably symmetric, illustrating the endogenous frequencies of positive and negative output growth. Output fluctuations arise naturally from the lulls and bursts in both the mental and physical capabilities of humans (31, 32). Moreover, the statistical regularities in the annual production change distribution indicate a striking resemblance to the growth rate distribution of countries, firms, and universities (33, 34).

To better account for individual growth factors, we next define the normalized production change

$$r_i'(t) \equiv [r_i(t) - \langle r_i \rangle]/\sigma_i(r) \quad \textbf{[3]}$$

which is measured in units of the fluctuation scale $\sigma_i(r)$ unique to each career. We measure the average $\langle r_i \rangle$ and the standard deviation $\sigma_i(r)$ of each career using the first L_i available years for each scientist *i*. $r_i'(t)$ is a better measure for comparing career uncertainty, because individuals have production factors that depend on the type of research, the size of the collaboration team, and the position within the team. Fig. 2*C* shows that $P(r')$, the pdf of r' measured in units of standard deviation, is well approximated by a Gaussian distribution with unit variance. The data collapse of each $P(r')$ onto the predicted Gaussian distribution (solid green curve) indicates that individual output fluctuations are consistent with a proportional growth model. We note that the remaining deviations in the tails for $|r'| \geq 3$ are likely signatures of the exogenous career shocks that are not accounted for by an endogenous proportional growth model.

The ability to collaborate on large projects, both in close working teams and in extreme examples as remote agents [i.e. Wikipedia (35)], is one of the foremost properties of human society. In science, the ability to attract future opportunities is strongly related to production and knowledge spillovers

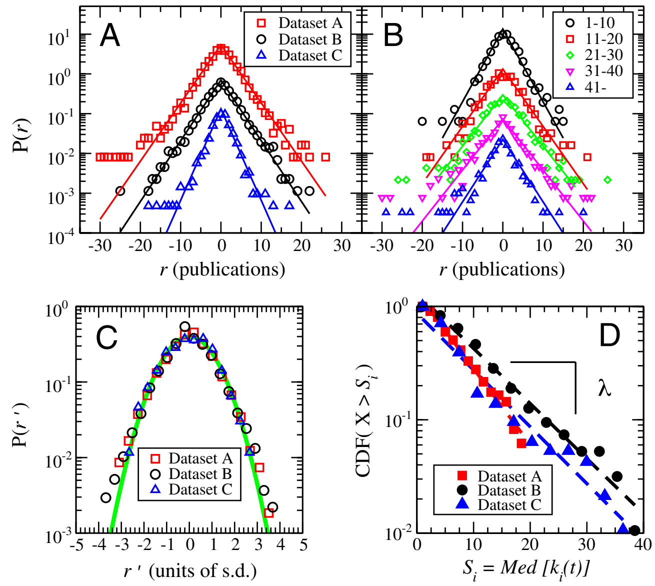

Fig. 2. Empirical evidence for the proportional growth model of career production. (*A*) pdf of the annual production change r in the number of papers published over a $\Delta t = 1$ y period. In the bulk of each $P(r)$, the growth distribution is approximately double-exponential (Laplace). (*B*) To test the stability of the distribution over career trajectory subintervals, we separate $r_i(t)$ values into 5 nonoverlapping 10-year periods and verify the stability of the Laplace $P(r)$. For each $P(r)$, we also plot the corresponding Laplace distribution (solid line) with standard deviation σ and mean $\mu \approx 0$ calculated using the maximum likelihood estimator method. To improve graphical clarity, we vertically offset each $P(r)$ by a constant factor. For visual comparison, we also plot a Normal distribution (dashed black curve) with $\sigma \equiv 1$ which instead decays parabolically on the log-linear axes. (*C*) Accounting for individual production factors by using the normalized production change r', the resulting pdfs $P(r')$ collapse onto a Gaussian distribution with unit variance. Deviations in the tails likely correspond to extreme "career shocks." (*D*) The cumulative distribution $CDF(X \geq S_i)$ is exponential, indicating that the unconditional distributions $P(r)$ in (*A*) and (*B*) follow from an exponential mixing of conditional Gaussian distributions $P(r|S_i)$.

(28, 36, 37) that are facilitated by the collaboration network (7, 12, 38–42). Indeed, there is a tipping point in a scientific career that occurs when a scientist's knowledge investment reaches a critical mass that can sustain production over a long horizon, and when a scientist becomes an attractor (as opposed to a pursuer) of new collaboration/production opportunities. To account for collaboration, we calculate for each author the number $k_i(t)$ of distinct coauthors per year and then define his/her collaboration radius S_i as the median of the set of his/her $k_i(t)$ values, $S_i \equiv Med[k_i(t)]$. We use the median instead of the average $\langle k_i(t)\rangle$ because extremely large $k_i(t)$ values can occur in specific fields such as high-energy physics and astronomy.

Given the complex scientific coauthorship network, we ask the question: what is the typical number of unique coauthors per year? Fig. 2*D* shows the cumulative distribution function $CDF(S_i)$ of S_i values for each dataset. The approximately linear form on log-linear axes indicates that S_i is exponentially distributed, $P(S_i) \sim \exp[-\lambda S_i]$. We calculate $\lambda = 0.15 \pm 0.01$ [A], $\lambda = 0.11 \pm 0.01$ [B], and $\lambda = 0.11 \pm 0.01$ [C]. The exponential size distribution has been shown to emerge in complex systems where linear preferential attachment governs the acquisition of new opportunities (43). This result shows that the leptokurtic "tent-shaped" distribution $P(r)$ in Fig. 2 follows from the exponential mixing of heterogenous conditional Gaussian distributions (44).

The exponential mixture of Gaussians decomposes the unconditional distribution $P(r)$ into a mixture of conditional Gaussian distributions

$$P(r|S_i) = \exp[-r^2/2VS_i^{\psi}]/\sqrt{2\pi V S_i^{\psi}}, \qquad [4]$$

each with a fluctuation scale $\sigma_i(r)$ depending on S_i by the scaling relation

$$\sigma_i^2(r) \approx V S_i^{\psi}. \qquad [5]$$

Hence, the mixture is parameterized by ψ

$$P_{\psi}(r) = \int_0^{\infty} P(r|S)P(S)dS \approx \sum_{i=1} P_i(r|S_i)P(S_i). \qquad [6]$$

The independent case $\psi = 0$ results in a Gaussian $P_{\psi}(r)$ and the linear case $\psi = 1$ results in a Laplace (double-exponential) $P_{\psi}(r)$. See *SI Appendix* and ref. 44 for further discussion of the ψ dependence of $P_{\psi}(r)$.

The Size-Variance Relation and Group Efficiency. The values of ψ for scientific and athletic careers follow from the different combination of physical and intellectual inputs that enter the production function for the two distinct professions. Academic knowledge is typically a nonrival good, and so knowledge-intensive professions are characterized by spillovers, both over time and across collaborations (36, 37), consistent with $\alpha_i > 1$ and $\psi > 0$. Interestingly, Azoulay, et al. show evidence for production spillovers in the 5–8% decrease in output by scientists who were close collaborators with a "superstar" scientists who died suddenly (28).

We now formalize the quantitative link between scientific collaboration (38, 39) and career growth given by the size-variance scaling relation in Eq. **5** visualized in the scatter plot in Fig. 3*B*. Using ordinary least squares (OLS) regression of the data on log-log scale, we calculate $\psi/2 \approx 0.40 \pm 0.03$ $(R = 0.77)$ for dataset [A], $\psi/2 \approx 0.22 \pm 0.04$ $(R = 0.51)$ [B], and $\psi/2 \approx 0.26 \pm 0.05$ $(R = 0.45)$ [C]. Interdependent tasks that are characteristic of group collaborations typically involve partially overlapping efforts. Hence, the empirical ψ values are significantly less than the value $\psi = 1$ that one would expect from the sum of S_i independent random variables with approximately equal variance V. Collectively, these empirical evidences serve as coherent motivations for the preferential capture growth model that we propose in the following section.

Alternatively, it is also possible to estimate ψ using the relation between the average annual production $\langle n_i\rangle$ and the collaboration radius S_i. The input-output relation $\langle n_i\rangle \sim S_i^{\psi}$ quantifies the collaboration efficiency, with $\psi = 0.74 \pm 0.04$ $(R = 0.87)$ for dataset [A] and $\psi = 0.25 \pm 0.04$ $(R = 0.37)$ for dataset [B]. If the autocorrelation between sequential production values $n_i(t)$ and $n_i(t+1)$ is relatively small, then we expect the scaling exponents calculated for $\langle n_i\rangle$ and $\sigma_i^2(r)$ to be approximately equal. This result follows from considering $r_i(t)$ as the convolution of an underlying production distribution $P_i(n)$ for each scientist that is approximately stable. Interestingly, the larger ψ values calculated for dataset [A] scientists suggests that prestige is related to the increasing returns in the scientific production function (45).

Next we use an alternative method to estimate the annual collaboration efficiency by relating the number of publications $n_i(t)$ in a given year to the number of distinct coauthors $k_i(t)$ over the same year. We use a single-factor production function,

$$n_i(t) \approx q_i[k_i(t)]^{\gamma_i}, \qquad [7]$$

to quantify the relation between output and labor inputs with a scaling exponent γ_i. We estimate q_i and γ_i for each author using OLS regression, and define the normalized output measure $Q_i \propto n_i(t)/k_i(t)^{\gamma_i}$ using the best-fit q_i and γ_i values calculated for each scientist i. Fig. 3*C* shows the efficiency parameter γ

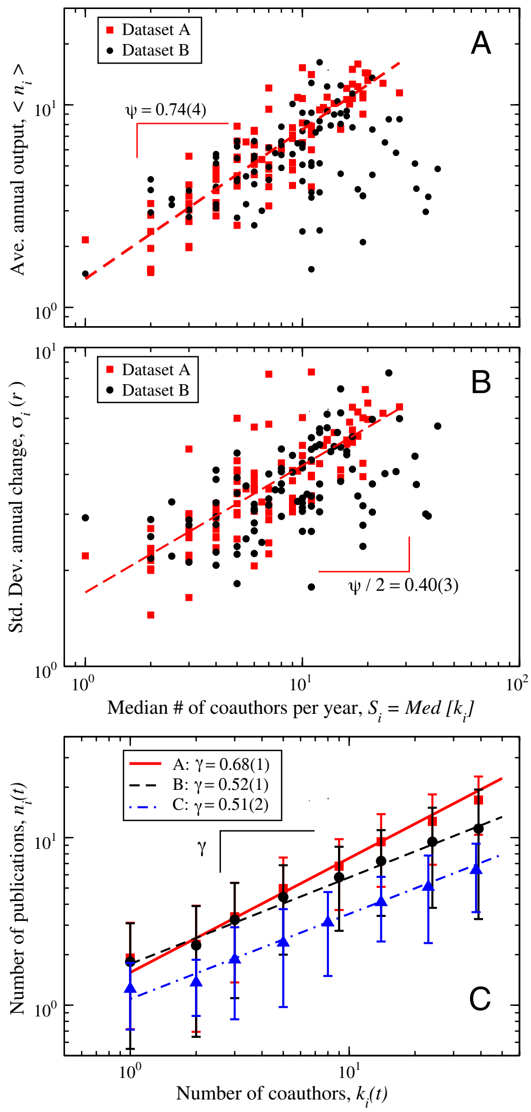

Fig. 3. Quantitative relations between career growth, career risk, and collaboration efficiency. The fluctuations in production reflect the unpredictable horizon of "career shocks" which can affect the ability of a scientists to access new creative opportunities. (*A*) Relation between average annual production $\langle n_i \rangle$ and collaboration radius $S_i \equiv Med[k_i]$ shows a decreasing marginal output per collaborator as demonstrated by sublinear $\psi < 1$. Interestingly, dataset [A] scientists have on average a larger output-to-input efficiency. (*B*) The production fluctuation scale $\sigma_i(r)$ is a quantitative measure for uncertainty in academic careers, with scaling relation $\sigma_i(r) \sim S_i^{\psi/2}$. (*C*) Management, coordination, and training inefficiencies can result in a $\gamma < 1$ corresponding to a decreasing marginal return with each additional coauthor input. The significantly larger γ value for dataset [A] scientists seems to suggest that managerial abilities related to output efficiency is a common attribute of top scientists.

calculated by aggregating all careers in each dataset, and indicates that this aggregate γ is approximately equal to the average $\langle \gamma_i \rangle$ calculated from the γ_i values in each career dataset: $\gamma = 0.68 \pm 0.01$ [A], $\gamma = 0.52 \pm 0.01$ [B], and $\gamma = 0.51 \pm 0.02$ [C]. Furthermore, the ψ and γ values are approximately equal, which is not surprising, because both scaling exponents are efficiency measures that relate the scaling relation of output $n_i(t)$ per input $k_i(t)$.

A Proportional Growth Model for Scientific Output. We develop a stochastic model as a heuristic tool to better understand the effects of long-term vs. short-term contracts. In this competition model, opportunities (i.e., new scientific publications) are captured according to a general mechanism whereby the capture rate $\mathscr{P}_i(t)$ depends on the appraisal $w_i(t)$ of an individual's record of achievement over a prescribed history. We define the appraisal to be an exponentially weighted average over a given individual's history of production

$$w_i(t) \equiv \sum_{\Delta t=1}^{t-1} n_i(t-\Delta t) e^{-c\Delta t}, \quad [8]$$

which is characterized by the appraisal horizon $1/c$. We use the value $c = 0$ to represent a long-term appraisal (tenure) system and a value $c \gg 1$ to represent a short-term appraisal system. Each agent $i = 1 \ldots I$ simultaneously attracts new opportunities at a rate

$$\mathscr{P}_i(t) = \frac{w_i(t)^\pi}{\sum_{i=1}^{I} w_i(t)^\pi} \quad [9]$$

until all P opportunities for a given period t are captured. We assume that each agent has the production potential of one unit per period, and so the total number of opportunities distributed per period P is equal to the number of competing agents, $P \equiv I$.

We use Monte Carlo (MC) simulation to analyze this two-parameter model over the course of $t = 1 \ldots T$ sequential periods. In each production period (i.e., representing a characteristic time to publication), a fixed number of P production units are captured by the competing agents. At the end of each period, we update each $w_i(t)$ and then proceed to simulate the next preferential capture period $t+1$. Because $\mathscr{P}_i(t)$ depends on the relative achievements of every agent, the relative competitive advantage of one individual over another is determined by the parameter π. In the *SI Appendix* we elaborate in more detail the results of our simulation of synthetic careers dynamics. We vary π and c for a labor force of size $I \equiv 1{,}000$ and maximum lifetime $T \equiv 100$ periods as a representative size and duration of a real labor cohort. Our results are general, and for sufficiently large system size, the qualitative features of the results do not depend significantly on the choice of I or T.

