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ECONOPHYSICS: WHAT CAN PHYSICISTS CONTRIBUTE TO ECONOMICS?

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In recent years, a considerable number of physicists have started applying physics concepts and methods to understand economic phenomena. The term "Econophysics" is sometimes used to describe this work. Economic fluctuations can have many repercussions, and understanding fluctuations is a topic that many physicists have contributed to in recent years. Further, economic systems are examples of complex interacting systems for which a huge amount of data exist and it is possible that the experience gained by physicists in studying fluctuations in physical systems might yield new results in economics. Much recent work in econophysics is focused on understanding the peculiar statistical properties of price fluctuations in financial time series. In this talk, we discuss three recent results. The first result concerns the probability distribution of stock price fluctuations. This distribution decreases with increasing fluctuations with a power-law tail well outside the Lévy stable regime and describes fluctuations that differ by as much as 8 orders of magnitude. Further, this nonstable distribution preserves its functional form for fluctuations on time scales that differ by 3 orders of magnitude, from 1 min up to approximately 10 days. The second result concerns the accurate quantification of volatility correlations in financial time series. While price fluctuations themselves have rapidly decaying correlations, the volatility estimated by using either the absolute value or the square of the price fluctuations has correlations that decay as a power-law and persist for several months. The third result bears on the application of random matrix theory to understand the correlations among price fluctuations of any two different stocks. We compare the statistics of the cross-correlation matrix constructed from price fluctuations of the leading 1000 stocks and a matrix with independent random elements, i.e., a random matrix. Contrary to first expectations, we find little or no deviation from the universal predictions of random matrix theory for all but a few of the largest eigenvalues of the cross-correlation matrix.

Keywords: Scaling, distributions, power-laws, returns, Lévy.

1. Introduction

The analysis of financial data using concepts and methods developed for physical systems has a long tradition [1–4] and has recently attracted the interest of physicists [5–9]. Possible reasons for this interest include the scientific challenge of understanding the dynamics of a strongly fluctuating complex system with a large number of interacting elements.

One can ask how physicists can contribute to the search for solutions to the puzzles posed by modern economics that economists themselves have not yet solved? One approach — in the spirit of experimental physics — is to begin with the empirical data that one can analyze in some detail, but without prior models. In economic systems such as financial markets, one has available a great deal of data. Moreover, if one has at one's disposal the tools of statistical physics and the computing power to carry out any number of approaches, this abundance of data is to great advantage. Thus, for many physicists, studying the economy means studying a wealth of data on a strongly fluctuating complex system. Indeed, physicists in increasing numbers are finding problems posed by economics sufficiently challenging to engage their attention [10–26].

Recent studies attempt to uncover and explain the peculiar statistical properties of financial time series such as stock prices, stock market indices or currency exchange rates. The dynamics of financial markets is difficult to understand not only because of the complexity of its internal elements but also due to the many intractable external factors acting on it, which may differ from market to market. Remarkably, the statistical properties of certain observables appear to be similar for quite different markets [27], consistent with the possibility that there may exist "universal" mechanisms.

The most challenging difficulty in the study of financial markets is that the nature of the interactions between the different elements comprising the system is unknown, as is the way in which external factors affect it. Therefore, as a starting point, one may resort to empirical studies to help uncover the regularities or "empirical laws" that may govern financial markets [28]. The interactions between the different elements comprising financial markets generate many observables such as the transaction price, the share volume traded, the trading frequency, and the values of market indices. Recent empirical studies are based on the analysis of price fluctuations. This talk reviews recent results on (a) the distribution of stock price fluctuations and its scaling properties, (b) time-correlations in financial time series, and (c) correlations among the price fluctuations of different stocks. Space limitations restrict us to focusing mainly on our group's work; a more balanced account can be found in two recent books [5, 6], other articles in these proceedings, and two other recent international conferences [7, 9]. Recent work in this field also focuses on applications such as risk control, derivative pricing, and portfolio selection [29], which shall not be discussed in this talk. The interested reader should consult, for example, Refs. [5, 6] and [30].

2. What is the Question?

The key question for physicists entering this field — or virtually any other field physicists might enter — is "how do I quantify things?" There are many things to quantify in economics.

An appropriate place to begin is with simple fluctuations. Consider a prototype fluctuating quantity: stock market indices. If we make a graph on log-linear paper of the value of the stock index as a function of time, we see fluctuations — sometimes even dramatic fluctuations such as the negative 25 percent fluctuation on Black Monday in 1987. When we first examine these fluctuations, those of us who have worked in statistical mechanics naturally think of a simple one-dimensional random walk — with the displacement of the random walk on the y-axis and the time on the x-axis. When we do such a plot on a graph, we immediately see that the curve produced does not agree with the empirical data for stock-price fluctuations.