The case with $\pi = 0$ corresponds to a random capture model that has (*i*) no appraisal and (*ii*) no preferential capture. Hence, in this null model, opportunities are captured at a Poisson rate $\lambda_p = 1$ per period. The results of this model (see *SI Appendix: Fig. S13*) show that almost all careers obtain the maximum career length T with a typical career trajectory exponent $\langle \alpha_i \rangle \approx 1$. Comparing to simulations with $\pi > 0$ and $c \geq 0$, the null model is similar to a "long-term" appraisal system ($c \to 0$) with sublinear preferential capture ($\pi < 1$). In such systems, the long-term appraisal time scale averages out fluctuations, and so careers are significantly less vulnerable to periods of low production and hence more sustainable because they are not determined primarily by early career fluctuations.

However, as π increases, the strength of competitive advantage in the system increases, and so some careers are "squeezed out" by the larger more dominant careers. This effect is compounded by short-term appraisal corresponding to $c \approx 1$. In such systems with superlinear capture rates and/or relatively large c, most individuals experience "sudden death" termination relatively early in the career. Meanwhile, a small number of "stars" survive the initial selection process, which is governed primarily by random chance, and dominate the system.

We found drastically different lifetime distributions when we varied the appraisal (contract) length (see *SI Appendix: Figs. S12–S16*). In the case of linear preferential capture with a long-term appraisal system $c = 0$, we find that 10% of the labor population terminates before reaching career age $0.94T$ (where T is the maximum career length or "retirement age"), and only 25% of the labor population terminates before reaching career age $0.98T$. On the contrary, in a short-term appraisal system with $c = 1$, we find that 10% of the labor population terminates

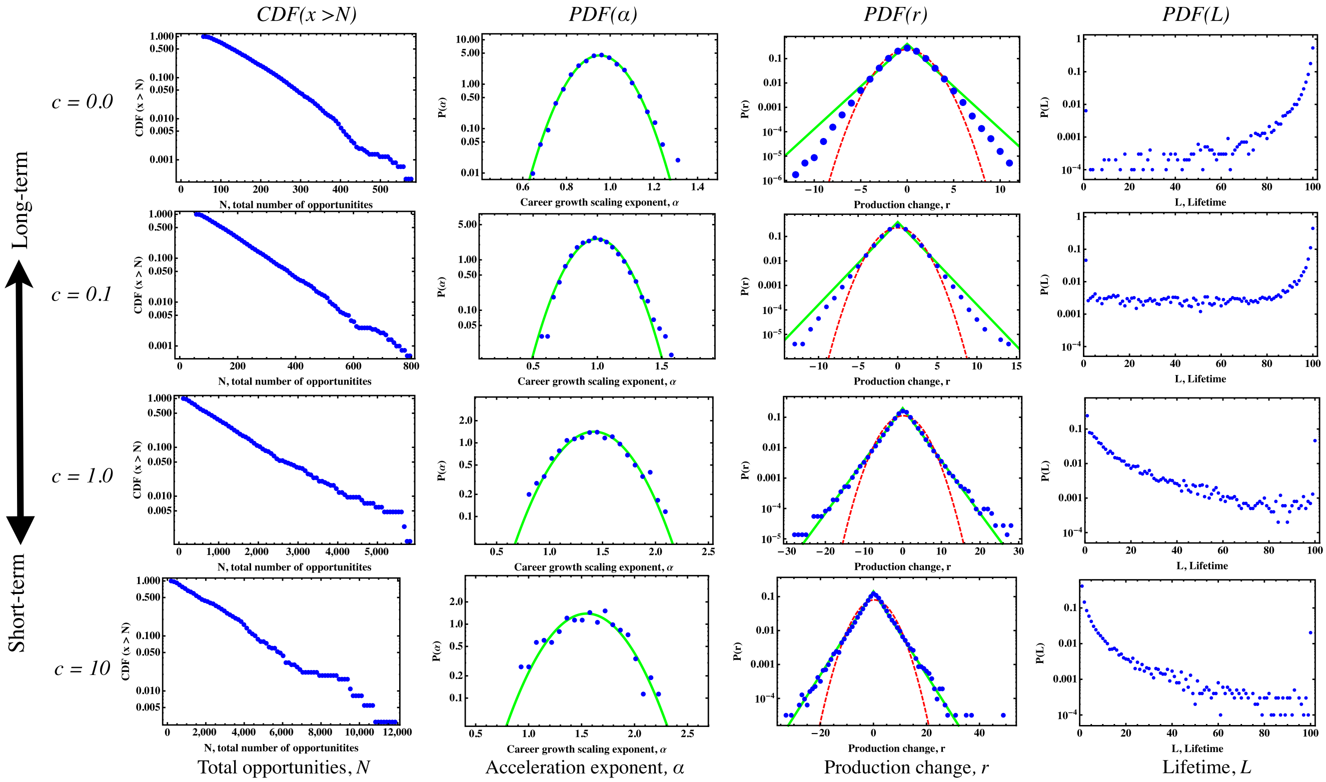

Fig. 4. MC simulation of the linear preferential capture model ($\pi = 1$) for varying contract length parametrized by *c*. We plot the probability distributions for (*i*) N_i, the total number of opportunities captured by the end period T, (*ii*) the growth acceleration exponent α_i, (*iii*) the single period growth fluctuation $r_i(t)$ including for comparison the Laplace (solid green) and Gaussian (dashed red) best-fit distributions calculated using the respective MLE estimator, and (*iv*) the career longevity L_i defined as the time difference between an agent's first and last captured opportunity. Results for $c \to 0$ systems shows that for a "long-term appraisal" scenario careers are less vulnerable to low-production phases, and as a result, most agents sustain production throughout the career. Conversely, results for $c \geq 1$ systems show that for a "short-term appraisal" scenario the labor system is driven by fluctuations that can cause career "sudden death" for a large fraction of the population. In this short-term appraisal model, there are typically a small number of agents who are able to capture the majority of the production opportunities with remarkably accelerating career growth reflected by significantly large $\alpha_i \geq 1$. Thus, a few "lucky" agents are able to survive the initial fluctuations and end up dominating the system. In *SI Appendix: and Figs. S12–S16*, we further show that systems with increased levels of competition ($\pi > 1$) mimic systems with short-term contracts, resulting in productivity "death traps" whereby most careers stagnate and terminate early.

before reaching age $0.01T$, and 25% of the labor population dies before reaching age $0.02T$ (see *SI Appendix: Table S1*). Hence, in model short contract systems, the longevity, output, and impact of careers are largely determined by fluctuations and not by persistence.

Fig. 4 shows the MC results for $\pi = 1$. For $c \geq 1$ we observe a drastic shift in the career longevity distribution $P(L)$, which becomes heavily right-skewed with most careers terminating extremely early. This observation is consistent with the predictions of an analytically solvable Matthew effect model (16) which demonstrates that many careers have difficulty making forward progress due to the relative disadvantage associated with early career inexperience. However, due to the nature of zero-sum competition, there are a few "big winners" who survive for the entire duration T and who acquire a majority of the opportunities allocated during the evolution of the system. Quantitatively, the distribution $P(N)$ becomes extremely heavy-tailed due to agents with $\alpha > 2$ corresponding to extreme accelerating career growth. Despite the fact that all the agents are endowed initially with the same production potential, some agents emerge as superstars following stochastic fluctuations at relatively early stages of the career, thus reaping the full benefits of cumulative advantage.

Discussion

An ongoing debate involving academics, university administration, and educational policy makers concerns the definition of professorship and the case for lifetime tenure, as changes in the economics of university growth have now placed *tenure* under the review process (3, 6). Critics of tenure argue that tenure places too much financial risk burden on the modern competitive research university and diminishes the ability to adapt to shifting economic, employment, and scientific markets. To address these changes, universities and other research institutes have shifted away from tenure at all levels of academia in the last thirty years towards meeting staff needs with short-term and non-tenure track positions (3).

For knowledge intensive domains, production is characterized by *long-term* spillovers both through time and through the knowledge network of associated ideas and agents. A potential drawback of professions designed around short-term contracts is that there is an implicit expectation of sustained annual production that effectively discounts the cumulative achievements of the individual. Consequently, there is a possibility that short-term contracts may reduce the incentives for a young scientist to invest in human and social capital accumulation. Moreover, we highlight the importance of an employment relationship that is able to combine positive competitive pressure with adequate safeguards to protect against career hazards and endogenous production uncertainty an individual is likely to encounter in his/her career.

In an attempt to render a more objective review process for tenure and other lifetime achievement awards, quantitative measures for scientific publication impact are increasing in use and variety (17–20, 24, 27, 46, 47). However, many quantifiable benchmarks such as the h-index (17) do not take into account collaboration size or discipline specific factors. Measures for the comparison of scientific achievement should at least account

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for variable collaboration, publication, and citation factors (19, 46, 47). Hence, such open problems call for further research into the quantitative aspects of scientific output using comprehensive longitudinal data for not just the extremely prolific scientists, but the entire labor force.

Current scientific trends indicate that there will be further increases in typical team sizes that will forward the emergent complexity arising from group dynamics (7, 12, 42), and overall, an incredible growth of science. There is an increasing need for individual/group production measures, such as the output measure Q, following from Eq. **7**, which accounts for group efficiency factors. Normalized production measures which account for coauthorship factors have been proposed in refs. 19, 46, but the measures proposed therein do not account for the variations in team productivity.

The complexity of large collaborations raises open questions concerning scientific productivity and the organization of teams. We measure a decreasing marginal return $\gamma < 1$ with increasing group size which identifies the importance of team management. A theory of labor productivity can help improve our understanding of institutional growth, for organizations ranging in size from scientific collaborations to universities, firms, and countries (33, 34, 44, 47–50).

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Supporting Information Appendix

Persistence and Uncertainty in the Academic Career

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I. DATA

To test the intriguing possibility that competition leads to common growth patterns in complex systems of arbitrary size S, we analyze the production dynamics of two professions that are dissimilar in many regards, but share the common underlying driving force of competition for limited resources. In order to establish empirical facts that we believe are independent of the details of a given competitive profession, we analyze a large dataset of production $n_i(t)$ values and corresponding growth fluctuation $r_i(t) \equiv n_i(t) - n_i(t-1)$ values. We define the appropriate measures for $n_i(t)$ to be (a) the annual number of papers published by scientist i and (b) the seasonal performance metrics of professional athlete i. While these two professions both display a high level of competition, they differ in their employment term structure and salary scale. In the case of academia, the tenure system rewards high performance levels with lifelong employment (tenure). In contrast, professional sports are characterized by relatively short contracts that emphasize continued performance over a shorter time frame and thereby exploit the high levels of athletic prowess in a player's peak years. The large number of careers in these two professions readily lend themselves to quantitative analysis because the data that quantify the career production trajectory are precisely defined and comprehensive throughout an individual's entire career. Furthermore, because of the generic nature of competition, we use these two distinct professions to compare and contrast the distribution of career impact measures across a cohort of competitors. The datasets we analyze are:

I : *Academia*:

We analyze the publication careers of 300 physicists which we categorize in 3 subsets each consisting of 100 individuals:

(A) Dataset A corresponds to the 100 most-cited physicists according to the citation shares metric [19] (with average h-index $\langle h \rangle = 61 \pm 21$). These 100 careers constitute 3,951 $r_i(t)$ values.

(B) Dataset B corresponds to the 100 other "control" scientists, taken approximately randomly from the same physics database (with average h-index $\langle h \rangle = 44 \pm 15$). In the selection process for dataset B, we only consider scientists who have published between 10 and 50 articles in PRL over the 50-year period 1958-2008. These 100 careers constitute 3,534 $r_i(t)$ values.

(C) Dataset C corresponds to 100 Assistant Professors (with average h-index $\langle h \rangle = 15 \pm 7$), where we select two physicists from each of the top-50 U.S Physics & Astronomy Departments (according to the U.S. News rankings). These Asst. Profs. are assumed to be early in their career and relatively accomplished given the difficulty in obtaining such a position in any given university. These 100 careers constitute 1,050 $r_i(t)$ values.

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In order to control for discipline-specific citation patterns, we select individuals in dataset A and B from set of all scientists who have published in *Physical Review Letters* (PRL) over the 50-year period 1958–2008. As a measure of output, we define $n_i(t)$ as the number of papers published in year t of the career of individual i, where year $t = 1$ corresponds to the year of the first publication on record for author i. We downloaded the complete publication records of the scientists in datasets A and B from ISI Web of Science (`http://www.isiknowledge.com/`) in Jan. 2010, and we downloaded the complete publication records of the scientists in dataset C from ISI Web of Science in Oct. 2010. We used the "Distinct Author Sets" function provided by ISI in order to increase the likelihood that only papers published by each given author are analyzed.

II : *Major League Baseball (MLB):*

We analyze 17,292 baseball players over the 90-year period 1920-2009 using comprehensive league data obtained from *Sean Lahman's Baseball Archive* accessed at `http://baseball1.com/index.php`. We separate the career data into two distinct subsets: non-pitchers (players not on record as having pitched during a game) and pitchers.

(A) For non-pitchers, we analyze two batting metrics: an "opportunity metric" - at-bats (AB), and a "success" metric - hits (H). Together, these 8,993 careers constitute 43,043 $r_i(t)$ values.

(B) For pitchers, we analyze two pitching metrics: an "opportunity metric" - innings-pitched measured in outs (IPO), and a "success" metric - strikeouts (K). Together, these 8,299 careers constitute 33,965 $r_i(t)$ values.

III : *National Basketball Association (NBA):*

We analyze 3,864 basketball careers, constituting 15,316 $r_i(t)$ values, over the 63-year period 1946–2008 using data obtained from *Data Base Sports Basketball Archive* accessed at `http://www.databasebasketball.com/`. We analyze two player metrics:

(A) an "opportunity metric" - minutes played (Min.), and

(B) a "success" metric - points scored (Pts.)

Since sports careers typically peak for athletes around age 30, we account for a time-dependent career trajectory which is dominant in most sports careers by "detrended" the measures for career growth fluctuations. In the case where we do not account for a individual fluctuation scale,

$$R_i \equiv [r_i(t) - \overline{r}(t)]/\sigma(t) \ . \tag{S1}$$

In this case we detrend with respect to the average production difference $\overline{r}(t)$ and the standard deviation of production difference $\sigma(t)$ which are calculated using all careers from a given sports league, conditional on the career year t.

In the case where we do account for individual variations, we first define $z_i(t) \equiv (r_i(t) - \langle r_i \rangle)/\sigma_i$ to be normalized with respect to the individual career scales $\langle r_i \rangle$ and σ_i which are the average and standard deviation of the production change of athlete career i. Then we define the detrended growth rate as

$$R'_i \equiv [z_i(t) - \langle z(t) \rangle]/\sigma_{z(t)} \ , \tag{S2}$$

where in this case we detrend with respect to the average $\langle z(t) \rangle$ and standard deviation $\sigma_{z(t)}$ calculated by collecting all $z_i(t)$ values for a given career year t. This detrending better accounts for the relatively strong time-dependent growth patterns in sports.

In this section we analyze the annual production of scientists measured as the number of papers published $n_i(t)$ over the period of a year. Using this measure does not account for the variability in the length of production, say in the number of pages, nor does it account for the impact of the paper, a quantity commonly approximated by a paper's citation number. Instead, we consider a simple definition that a scientific product is a final output of a collection of inputs. Furthermore, in science it is assumed that the peer review process establishes a quality threshold so that only manuscripts above a certain quality and novelty standard can be published and incorporated into the scientific body of knowledge.

Prior theories of scientific production have also used the number of publications as a proxy for scientific output. In particular, the Shockely model [14] proposed a simple multiplicative factor model for the production $n_i(t)$ which predicts a log-normal distribution for $P(n)$. An alternative null model for $n_i(t)$ is the Poisson process, which assumes that each individual is endowed with a rate parameter ω related to an individual's production factors. This model predicts a Poisson distribution for $P(n)$. However, a shortfall of these models is that multiplicative parameters in the Shockley model and the rate parameter ω are difficult to measure, especially if the set of individuals span a large range of production factors, and moreover, if the careers are non-stationary.

Fig. S8 shows the unconditional probability distribution $P(n)$ calculated by aggregating all $n_i(t)$ values for all scientists and all years into an aggregate dataset. Naively, the distributions are well-fit by the Log-normal distribution, and so there is an apparent agreement with the multiplicative factor Shockley model. However, the distribution $P(n) = \sum_{i=1}^{100} P(n|S_i)$ is the aggregate distribution constructed from 100 individual career trajectories $n_i(t)$, each with varying size S_i. Indeed, we demonstrate in Figs. 1 and S1 to be non-linear, with time-dependent residuals around the moving average. Hence, it is not possible from the unconditional pdf $P(n)$ to determine if the process underlying scientific production corresponds to a simple multiplicative process or a Poisson process.

In order to better account for the variable size S_i of each career which affects the rate at which an individual is able to capture publication opportunities, we plot in Fig. S7 the pdf of the normalized output

$$Q_i = \frac{n_i(t)}{f_i(k)} \ . \tag{S3}$$

We calculate the normalization factor $f_i(k) = q_i[k_i(t)]^{\gamma_i}$ for each individual i by estimating the parameters q_i and γ_i for each scientist i from the single-factor model

$$n_i = q_i k_i^{\gamma_i} \ . \tag{S4}$$

where $n_i(t)$ is the annual production in year t and $k_i(t)$ is the total number of distinct coauthors in year t. Hence, Q_i represents the production factor above $Q > 1$ or below $Q < 1$ what would be expected from the author i given the fact that he/she had additional inputs from $k_i(t)-1$ individuals that year. This model assumes that the major component contributing to production is the collaboration degree k of the research output, and also assumes that the input of each coauthor contributes equally to the final output. Clearly, these assumptions neglect some important idiosyncratic details affecting scientific publication, but given the incomplete information associated with every publication, it is a decent approximation. We estimate q_i and γ_i by performing a linear regression of $\log n_i$ and $\log k_i$ using the first L_i years of each career, neglecting years with $n_i = 0$. We use $L_i = 35$ years for dataset [A] and [B] scientists, and $L_i = 10$ years for dataset [C] scientists.

In Fig. 3(c) we approximate γ using all $n(t)$ within each dataset with $k \leq 50$, and performing a regression of the model

$$\ln n = \ln q + \gamma \ln k + \epsilon \tag{S5}$$

to estimate γ, where ϵ is the residual due to other unaccounted production factors. For each dataset we find that the aggregate efficiency parameter γ is approximately equal to the average $\langle \gamma_i \rangle$ calculated from the 100 γ_i values in each career dataset: $\gamma = 0.68 \pm 0.01$ [A], $\gamma = 0.52 \pm 0.01$ [B], and $\gamma = 0.51 \pm 0.02$ [C]. Furthermore, the $\psi \approx \gamma$ since the size-variance scaling parameter ψ is also an efficiency measure that relates the scaling of output n to input k.

As a result of this analysis, we quantify the scaling exponent $\gamma < 1$ of the decreasing marginal returns in the scientific production function for projects with $k \leq 50$. This likely stems from the inefficient management costs associated with large group collaborations which typically manifest in a larger production timescale. In fact, for years with $k \geq 50$ coauthors, scientific output shows decreasing returns to scale. Interestingly, the star scientists in dataset [A] display significantly larger efficiency, quantitatively showing the importance of management skills in scientific success.

The normalized production values are normalized to units of "expected production" conditional on the k_i inputs for author i. We aggregate all data from each dataset and show in Fig. S7 that the Q values are well-described by the Gamma distribution

$$P(Q) = Q^{m-1} \frac{\exp[-Q/\theta]}{\theta^m \Gamma(m)} \tag{S6}$$

where m is the shape parameter and θ is the scale parameter. Surprisingly, we find that dataset [A] and [B] have approximately equal Gamma parameters, indicating that besides their production efficiency, top scientists are virtually indistinguishable with average normalized output $\langle Q \rangle = m\theta > 1$. For each dataset we calculate the Gamma parameters using the maximum likelihood estimator method: $m = 5.45$ and $\theta = 0.21$ [A], $m = 5.60$ and $\theta = 0.20$ [B], and $m = 7.00$ and $\theta = 0.15$ [C]. We leave it as an open question to determine why the Gamma distribution describes so well the production statistics. We ponder the intriguing possibility that the stochastic dynamics underlying individual production corresponds to an increasing Lévy process with variable jump length which is known to produce a Gamma distribution.

II. QUANTIFYING THE CAREER TRAJECTORY

The reputation of an individual is typically cumulative, based on the total sum of achievements, which we approximate by the cumulative output $N_i(t)$ (e.i. number of papers published by year t). In Figs. 1 and S1 we plot $N_i(t)$ for several individuals. The careers presented in Fig. 1 are more linear, indicating quantifiable career trajectory that has the approximate form

$$N_i(t) = \sum_{t'=1}^{t} n_i(t') \approx A_i\ t^{\alpha_i}\ ,\quad t < T_i \tag{S7}$$

where $n_i(t)$ are the number of papers in year t of the scientist's career which begins with $t \equiv 1$ in the year of his/her first publication, and begins to decline around time T_i which is the time horizon over which the scaling regularity holds before termination and aging effects begin to dominate the career. In our analysis of academic career trajectories $N_i(t)$, we only analyze $N_i(t)$ for $t \leq 40$ years in order to account for such termination affects.

The smooth career trajectories which appear as a linear curve when plotted on log-log scale are characterized by an amplitude parameter A_i and a scaling exponent α_i. However, as indicated by Fig. S1, there are also non-stationary $N_i(t)$ which are dominated by "career shocks" that significantly alter the career trajectory. Such career shocks have been demonstrated using publication impact measures (e.i. citations, and h-index sequences) [20, 23, 26], and here we show that they even occur at the more fundamental level of individual production dynamics.

In order to analyze the characteristic properties of $N_i(t)$ for all 300 scientists analyzed, we define the normalized trajectory $N'_i(t) \equiv N_i(t)/\langle n_i \rangle$, where $\langle n_i(t) \rangle$ is the average annual production rate of author i, and so by construction $N'_i(L_i) = L_i$. Fig. S2(A) shows the characteristic production trajectory obtained by averaging the 100 individual $N'_i(t)$ for each dataset,

$$\langle N'(t) \rangle \equiv \Big\langle \frac{N_i(t)}{\langle n_i \rangle} \Big\rangle \equiv \frac{1}{100} \sum_{i=1}^{100} \frac{N_i(t)}{\langle n_i \rangle}\ . \tag{S8}$$

The standard deviation $\sigma(N'(t))$ is shown in Fig. S2(B), which has a broad peak that is a likely signature of career shocks that can significantly alter the career trajectory. The characteristic trajectory for each dataset are well-approximated by the scaling relation

$$\langle N'(t) \rangle \sim t^{\overline{\alpha}} \tag{S9}$$

with characteristic scaling exponents $\overline{\alpha} > 1$ that are significantly greater than unity: $\overline{\alpha} = 1.28 \pm 0.01$ for Dataset A, $\overline{\alpha} = 1.31 \pm 0.01$ for Dataset B, and $\overline{\alpha} = 1.15 \pm 0.02$ for Dataset C. This fact implies that there is a significant cumulate advantage in scientific careers which allows for the career trajectory to be accelerating. In Fig. S2(C) and S2(D) we plot the analogous $\langle N'(t) \rangle$ curves for professional sports metrics, where for this profession, $\overline{\alpha} \approx 1$ for all measures analyzed. This is likely due to the fact that annual production in professional sports is capped by the limited number of opportunities provided by a season, whereas in academics, the number of publications a scientist can publish is in principle unlimited. Also, in more labour-intensive activities are likely to experience smaller returns since physical labor is non-cumulative with less spillover through time.

In Fig. S3 we plot each individual career trajectory using the rescaled time $t'_i = t^{\alpha_i}$ as an additional visual test of the scaling model given by Eq. S7 . We show that on average, all curves $i = 1..300$ approximately collapse onto the expected curve $N_i(t)/A_i = t'$, where the residual difference $\epsilon_i(t') \equiv N_i(t)/A_i - t'$ are likely due to career shocks of various magnitudes. We plot the average and standard deviation of each set of 100 $N_i(t)/A_i$ curves which show that most of the shocks $\epsilon_i(t')$, with some significant exceptions, lie within the 1σ standard deviation denoted by the error bars. In Fig. S4 we plot the probability distributions $P(\alpha_i)$ for each academic dataset. For each dataset, the average value $\langle \alpha_i \rangle$ is in good agreement with $\overline{\alpha}$, the scaling parameter calculated for the corresponding trajectory $\langle N'(t) \rangle$.

III. EXPONENTIAL MIXING OF GAUSSIANS

The idea that entities are independent and identically distributed is an unrealistic assumption commonly made in analyses of complex systems. The unconditional pdf $P(r)$ is commonly analyzed in empirical studies where insufficient data are present to define normalized r'_i measures for each sample constituent i. Nevertheless, when modeling the evolution of complex based on empirical data corresponding to distinct subunits (such as individual careers, companies, or nation regions), unconditional quantities that account for variations in underlying production factors should be used.

In the case of scientific output, there are many production factors that combine together and determine the amount of human efforts needed to produce a unit of production. In general, consider the value $f_{i,j}$ of individual i corresponding to his/her relative abilities in the production factor $j = 1...J$ corresponding to a variety of attributes: knowledge, genius, persistence, reputation, mental and physical health, communication skills, organization skills, and access to technology, equipment and data, etc. In this study, we compare scientists who publish in similar journals. Still, the scientific input required for each scientific output can vary by a large amount, largely depending on the technology needed to perform the analysis, ranging from particle accelerators to just a pencil and paper.

In a very generalized representation, an unconditional distributions $P(r)$, such as shown in Fig. 2(a-d) for production change r, may follow from a mixture of conditional Gaussian distributions $P(r|S_i)$

$$P_\psi(r) = \int_0^\infty P(r|S)P(S)dS \approx \sum_{i=1}^{I} P_i(r|S_i)P(S_i) \ . \tag{S10}$$

The underlying conditional distributions are characterized by the average $\langle r\rangle_{S_i}$ and variance $\sigma_i^2 \equiv VS_i^\psi$

$$P(r|S_i) = \exp[-(r-\langle r\rangle)^2/2VS_i^\psi]/\sqrt{2\pi VS_i^\psi} \ . \tag{S11}$$

which are each parameterized by the characteristic collaboration size S_i. In cases where the average change $\langle r\rangle \approx 0$, then the distribution $P(r|S_i)$ is characterized by only the fluctuation scale $\sigma_i(r)$. Fig. S5 demonstrates that the normalized production change $r'_i(t) = (r - \langle r_i\rangle)/\sigma_i$ is distributed according to a Gaussian distribution. Hence, using normalized variables, we have mapped the process to a universal scaling distribution $P(r|S_i)$.

When the distribution $P(S_i)$ is exponential,

$$P(S_i) = \lambda e^{-\lambda S_i} \tag{S12}$$

then mixture is termed an "exponential mixture of Gaussians" [43], where the units have characteristic size $\overline{S_i} = 1/\lambda$. Fig. S10 shows that the distribution of collaboration radius S_i is approximately exponential for each dataset, supporting the case for exponential mixing. Using the cumulative distribution of S for each data set we calculate $\lambda = 0.15 \pm 0.01$ [A], $\lambda = 0.11 \pm 0.01$ [B], and $\lambda = 0.11 \pm 0.01$ [C]. While the tail behavior of $P(r)$ can be used to better discriminate the value of ψ, we do not have sufficient data in this analysis to perform a more rigorous test of the tail dependencies, or in general, to investigate the distribution of significantly large $r_i(t)$ values.

The scaling relation $\sigma_i(r) \sim S_i^{\psi/2}$ determines the functional form of the aggregate $P_\psi(r)$. Clearly, $\sigma(r)$ increases for $\psi > 0$ values, whereas for values $\psi < 0$, $\sigma(r)$ decreases with size S_i. This latter case is empirically observed for countries and firms [48], whereby in general, large economic entities are able to decrease growth volatility by increasing and diversifying their portfolio of growth products. In our analysis of scientific careers we define $S_i \equiv Med[k_i(t)]$, the median number of distinct coauthors per year, as a proxy for the ability of the career to attract new opportunities, and hence, as a proxy for the size S_i of an academic career. For professional athletes, we define the career size as the average number of points scored over the career $S_i \equiv \langle p_i(t)\rangle$. In Fig. 3 we calculate $\psi/2 \approx 0.40 \pm 0.03$ (regression coefficient $R = 0.77$) for dataset [A], $\psi/2 \approx 0.22 \pm 0.04$ ($R = 0.51$) [B], and $\psi/2 \approx 0.26 \pm 0.05$ ($R = 0.45$) [C].