If this approach is unhelpful, what do we do next? How are we to understand these fluctuations? For that matter, why would we want to?

3. Why Do We Care?

There are many practical reasons for wanting to understand stock-price fluctuations. The first reason is painfully obvious: although not everyone owns stocks, everyone is powerfully affected by stock markets and financial systems. If a country goes bankrupt and the food supply fails, the poorest citizens — who probably are not stockholders — still suffer. Beginning to understand financial fluctuations means beginning to understand risk, and if we can begin to quantify risk, then perhaps we can develop ways to manage risk.

The second reason for wanting to understand stock-price fluctuations is simply that people are interested in the topic. Within the physics community, econophysics is a fast-growing subfield. The business community has become interested, and recently there was an article on econophysics in the *Wall Street Journal*.

The third reason for wanting to understand stock-price fluctuations is intellectual. The economy is a complex system, but is unlike other complex systems we study, and offers unique intellectual opportunities. In most research on complex systems, we start with a general statement, invent some sort of theoretical model to test the general statement, compare the data produced by the model with the general statement, and only then, perhaps, compare the theoretical data with whatever real-world data might be available. When the subject of research is the economy, however, the situation is totally different. The economy is data-dominated. There are huge quantities of data on the economy. Virtually *every* economic transaction today is recorded. Many of these data are available on the internet. This "complex system" has already been extensively analyzed, of course, and most of this analysis has been done by economists. What can we physicists add to what has already been done? How might our approach differ from that of economists?

4. What Do We Do?

We start with the initial set of data: a simple time series giving the value of some stock average as a function of time. How can we improve on the biased random walk model to describe this set of data? We analyze a fundamental variable: the change (g) in the value of this index over some time window (Δt). Obviously the value of g depends on the size of Δt, and on where in time we look at g. If what we see in this initial data set does not conform to the behavior of a simple random walk, perhaps we can find some correlations.

Our results show that when we examine the correlations in the price changes themselves, g, using a log-linear scale (with the logarithmic y-axis showing the price changes and the linear x-axis showing the time), an approximate straight line is produced (see Ref. [31] and citations therein). This line extends out to about 20 minutes on the linear time scale, at which point it hits the noise level. A straight line on log-linear paper indicates an exponential decrease, which is consistent with these data in this autocorrelation function — with a time constant of about four minutes. In order to make any money with this observed correlation, one would have to react on a time scale significantly shorter than four minutes.

When we examine the correlations in the *absolute value* of g, we discover not exponential behavior but power-law behavior. On the logarithmic y-axis we now have the autocorrelation function of the absolute values of g. This time the data are only approximately straight over a little more than one decade. Although they are quite noisy, they still approximate a power-law slope of 0.3.

To study this in more detail, we can analyze the power spectrum of the correlation function. The slope of the power-law changes at approximately one day — one slope with a value of 0.3 for time scales shorter than approximately one day and another slope with a value of 0.9 for time scales longer than approximately one day. The crossover at approximately one day is a genuine property of the data. If we shuffle the data before we do the analysis, we end up with white noise.

To summarize thus far: because the experimental data display correlations with a very short range ($\approx$ 4 minutes), we cannot understand the time-series and the fluctuations — and we cannot quantify them — simply by using second-order correlation functions to explain the data.

What has been done traditionally in this field? In 1963, Benoit Mandelbrot published a paper in which he analyzed the fluctuations in the market price of cotton [4]. Available to him were 1000 data points in three different data sets, which he used to analyze the histogram of the price changes of this single commodity. He plotted the cumulative distribution function and then turned his attention to the behavior of the tails of the distribution. He was familiar with Pareto's work on power-law tails, so he decided to use log-log plots of his data sets. For the three data sets, he ended up with six curves — three for the positive tails and three for the negative. Since the six all approximate a straight line, we know the tails display a power-law behavior. Mandelbrot described the tails using an exponent, α, of approximately 1.7. This exponent falls within the range of that predicted by a Lévy distribution, so Mandelbrot concluded that fluctuations in the price of cotton could be described by a Lévy distribution. He also concluded that the exponent was the same for time windows ranging from one day to one month.

Although Mandelbrot's paper was seminal in the field, because he had only daily and monthly data — and not shorter time-windows of Δt — the data fluctuations span only two orders of magnitude. Since we now have much more data available to us, how do we extend the work Mandelbrot started?

This has been done by Rosario Mantegna, who has followed the same prescription as Mandelbrot and has analyzed not fluctuations in cotton prices, but fluctuations in the S&P 500 stock index, the weighted average of the 500 largest firms in the US. He studied them not in daily time intervals, but in one minute time intervals (roughly 300 times smaller) [18]. His data span six years. Thus he was dealing with not 1000 records, but on the order of magnitude of one million records — three orders of magnitude more data.