The role of mental, physical, and group spillovers is quite different in professional sports. Athletes attract future opportunities largely through their historical track record, which is heavily weighted on performance in the near past, and less on the cumulative history. Hence, for this performance-based labor force, we use a simple definition of "team value" to define the career size S_i. This quantity is easier to define for basketball, since there are smaller differences between players of different team position than in other sports. For NBA player i we define S_i as the average number of points scored per year, $S_i \equiv \langle p_i\rangle$. Fig. S9 shows a crossover value S_c which we interpret to reflect the fact that sports players typically fall into one of two categories: starters (everyday players) and replacement (game filler) players. We calculate $\psi/2 \approx 0.38 \pm 0.02$ for emerging and "second string" careers with $S_i < S_c$, and a decreasing size variance relation ($\psi < 0$) for high-value careers with $S_i > S_c$. Similar values occur in the MLB. These two ψ regimes reflect the crucial balance of risk and reward in short-term contract professions.

A variety of pdfs $P_\psi(r)$ can result from the exponential mixture of Gaussians

$$P_\psi(r) = \int_0^\infty \lambda e^{-\lambda S} \frac{1}{\sqrt{2\pi\sigma^2(r)}} \exp[-r^2/2\sigma^2(r)]dS \tag{S13}$$

depending on the value of ψ which quantifies the size-variance relation. The functional form of $P_\psi(r)$ can vary in both the bulk and the tails of the distribution [43]. A simple result which follows from the case $\psi = 1$ is the Laplace (double-exponential) distribution

$$P_{\psi=1}(r) = \sqrt{\frac{\lambda}{2V}} \exp\Big[- \sqrt{\frac{2\lambda}{V}}|r|\Big] \ . \tag{S14}$$

This distribution is a member of the family of Exponential power distributions which follow from the range of values $\psi \geq 0$ [43]. In general, if the scaling values are in the range $\psi \geq 0$, then the exponential mixture leads to an Exponential power distribution

$$P(r) = \frac{\beta}{\sqrt{2}\sigma\Gamma(1/\beta)} \exp[-\sqrt{2}(|r|/\sigma)^\beta] \tag{S15}$$

with shape parameter β in the range $\beta \in (0, 2]$ [43]. The pure exponential $P(r)$ with $\beta = 1$ corresponds to the case $\psi = 1$. The pure Gaussian $P(r)$ with $\beta = 2$ corresponds to the case $\psi = 0$.

Furthermore, if the annual production is logarithmically related to an underlying production potential, $n_i(t) \propto \ln U_i(t)$, then $r_i(t) \propto \ln U_i(t) - \ln U_i(t-1)$ quantifies the logarithmic change ("growth rate") of $U_i(t)$. This forms the analogy with growth dynamics of large institutions with size $S \gg 1$. For example, in the case of financial securities such as the stock of a company i, the growth rate $r_i(t)$ measure the logarithmic change in the market's expectations of the company's future earnings potential captured by the market capitalization and price [49]. As a result, distributions $P(r)$ of career growth fluctuation r, which we plot in Figs. 2 (a-d), can be seen as a bridge between the micro level and the macro level of economic growth fluctuation. A theory of micro growth processes can help improve the growth forecasts for economic organizations ranging in size from scientific collaborations to universities and firms [32, 33, 43, 46–49].

IV. NONLINEAR PREFERENTIAL CAPTURE MODEL

Here we describe a stochastic system in which a finite number of opportunities are distributed to a system of individual competing agents $i = 1...I$. The opportunities are distributed in batches of P opportunities per arbitrary time interval. This model has two parameters.

(i) π determines the preferential capture mechanism (the value $\pi = 1$ corresponds to the traditional "linear" preferential attachment model) and

(ii) c determines the performance timescale $1/c$ which is incorporated into the calculation of the capture rates of each individual. The value $c = 0$ corresponds to a long-term memory and $c \gg 1$ corresponds to short-term memory.

We use this simple model to show that a system governed by a preferential capture can become dominated by fluctuations when c is large. The value $1/c$ quantifies the "performance appraisal timescale": a small c corresponds to a labor system with long contracts, or some alternative mechanism that provides employment insurance through periods of low production, so that the ability to attract future opportunities is largely based on the cumulative record of career achievement. Conversely, a large c corresponds to a labor system with short contracts in which the ability to attract future opportunities is largely based on the accomplishments in the near past, requiring an agent to maintain relatively high levels of production in order to survive. In this latter case, we find that (natural) fluctuations in the annual production can cause a significant fraction of the careers to "fizzle out" leaving behind only a few "super careers" who attract almost all of the opportunities. In other words, short contracts can tip the level of competition into dangerous territory whereby careers are largely determined by fluctuations and not persistence.

A. System of competing agents

1) The system consists of $I \equiv 1000$ agents competing for P opportunities that are allocated in a single period. There is no entry, hence the number I is kept constant. Also, P is also kept constant, so there is no growth in the labor supply.

2) We run the Monte Carlo (MC) simulation for $T \equiv 100$ time periods and all agents are by construction from the same age cohort (born at same time).

3) Each time period corresponds to the allocation of $P \equiv \sum_{i=1}^{I} n_{0,i}$ opportunities, sequentially one at a time, to randomly assigned agents i, where $n_{0,i} \equiv 1$ is the potential production capacity of a given individual.

4) The assignment of a given opportunity is proportional to the time-dependent weight (capture rate) $w_i(t)$ of each agent. Hence, the assignment of 1 opportunity to agent i at period t results in the production (achievement) $n_i(t)$ to increase by one unit: $n_i(t) \rightarrow n_i(t) + 1$. In the next time period $t + 1$, we update the weight $w_i(t + 1)$ to include the performance $n_i(t)$ in the current period.

B. Initial Condition

The initial weight at the beginning of the simulation is $w_i(t = 0) \equiv n_c$ for each agent i with $n_c \equiv 1$. The value $n_c > 0$ ensures that competitors begin with a non-zero production potential, and corresponds to a homogenous system where all agents begin with the same production capacity. Hence, we do not analyze the more complicated model wherein external factors (i.e. collaboration factors) can result in a heterogeneous production capacity across scientists. By construction, each agent begins with one unit of achievement $n_i(t = 1) \equiv 1$.

C. System Dynamics

1) In each Monte Carlo step we allocate one opportunity to a randomly chosen individual i so that $n_i(t) \rightarrow n_i(t)+1$

2) The individual i is chosen with probability $\mathcal{P}_i(t)$ proportional to $[w_i(t)]^\pi$

$$\mathcal{P}_i(t) = \frac{w_i(t)^\pi}{\sum_{i=1}^{I} w_i(t)^\pi} \tag{S16}$$

where the value $w_i(t)$ is given by an exponentially weighted sum over the entire achievement history

$$w_i(t) \equiv \sum_{\Delta t=1}^{t-1} n_i(t - \Delta t) e^{-c\Delta t} \ . \tag{S17}$$

The parameter $c \geq 0$ is a memory parameter which determines how the record of accomplishments in the past affect the ability to obtain new opportunities in the current period, and therefore, the future. The limit $c = 0$ rewards long-term accomplishment by equally weighting the entire history of accomplishments. Conversely, when $c \gg 1$ the value of $w_i(t)$ is largely dominated by the performance $n_i(t-1)$ in the previous period, corresponding to increased emphasis on short-term accomplishment in the immediate past. Intermediate values $0 < c < 1$ weight more equally the immediate past and the entire history of accomplishment.

3) The exponent π determines how the relative ability to attract opportunities $\mathcal{P}_i/\mathcal{P}_j = [w_i(t)/w_j(t)]^\pi$ depends on the weights $w_i(t)$ and $w_j(t)$ between two individuals i and j. The linear capture case follows from $\pi = 1$, uniform capture $\pi = 0$, super linear capture $\pi > 1$, and sub-linear capture $\pi < 1$.

4) At the end of each time period, the weight $w_i(t)$ is recalculated and used for the entirety of the next MC time period corresponding to the allocation of the next $I \times n_c$ achievement opportunities.

D. Model Results

We simulate this system for a realistic labor force size $I = 1000$ with the assumption that in any given period, an individual has the capacity for one unit of production ($n_c \equiv 1$). We evolve the system for $T = 100$ periods corresponding to $I \times n_c \times T$ Monte Carlo time steps. The timescale T represents the (production) lifetime of individuals with finite longevity. In this model we do not include exogenous shocks (career hazards) that can result in career death [16]. Here we analyze four quantities:

1) The distribution $P(N)$ of the total number of opportunities $N_i(T) \equiv \sum_{t=1}^{T} n_i(t)$ captured by agent i over the course of the $T-$ period simulation.

2) The distribution $P(\alpha)$ of the career trajectory scaling exponent α_i defined in Eq. S7 which quantifies the (de)acceleration of production over the course of the career.

3) The distribution $P(r)$ of production outcome change r defined in Eq. 2 which quantifies the size of endogenous production shocks.

4) The distribution $P(L)$ of career length L_i which measures the active production period of each career starting from $t = 0$. We define activity as the largest period value L_i for which $n_i(L_i) = 0$, which in other words, corresponds to truncating all 0 production values from the end of the trajectory $n_i(t)$ and defining L_i as the length of this time series.

We display these four distributions, from left to right, for varying π and c values, in each panel of Figs. S12 – S16. Empirical distributions calculated from MC simulations are plotted as blue dots, with benchmark distributions described below plotted as solid green curves. For each π and c value we simulate 10 MC systems, and combine the results into aggregate distributions which are shown. For simulations with $\pi > 1$ the pdf data are aggregated over the results of 50 MC simulations. We list below some of our main observations.

For $\pi = 1$, independent of c, we observe exponential $P(N)$, consistent with the prediction of the linear preferential capture model in the case of no firm entry ($b = 0$) in the model of Kazuko et al. [42]. However, the distribution $P(L)$ and the distribution $P(\alpha)$ does depend strongly on c, reflecting the possibility of career "sudden death" for large c.

For the $P(\alpha)$ distributions (middle-left panels), the solid green line is a best-fit Gaussian distribution (using the MLE method) for the set of α_i values computed for careers that did not undergo "sudden death."

For the $P(r)$ distributions (middle-right panels), the solid green curve corresponds to a best-fit Laplace distribution (using the MLE method) and the dashed red curve corresponds to a best-fit Guassian distribution (using the MLE method) which we show only for benchmark comparison. Typical empirical distributions (values shown as blue dots) range from being distributions that are Gaussian to distributions that are Laplacian in the bulk but with heavy tails.

For the $P(L)$ distributions (right most panels), we note that the most likely career length L is typically either $L = 1$ or $L = T$ for all systems analyzed. However, there are likely c and π parameter values corresponding to $P(L)$ that is uniform distributed over the entire range of L values, which may be an interesting class of system to analyze in future analyses since such a system promotes diversity across the entire longevity spectrum. The system we show for $\pi = 1.2$ and $c = 1$ appears to be close to this scenario.

Fig. S12 shows the null model with no preferential capture ($\pi = 0$). We confirm that the careers in this model are driven by a stochastic accumulation process that is equivalent to a Poisson process with rate $\lambda_p \equiv 1$. In this homogenous system, each career gains on average one opportunity each time period, so that at the end of the simulation, the distribution $P(N)$ is a Poisson distribution with $\langle N \rangle = \lambda_p T$ (shown as the solid blue line) which fits the model data excellently. For these careers, the typical $\alpha = 1$, the production changes are well-approximated by a Gaussian distribution, and most careers are sustained for the maximum possible lifetime corresponding to T periods.

Fig. S13 shows the system with $c = 0$ corresponding to comprehensive career appraisal corresponding to a long-term memory system. We analyze this system for 4 values of $\pi = 0.8, 1.0, 1.2, 1.4$. This "long-term memory" scenario corresponds to a long-term contract profession whereby careers are less vulnerable to periods of low production. As a result, most careers sustain production throughout the career.

Fig. S14 shows the system with $c = 0.1$ corresponding to an effective memory timescale of $1/c = 10$ periods. We analyze this system for 4 values of $\pi = 0.8, 1.0, 1.2, 1.4$. This "medium-term memory" scenario yields a rich variety of careers for $\pi = 1$, but for $\pi = 1.2$ the system becomes quickly dominated by "rich-get-richer" effects which results in careers being vulnerable to low production fluctuations.

Fig. S15 shows the system with $c = 1$ corresponding to an effective memory timescale of $1/c = 1$ period. We analyze this system for 4 values of $\pi = 0.8, 0.9, 1.0, 1.1$. For all values of π analyzed, we observe a system that is dominated by careers that are cut short by the high levels of competition induced by the relatively high value placed on continued production.

Fig. S16 shows the extreme case of a "no memory" scenario in which $w_i(t) \approx n_i(t-1)$ whereby most careers experience sudden death due to endogenous negative production shocks early in their career. The lucky few careers who survive this period end up as rich-get-richer "superstars." This behavior occurs for all systems analyzed using 4 values of $\pi = 0.8, 0.9, 1.0, 1.05$.

E. Discussion of the model in relation to the Academic labor market

One serious drawback of short-term contracts are the tedious employment searches, which displace career momentum by taking focus energy away from the laboratory, diminishing the quality of administrative performance within the institution, and limiting the individual's time to serve the community through external outreach [3, 6]. These momentum displacements can directly transform into negative productivity shocks to scientific output. As a result, there may be increased pressure for individuals in short-term contracts to produce quantity over quality, which encourages the presentation of incomplete analysis and diminishes the incentives to perform sound science. These changing features may precipitate in a "tragedy of the scientific commons."

Aside from promoting circumspect research, job security in academia diminishes the incentives for scientists to "save and store" their knowledge for future liquidation in the case of employment emergency, and thus promotes the institution of "open science" [1]. However, a policy shift towards short-term contracts, along with the heightened value of intellectual property, may alter the course of publicly funded "open science." This scientific commons emerged from the noble courts during the Renaissance as a hallmark of the scientific revolution and now faces pressure from what has been termed "intellectual capitalism," with the vast privatization of knowledge and innovation ("closed science") occurring in public universities and corporate R&D [1]. An academic system that is dominated by short term contracts, stymied by production incentives that favor quantity over quality, and jeopardized at the level of the "open knowledge" commons, presents a new institutional scenario revealing selection pressures that could alter the birth and death rates of high-impact careers.

The purpose of this stochastic model is to show how careers can become very susceptible to negative production shocks if the labor market is driven by a preferential capture mechanism with $\gamma > 1$ whereby early success of an individual can lead to future advantage. However, this model also shows that the onset of a fluctuation-dominant (volatile) labor market can also be amplified when the labor market is governed by short-term contracts reinforced by a short-term appraisal system. In such a system, career sustainability relies on continued recent short-term production, which can encourage rapid publication of low-quality science. In professions where there is a high level of competition for employment, bottlenecks form whereby most careers stagnate and fail to rise above an initial achievement barrier. Instead, these careers stagnate, and in a profession that shows no mercy for production lulls, these careers undergo a "sudden death" because they were "frozen out" by a labor market that did not provide insurance against endogenous fluctuations. Such a system is an employment "death trap" whereby most careers stagnate and "flat-line" at zero production. However, at the same time, a small fraction of the population overcomes the initial selection barrier and are championed as the "big winners", possibly only due to random chance.

Table demonstrates how the life expectancy decreases with increasing c even for the linear preferential capture model corresponding to $\pi = 1$. With increasing c, the model simulates systems with shorter contracts (shorter appraisal "memory" timescales), and so larger percentages of the population die before characteristic ages $T_c(p)$, values that decrease with increasing c for a given p.

$T_c(p)$ as a % of T, (% T)

	$p = 0.1$	$p = 0.25$	$p = 0.5$	$p = 0.75$
$c = 0$ (long term)	$0.94T$	$0.98T$	$1.00T$	$1.00T$
$c = 0.1$	$0.20T$	$0.79T$	$0.99T$	$1.00T$
$c = 1.0$	$0.01T$	$0.02T$	$0.05T$	$0.15T$
$c = 10.0$ (short term)	$0.01T$	$0.01T$	$0.02T$	$0.06T$

TABLE S1: Decrease in career life expectancy as a result of short-term contract length in the $\pi = 1$ linear preferential capture model. The fraction p of the population that experienced career termination before the crossover age $T_c(p)$: "p percent of the population died before reaching the age $L = T_c(p)$." As c increases (recall the appraisal "memory" timescale is $1/c$) towards a short-term contract scenario, a significant fraction of the population (increasing p) dies before reaching a smaller and smaller $T_c(p)$. The empirical value of $T_c(p)$ is given as a percentage of the maximum career length T corresponding to the stopping time of the Monte Carlo simulation. The value $T_c(p)$ is calculated using the equality $p = CDF(T < T_c(p))$, where $CDF(T < L)$ is the cumulative distribution function of career length L. To estimate $CDF(T < L)$, we combine an ensemble of 10 MC simulations for each c value. In the model simulations we use $T \equiv 100$ periods.

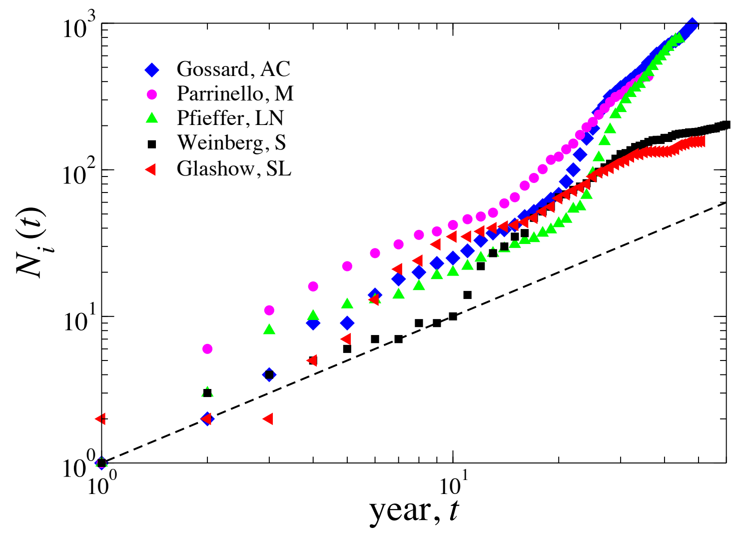

FIG. S1: Positive career shocks likely associated with reputation boosts. Examples of career production trajectories $N_i(t)$ that have significant deviations from the scaling hypothesis in Eq. S7 . These significant deviations likely follow extraordinary scientific discoveries (and the publicity and reputation that are typically rewarded) which can vault a career and result in lasting benefits to the individual.

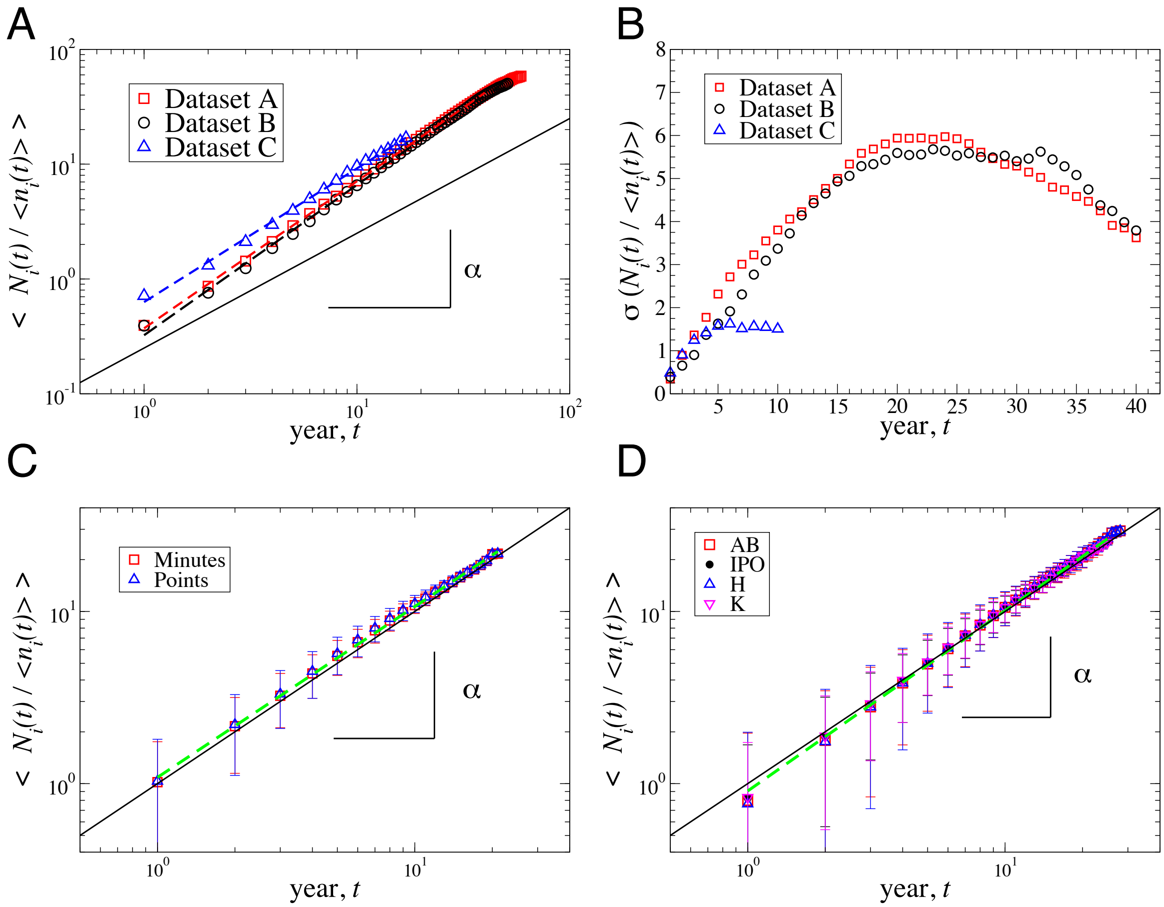

FIG. S2: Regularities in the career trajectory $N_i(t)$. We analyze the normalized career trajectory $N'_i(t) \equiv N_i(t)/\langle n_i \rangle$ which allows us to aggregate $N_i(t)$ with varying publication rates $\langle n_i \rangle$. As a result, we can better quantify the scaling exponent $\overline{\alpha}$ which quantifies the acceleration of the typical career over time. We calculate $\overline{\alpha}$ using OLS regression on log-log scale of the average normalized career trajectory $\langle N'(t) \rangle \equiv \left\langle \frac{N_i(t)}{\langle n_i \rangle} \right\rangle$. For reference, each $N'_i(t)$ trajectory in panels A, B, and C has a corresponding best-fit curve that is a dashed line. **(A)** For the scientific careers, we calculate $\overline{\alpha}$ values: 1.28 ± 0.01 for Dataset A, 1.31 ± 0.01 for Dataset B, and 1.15 ± 0.02 for Dataset C. These values are all significantly greater than unity, $\overline{\alpha} > 1$, indicative of a systematic cumulative advantage effect in science. **(B)** The standard deviation $\sigma N'(t)$ has a broad peak, likely related to career shocks that can significantly alter the career trajectory. **(C)** The average normalized career trajectory for NBA careers has $\overline{\alpha} \approx 1$ **(D)** The average normalized career trajectory for MLB careers has $\overline{\alpha} \approx 1$. For visual comparison, the solid straight black line in panels A,B and C correspond to a linear function with $\alpha = 1$.

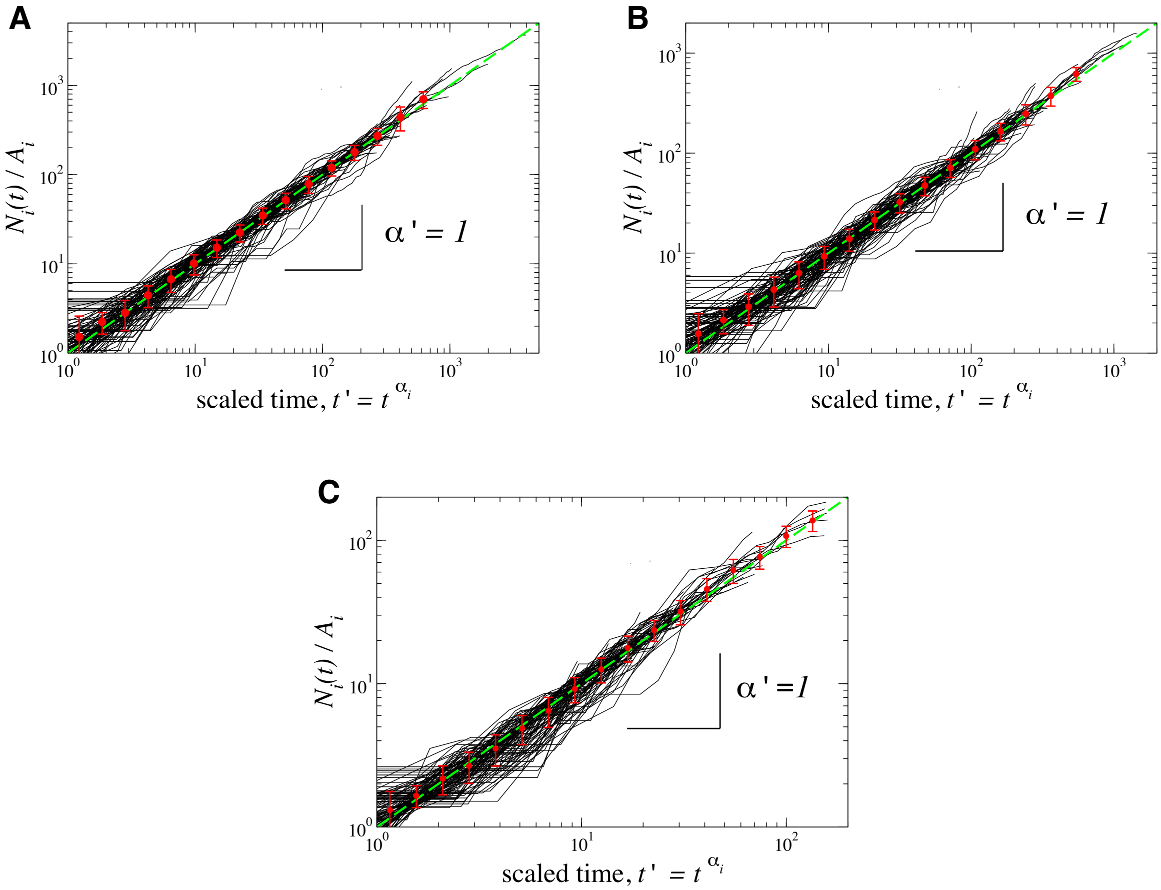

FIG. S3: Using scaling methods to show approximate data collapse of each $N_i(t)$. Normalized trajectory $\tilde{N}_i(t) \equiv N_i(t)/A_i$ plotted using the scaled time $t' \equiv t^{\alpha_i}$ for each career over the time horizon $t \in [1, 40]$ years. We plot the 100 $\tilde{N}_i(t)$ curves belonging to datasets [A], [B], and [C] in the corresponding panels. There is approximate data collapse of all the normalized trajectories $\tilde{N}_i(t)$ along the dashed green line corresponding to the rescaled career trajectory $\tilde{N}_i(t) = t'$ with $\alpha' \equiv 1$ by construction. We also plot in red the corresponding average value $\langle \tilde{N}_i(t) \rangle$ with 1σ error bars for logarithmically spaced t' intervals. Deviations from $\langle \tilde{N}_i(t) \rangle$ are indicative of career shocks which can significantly alter the career trajectory.

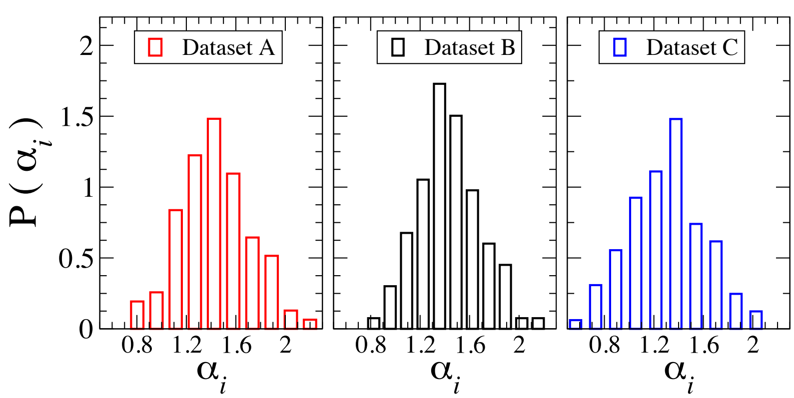

FIG. S4: Increasing returns to scale $\alpha > 1$. Probability distribution of the individual α_i values calculated for each career using the scaling model $N_i(t) \sim t^{\alpha_i}$ over time horizon $t \in [1, 40]$ years. The average $\langle \alpha_i \rangle$ and standard deviation $\sigma(\alpha_i)$ for each dataset are: 1.42 ± 0.29 [A], 1.44 ± 0.26 [B], 1.30 ± 0.31 [C]. The distribution of α_i values indicate that career trajectories are typically accelerating ($\alpha_i > 1$), most likely the result of a cumulative advantage effect.