He has found that, out to approximately 5 or 6 standard deviations, the S&P 500 data conform to Mandelbrot's Lévy distribution (but with a parameter $\alpha \approx 1.4$ instead of $\alpha \approx 1.7$). Beyond approximately 5 or 6 standard deviations, the data deviate below the prediction of the Lévy distribution. This is probably a good thing; if the Lévy distribution held out to, say, 20 standard deviations, one would have many more Black Mondays. The fact that the data are truncated is also good for an intellectual reason: Lévy distributions have an infinite variance, and this truncation restores the finite variance. So the data are consistent with a truncated Lévy distribution ("flight") [32].

How robust is this truncated Lévy flight behavior? Skeltorp used the same approach to study the behavior of the OVX index of the Norwegian stock market with equivalent time-interval ranges. The results are the same — the same Lévy distribution seems to hold for 4 to 5 standard deviations, and then the data begin to fall below the Lévy distribution. So this behavior does indeed seem robust.

Mantegna's work has been extended by Gopikrishnan, who, instead of working with stock averages, worked with individual stocks (see Ref. [33] and citations therein). Instead of the ≈ 2000 data points of Mandelbrot, or the ≈ 1.5 million data points, of Mantegna, Gopikrishnan used ≈ 40 million data points. He took the 1000 largest stocks, which, if simply averaged, look benign — but individually they exhibit much larger fluctuations. When working with individual stocks, Gopikrishnan found the most useful minimum time window to be five minutes. Because the plots from these short time windows of individual stocks are extremely noisy, the cumulative distribution function (the integral of the pdf) is used. If you plot the cumulative distribution function on a sheet of log-log paper, you see an approximate straight line with a slope of ≈ -3. The slope for each stock will be slightly different. If we make a histogram of all of the slopes of the stocks, we get a bell-shaped curve centered around the value of 3.

This is interesting because, in the previous work by Mandelbrot in which he describes cotton-price fluctuations as a Lévy distribution, the bell-shaped curve is centered around some value between 0 and 2 — which is one of the characteristics of a Lévy distribution. The fact that Gopikrishnan's bell-shaped curve is centered around 3 means that the data are *not* described by a Lévy distribution in the tails. Thus this surprising discovery disagrees with the classic work of Mandelbrot.

To check this discovery, Gopikrishnan and his collaborators spent a year and a half checking their work. They found that the cumulative distribution function on log-log paper is a function of the scale of returns of 100 standard deviations follows a power law, i.e., has a slope of ≈ -3, which — irrespective of error bars — excludes the value 2. The data are described by this power law distribution out to 10^{-8}.

One physical implication of this is that "shocks" in the economy are not isolated points that have to be added to the data. They are actually part of the data set. If we return to the stock average provided by the S&P 500, we see the same cumulative distribution function with the Lévy distribution in the center and a crossover — not to an exponentially truncated tail, but to a power-law truncated tail, with $\alpha \approx 3$.

In conclusion, we can say that the price fluctuations of individual stocks are consistent with this relatively simple power-law distribution and are at odds with Mandelbrot's Lévy distribution. They are consistent with the power-law distribution over fully 100 standard deviations. This distribution preserves its functional form in time windows ranging from one minute up to almost 4 orders of magnitude. We can also say that the amplitudes of price changes, i.e., the absolute values of price changes, display long-range correlations — even when the price changes themselves do not.

5. Correlations among Different Units

Recently, the problem of understanding the correlations among the returns of different stocks has been addressed by applying methods of random matrix theory to the cross-correlation matrix [34, 35]. Aside from scientific interest, the study of correlations between the returns of different stocks is also of practical relevance in quantifying the risk of a given portfolio [29]. Consider, for example, the equal-time correlation of stock returns for a given pair of companies. Since the market conditions may not be stationary, and the historical records are finite, it is not clear if a measured correlation of returns of two stocks is just due to "noise" or genuinely arises from the interactions among the two companies. Moreover, unlike most physical systems, there is no "algorithm" to calculate the "interaction strength" between two companies (as there is for, say, two spins in a magnet). The problem is that although every pair of companies should interact either directly or indirectly, the precise nature of interaction is unknown.