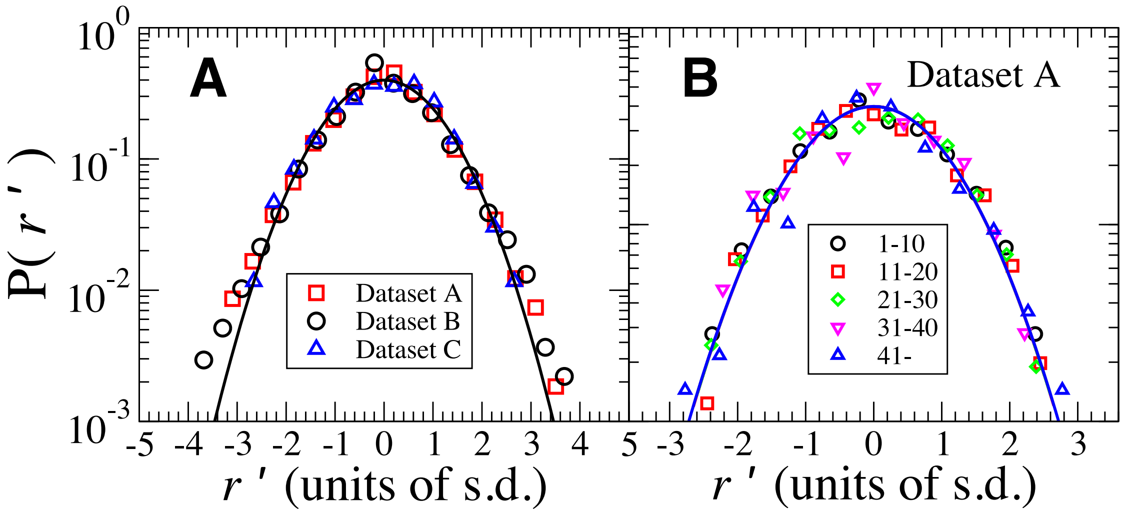

FIG. S5: Universal patterns in underlying production fluctuations of scientists. Accounting for variable individual publication factors, such as academic subfield or group collaboration size, we find that the normalized annual production change $r'_i(t) \equiv [r_i(t) - \langle r \rangle_i]/\sigma_i$ is distributed according to a Gaussian distribution, with $\langle r' \rangle = 0$ and $\sigma(r') = 1$ by construction (solid lines show best-fit Guassian distributions using the maximum likelihood estimator method). This results indicates that the Laplace distribution shown in Fig. 2 results from a mixture of Gaussian distributions $P_i(r = \sigma_i r')$ that indicate that annual production is consistent with a proportional growth model..

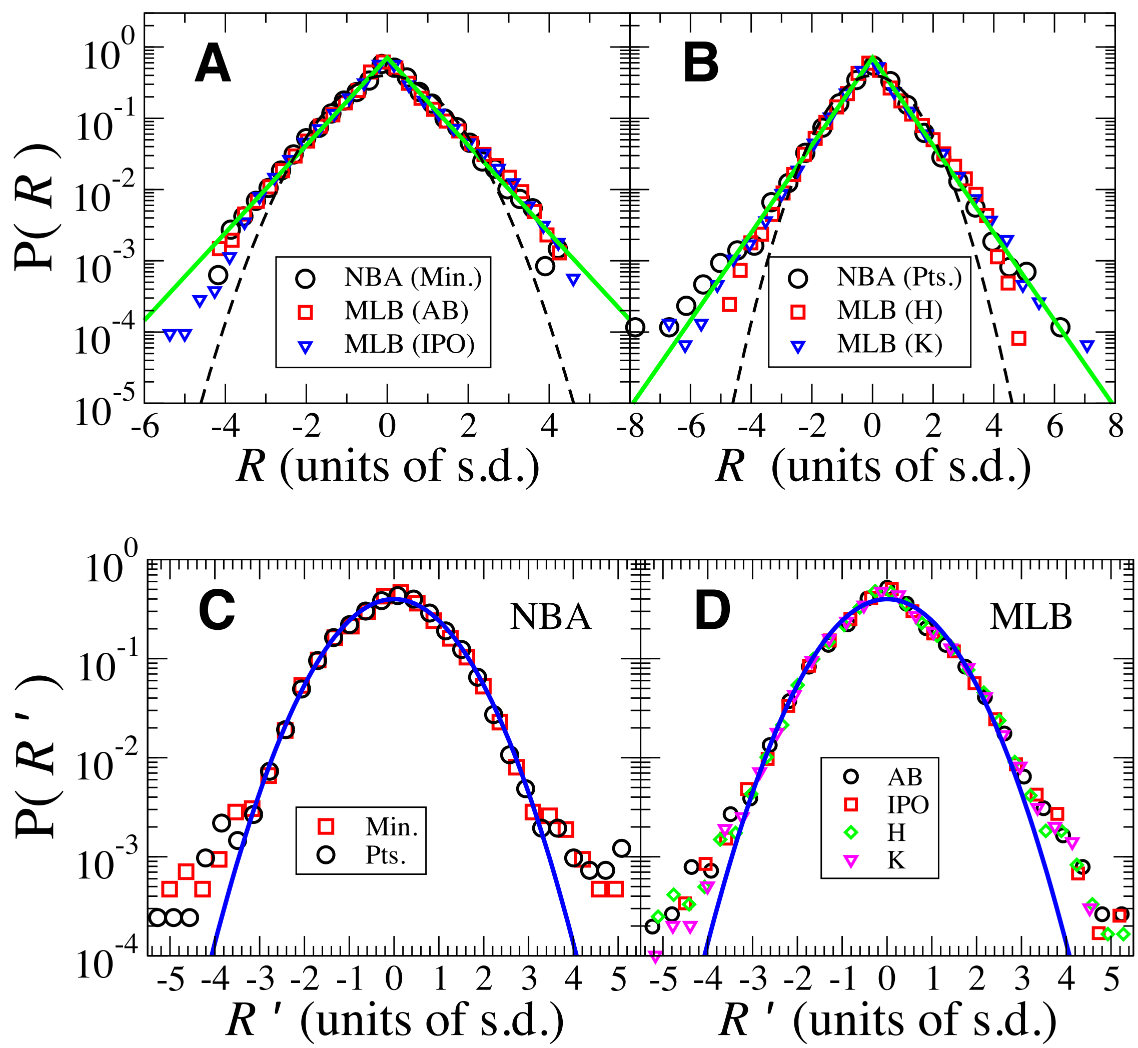

FIG. S6: Universal patterns in the production fluctuations of athletes. For athlete careers in the NBA and MLB we define production change for (A,C) the change in the number of in-game opportunities and (B,D) the change in the number of in-game successes. (A,B) Since the detrended production change R is defined to have standard deviation $\sigma \equiv 1$, the pdfs $P(R)$ approximately collapse onto a universal "tent-shaped" Laplace pdf (solid green line). (C,D) For sports careers, we also define a measure R' which account for variable individual production factors, such as propensity for injury, team position, etc. As a result normalized annual growth rate $R'_i \equiv [z_i(t) - \langle z(t)\rangle]/\sigma_{z(t)}$ is normalized twice, once to account for age factors and once to account for individual factors. The quantity $z_i(t) \equiv (r_i(t) - \langle r_i\rangle)/\sigma_i$ is normalized with respect to individual factors, where $\langle r_i\rangle$ and σ_i are the average and standard deviation of the production change of career i. Then, we aggregate all $z_i(t)$ values for a given career year t in order to calculate the average $\langle z(t)\rangle$ and standard deviation $\sigma_{z(t)}$ over all careers. The final quantity R'_i represents a normalized annual production change which is distributed in the bulk according to a Gaussian distribution, with $\langle R'\rangle \approx 0$ and $\sigma(r') \approx 1$ by construction (solid lines show best-fit Guassian distributions using the maximum likelihood estimator method). This results indicates that the tent-shaped distributions in (A,B) results from a mixture of conditional Gaussian distributions $P_i(R = \sigma_i R')$ that indicate that annual production is consistent with a proportional growth model.

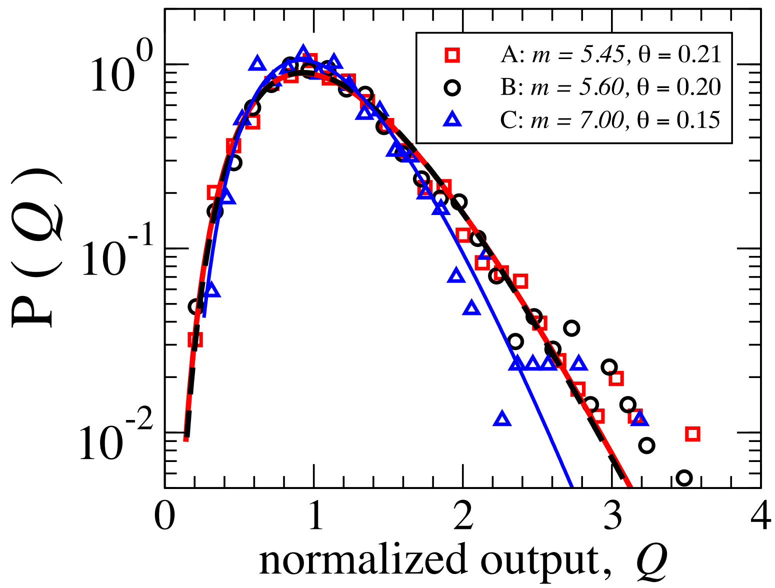

FIG. S7: Universal micro-scale output distribution $P(Q)$ which accounts for coauthorship variability. The normalized output $Q \propto n_i/k_i^{\gamma_i}$ is a residual output after we quantitatively account for the collaboration size k_i corresponding to the number of distinct coauthors of author i. Each pdf is well-approximated by the Gamma distribution $P(Q) \propto Q^{m-1} \exp[-Q/\theta]$ which suggests that production at the micro scale is governed by a Gamma Lévy process. We calculate the Gamma distribution parameters using the maximum likelihood estimator method (distributions shown by solid and dashed curves), and find an insignificant difference between [A] and [B] scientists with Gamma shape parameter m and scale parameter θ. However, for dataset [C] scientists, the output distribution is more skewed towards smaller Q values, possibly reflecting the relative advantage that senior scientists gain due to reputation, experience, and knowledge spillover factors.

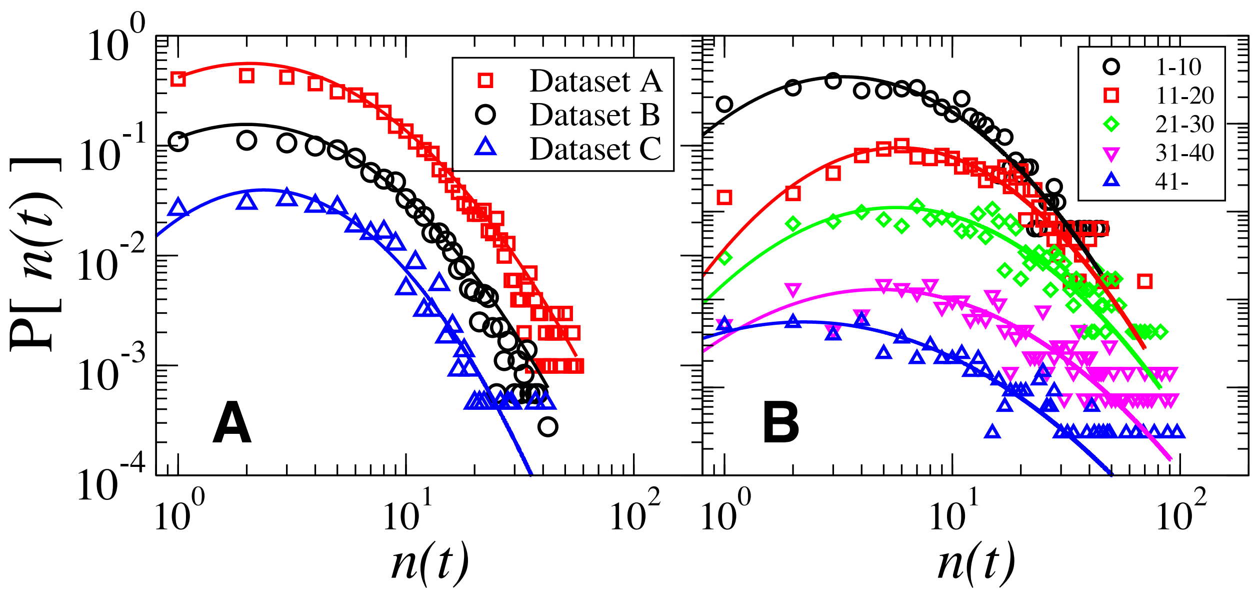

FIG. S8: Aggregate production distributions can be deceiving. Unconditional distribution of annual publication rate $n(t)$ appears as log-normal distributions because it is a mixture of underlying distributions that depend strongly on collaboration factors. We define $n_i(t)$ as the number of papers published in **(A)** $\Delta t = 1$ and **(B)** $\Delta t = 2$ year periods, which reduces the finite-size effects arising from the calendar year labeling of publication dates. (A) We combine $n_i(t)$ values for all values of t, and find excellent agreement between the empirical $P(n(t))$ data points and the log-normal model. We use the maximum likelihood estimator method to calculate the log-normal parameters $\sigma_L \equiv \sigma(\ln n)$ and $\mu = \langle \ln n \rangle$. **(B)** In order to analyze the time-dependence of $P(n(t))$, we separate $n_i(t)$ values from Dataset A into 5 subsets, depending on the range t years into the career, as indicated in the figure legend. We offset each pdf by a constant factor in order to distinguish each pdf, which are also well-approximated by log-normal distributions (shown as solid curves).

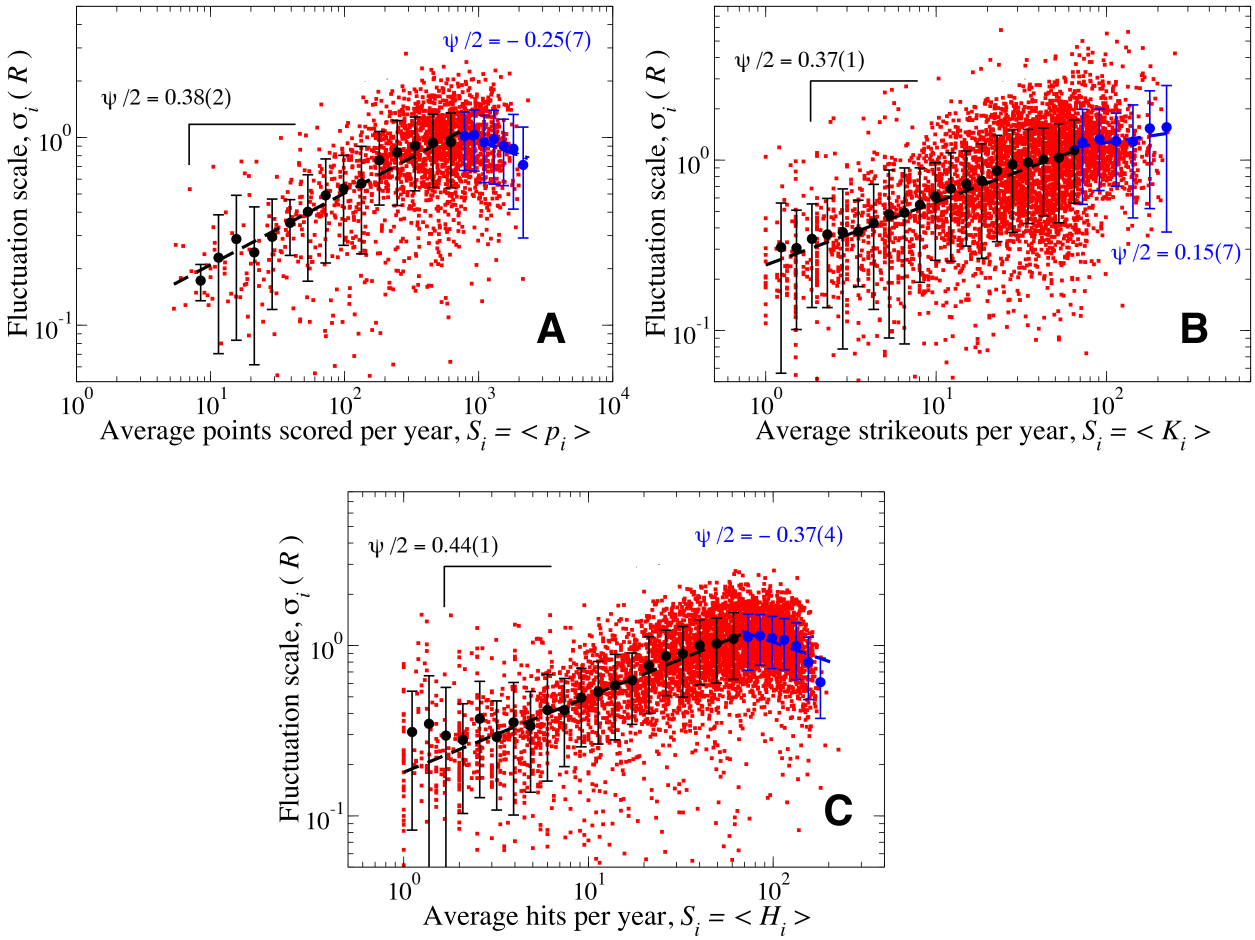

FIG. S9: Quantifying the growth fluctuations of sports careers. The size variance relation for sports careers is similar to academic careers for small S_i. However, for relatively large S_i the relation becomes decreasing corresponding to $\psi < 0$, analogous to what is found for firm growth [33, 47–49]. The decreasing relation for $S_i > S_c$ likely follows from the fact that in sports, there is a hard upper limit to the number of opportunities available to a player in a given year. Hence, individuals with large S_i are likely the starters on their teams, since it is neither economical nor in the strategy of winning to keep players above a threshold value S_c out of the game, and so these players typically remain as positional starters except for episodic leaves of absence due to injury. Hence, these players experience smaller $\sigma_i(r)$ due to limitations to their potential for further career growth. However, players with $S_i < S_c$ are typically on the fringe of being released or provide alternative value to the team, and so these individuals experience larger fluctuations in team play because they are easily dispensable, especially in a profession dominated by short-contracts lasting sometimes less than a year. For each dataset, we use careers with career length $L_i \geq 3$ seasons. **(A)** NBA basketball players: Units of $\sigma_i(R)$ are normalized minutes played. We define the scaling relation $\sigma_i(R) \sim \langle p_i \rangle^{\psi/2}$ between the average number of points scored per season $\langle p_i \rangle = \sum_{t=1}^{L_i} p_i(t)/L_i$ and the standard deviation $\sigma_i(R)$. In this way, we utilize the average points per season as the proxy for the ability of a player to obtain future opportunities which are realized as minutes played. Using $S_c \equiv 720$ points, we calculate $\psi/2 = 0.38 \pm 0.02$ (regression coefficient $R = 0.50$ and ANOVA F-test significance level $p \approx 0$) for $S_i < S_c$ and $\psi/2 = -0.25 \pm 0.07$ ($R = 0.15$ and $p \approx 10^{-3}$) for $S_i > S_c$. **(B)** MLB pitchers: Units of $\sigma_i(R)$ are normalized IPO (innings pitched in outs). Interestingly, $\sigma_i(R)$ continues to increase for $S_i > S_c$, possibly due to the relatively high career risk attributed to throwing arm injury. Using $S_c \equiv 65$ strikeouts, we calculate $\psi/2 = 0.37 \pm 0.01$ ($R = 0.48$ and $p \approx 0$) for $S_i < S_c$ and $\psi/2 = +0.15 \pm 0.07$ ($R = 0.07$ and $p \approx 0.02$) for $S_i > S_c$. **(C)** MLB batters: Units of $\sigma_i(R)$ are normalized AB (at bats). Using $S_c \equiv 68$ hits, we calculate $\psi/2 = 0.44 \pm 0.01$ ($R = 0.59$ and $p \approx 0$) for $S_i < S_c$ and $\psi/2 = -0.37 \pm 0.03$ ($R = 0.21$ and $p \approx 0$) for $S_i > S_c$. The dashed black (blue) line in each panel is a least squares linear regression on log-log scale for all data values with S_i less (greater) than S_c. The data shown with error bars represent the average $\langle \sigma_i(R) \rangle$ and corresponding 1 standard deviation values calculated using equally spaced S_i bins on the logarithmic scale.

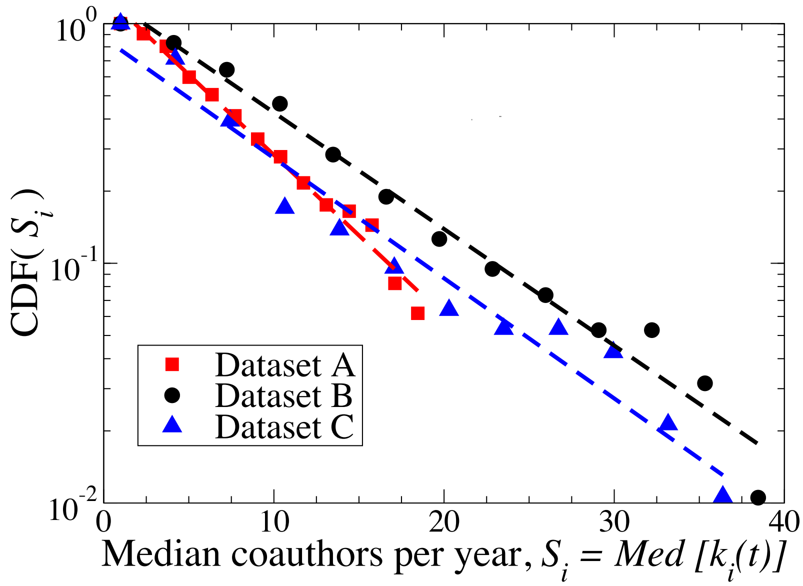

FIG. S10: Exponential distributions of coauthor radius in Physics. We test the hypothesis that the distributions $P(r)$ for annual production change r (shown in Fig. 2) follow from an exponential mixing of Gaussians with varying fluctuation scale $\sigma_i \propto Med[k_i(t)]^{\psi/2}$. An important criteria for this model is that the distribution of $S_i \equiv Med[k_i(t)]$ is exponential, $P(S_i) \sim \exp[-\lambda S_i]$. We plot the cumulative distribution function (CDF) $P(x > S_i)$ for each dataset, and confirm that the distributions are approximately linear on log-linear axes. Using linear regression, we calculate $\lambda = 0.15 \pm 0.01$ [A], $\lambda = 0.11 \pm 0.01$ [B], and $\lambda = 0.11 \pm 0.01$ [C].

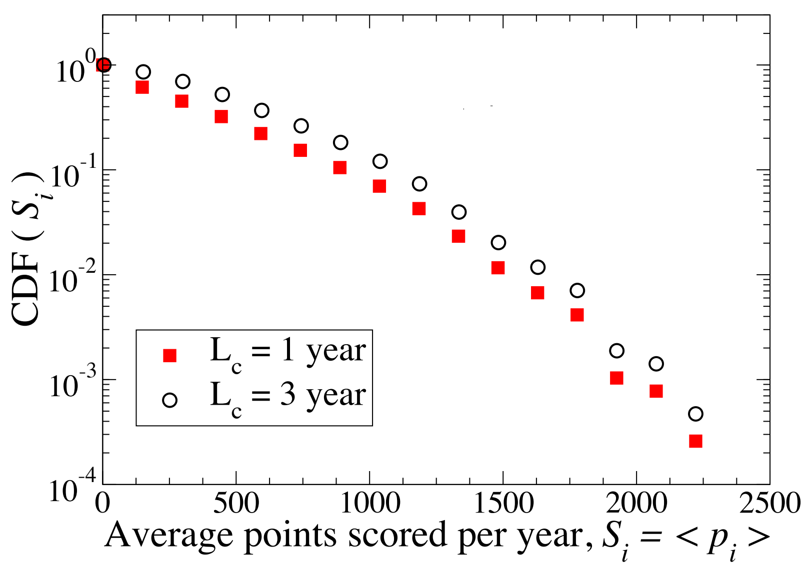

FIG. S11: Approximately exponential distribution of scoring value in the NBA. We further test the hypothesis that the distributions $P(R)$ for annual production change R in professional sports (shown in Fig. 2 C and D) follow from an exponential mixing of Gaussians with varying fluctuation scale $\sigma_i \propto \langle p_i \rangle^{\psi/2}$. An important criteria for this model is that the distribution of "team value" $\langle p_i \rangle$ is exponential, $P(\langle p_i \rangle) \sim \exp[-\lambda \langle p_i \rangle]$. We plot the cumulative distribution function (CDF) $P(x > \langle p_i \rangle)$ for each dataset, and confirm that the distributions are approximately linear on log-linear axes. We show the CDFs calculated using all careers with career length $L_i \geq L_c$ years, for $L_c = 1, 3$ years.

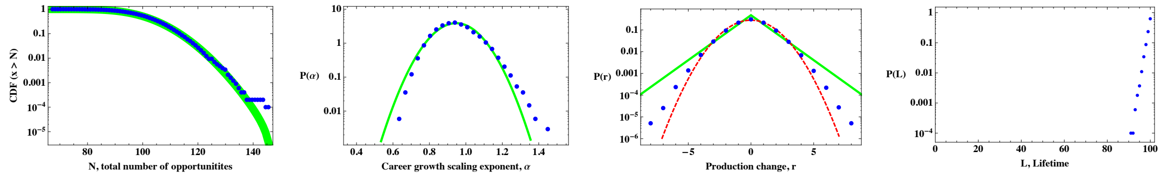

FIG. S12: A production output null model with $\pi = 0$ agrees with the predictions of a Poisson process. (Far left) The cumulative distribution $CDF(x > N)$ is in excellent agreement with the prediction of a Poisson process with rate $\lambda_p = 1$ and corresponding average $\langle N \rangle = \lambda_p T = 100$. The solid green curve is the corresponding Poisson CDF using $\langle N \rangle \equiv 100$. (Middle left) Furthermore, the typical scaling exponent $\langle \alpha \rangle = 1$ which is also consistent with Poisson trajectories. (Middle right) The distribution of production changes is close to Gaussian. (Far right) The typical career length L_i spans the entire system length T, indicating low levels of career risk.

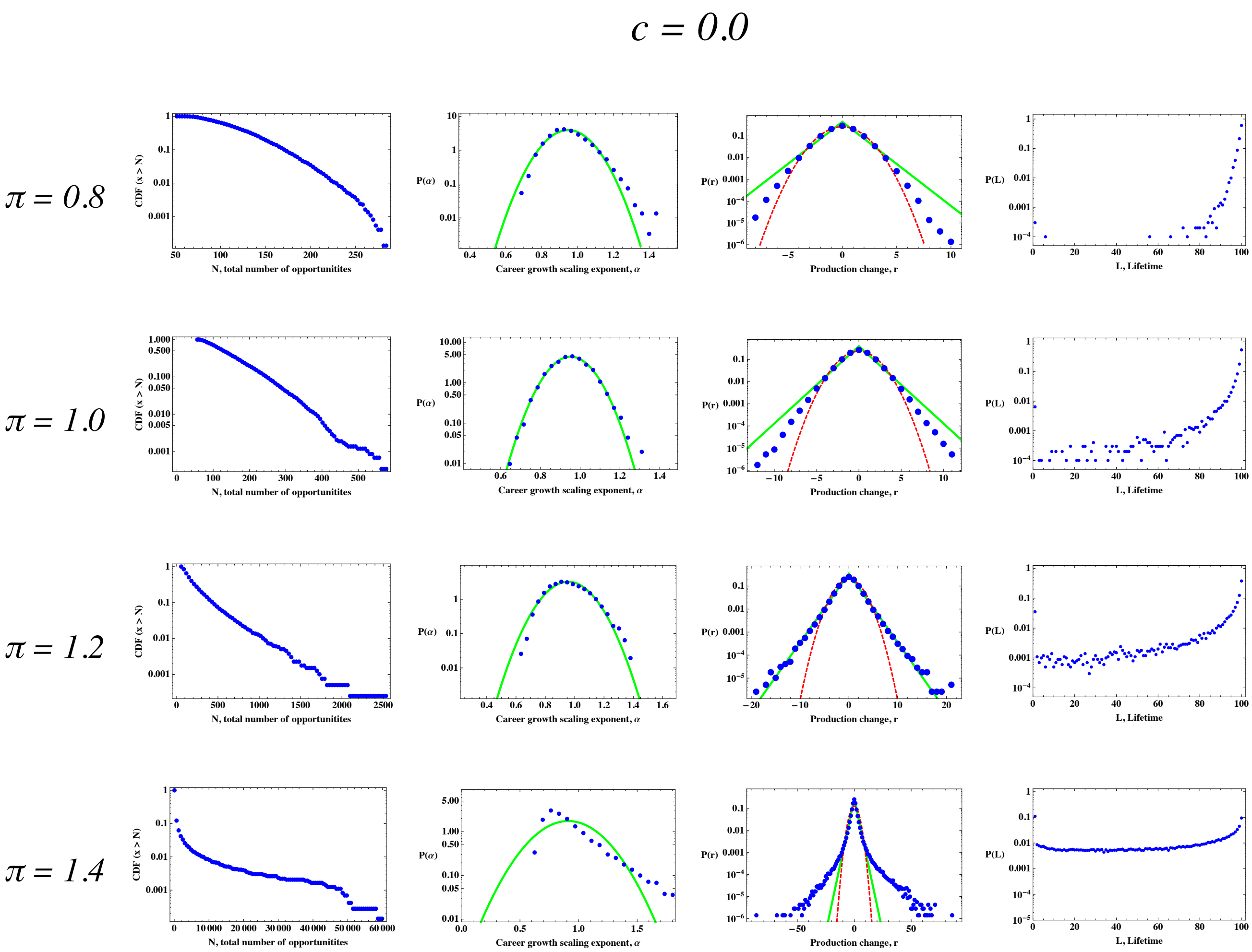

FIG. S13: The production output model with $c = 0$. Results of MC simulations for a "long-term appraisal" scenario. Careers are less vulnerable to low-production phases, and as a result, most agents sustain production throughout the career for a relatively large range of π values.

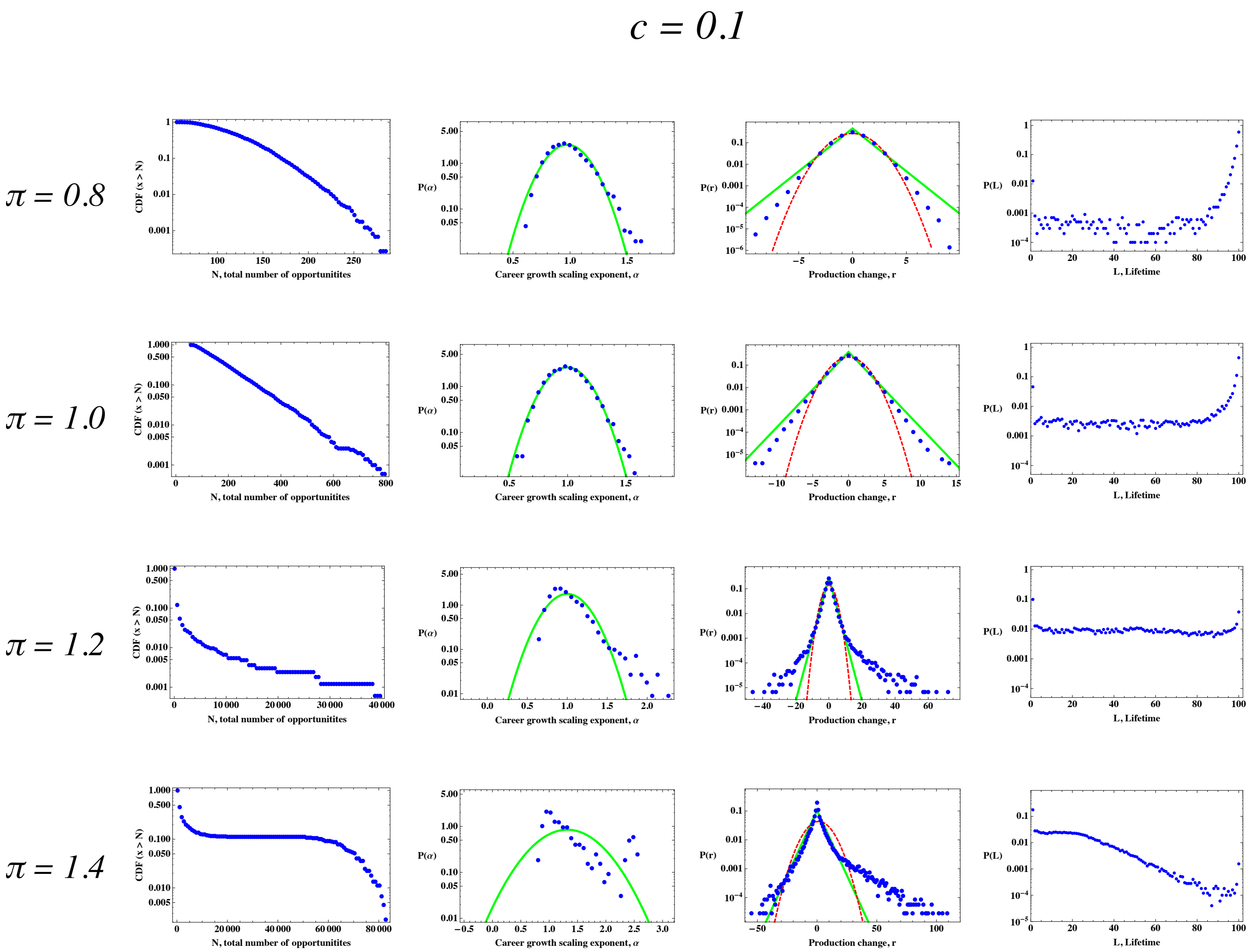

FIG. S14: The production output model with $c = 0.1$. Results of MC simulations for a "medium-term appraisal" scenario. The corresponding memory time scale is approximately 10 time periods, and so only for significantly large $\pi = 1.4$ do we observe a labor market scenario in which there is a significant death rate and just a few "big winners" corresponding to those agents with $\alpha \geq 1$.

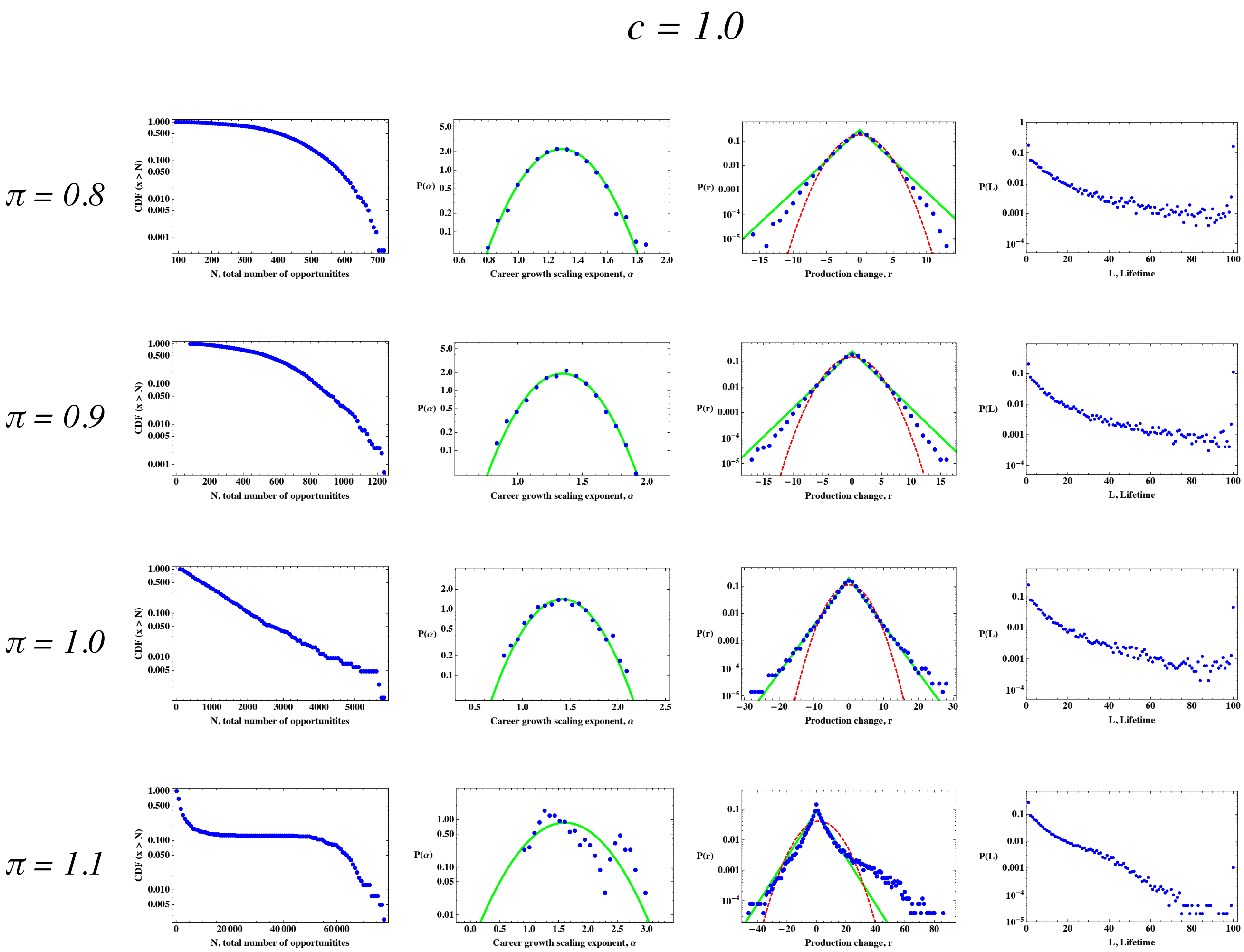

FIG. S15: The production output model with $c = 1.0$. Results of MC simulations for a "short-term appraisal" scenario. The corresponding memory time scale is approximately 1 time period. Even for $\pi < 1$, the system is driven by fluctuations that can cause career "sudden death" for a large fraction of the population. For $\pi > 1$ we observe a very quick transition to a significant death rate and just a few "big winners" corresponding to those agents with $\alpha \geq 1$.

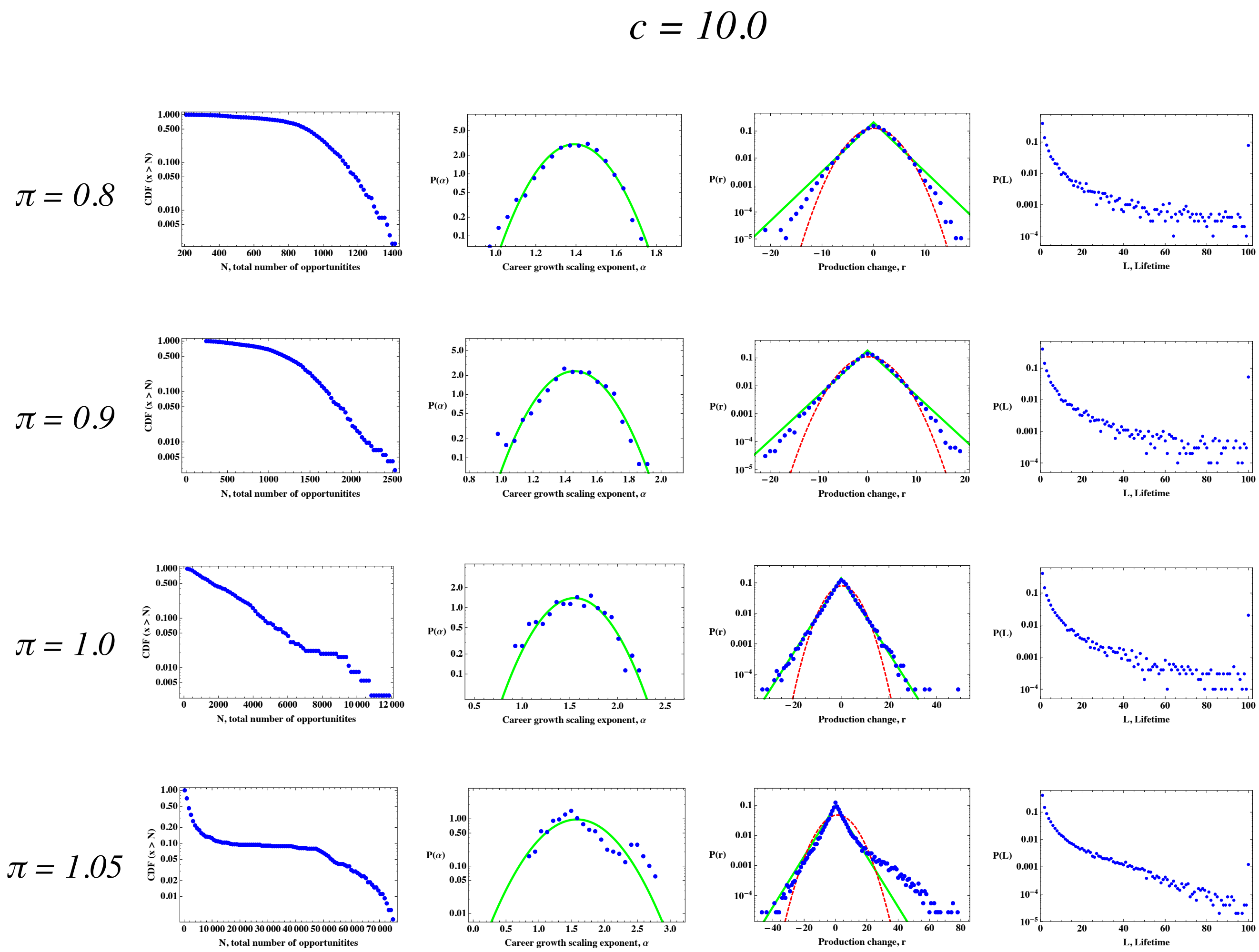

FIG. S16: The production output model with $c = 10.0$. Results of MC simulations for a "zero-memory appraisal" scenario wherein only the previous period matters, $w_i(t) = n_i(t-1)$. Even for linear preferential capture $\pi = 1$, the systems shows "no mercy" for careers that are stagnant for possibly just one period. As a result, just a few "lucky" agents are able to survive the initial fluctuations and end up dominating the system. For π values close to unity, $\pi \to 1$, the systems quickly becomes an employment "death trap" whereby most careers stagnate and "flat-line."