In some ways, the problem of interpreting the correlations between individual stock-returns is reminiscent of the difficulties experienced by physicists in the fifties, in interpreting the spectra of complex nuclei. Large amounts of spectroscopic data on the energy levels were becoming available but were too complex to be explained by model calculations because the exact nature of the interactions were unknown. Random matrix theory (RMT) was developed in this context, to deal with the statistics of energy levels of complex quantum systems [36–38]. With the minimal assumption of a random Hamiltonian, given by a real symmetric matrix with independent random elements, a series of remarkable predictions were made and successfully tested on the spectra of complex nuclei [36]. RMT predictions represent an average over all possible interactions [37]. Deviations from the *universal* predictions of RMT identify system-specific, non-random properties of the system under consideration, providing clues about the underlying interactions [38].

Recently, Plerou and her collaborators analyzed the cross-correlation matrix $\mathbf{C} \equiv C_{ij} \equiv \langle G_i G_j \rangle - \langle G_i \rangle \langle G_j \rangle / \sigma_i \sigma_j$ of the returns at 30-minute intervals of the largest 1000 US stocks for the two-year period 1994–1995. They analyze the statistical properties of $\mathbf{C}$ by applying techniques of random matrix theory (RMT) [34, 35]. First, they test the eigenvalue statistics of the cross-correlation matrix for universal properties of real symmetric random matrices such as the Wigner surmise for the eigenvalue spacing distribution and eigenvalue correlations. Remarkably, they find that eigenvalue statistics of the correlation matrix agree well with the universal predictions of random matrix theory for real symmetric random matrices, in contrast to our naive expectations for a strongly interacting system.

Deviations from RMT predictions represent genuine correlations. In order to investigate deviations, we compute the distribution of the eigenvalues of the $\mathbf{C}$ and compare with the prediction [34] for uncorrelated time series [39]. We find that the statistics of all but a few of the largest eigenvalues in the spectrum of $\mathbf{C}$ agree with the predictions of random matrix theory, but there are deviations for a few of the largest eigenvalues [34, 35]. The deviations of the largest few eigenvalues from the random matrix result are also found when one analyzes the distribution of eigenvector components. Specifically, the largest eigenvalue which deviates significantly (25 times larger than random matrix bound) has almost all components participating equally and thus represents the correlations that pervade through the entire market. This result is in agreement with the results of Laloux and collaborators [34] for the eigenvalue distribution of $\mathbf{C}$ on a daily time scale.

6. How Economic Organizations Grow and Shrink

Most current research on the economy starts off by dividing the economy into sectors: food, automotive, computer, entertainment, and so on. Then the interactions between firms within a sector are analyzed, e.g., how does the behavior of General Motors affect the behavior of Ford? The assumption is that firms that compete directly affect each other's behavior much more strongly than firms that do not.

Physicists look at this model and immediately see a similarity between it and something we were playing with 30 years ago during the early days of critical phenomena research, when we divided a set of spins into cluster subsystems. We treated exactly the interactions of a spin within the small cluster of spins surrounding it, and approximated the interactions between spins in that cluster and spins in more distant clusters. We eventually abandoned these "effective field theories of magnetism" because they failed to give information that was in accord with accurate experiments.

What is the analogy with research on the economy? Direct interactions within sectors (clusters) are obvious; if General Motors has quality-control problems, their customers will start buying Fords. Indirect interactions between sectors (clusters) may not be immediately obvious, but they are there; Ford has to hire more workers to meet the rising demand for their cars and the McDonald's outlet across the street from the assembly plant has to expand to accommodate the much larger lunchtime crowd.

It is a kind of spin-glass with both ferromagnetic and antiferromagnetic interactions, both short-range and long-range. Unlike the spin-glass, however, we have no *a priori* way to plausibly choose which interactions to study.

There are data available on the approximately 4000 publicly-traded firms listed in the stock markets — by law they are required to make a great deal of information public. One category of data that is often of interest is whether a firm is growing or shrinking. The sales of a company this year divided by the sales last year is one possible measure of the growth rate.

In making a histogram of growth rates, we could put all 4000 companies together and make one single histogram. Instead, however, we divide the 4000 companies into 20 "bins" according to each company's size. When we do that, we get different histograms for different sizes. Smaller companies can grow — or shrink — more rapidly than larger companies. It is highly unlikely that Ford would grow or shrink by a factor of 10 in a single year. On the other hand, a small company can — and often will — grow or shrink by a factor of 10 in a single year. So each of the 10 histograms has a different characteristic standard deviation or "width." The width is found to be a decreasing function of the size of the firm — the larger the firm, the smaller the width.

If we make a plot with the width on the y-axis and the firm size — measured in amount of sales — on the x-axis, we find we have an approximate straight line over eight or nine decades. If we make another plot, this time using number of employees as the measure of firm size, we find we have another approximate straight line, this time over roughly five decades. Remarkably, the slopes of these two straight lines are identical. After checking this empirical law of economics by applying it to a number of financial markets in countries other than those in the US, we find that it is quite a robust law [40].

This law has also been applied to not just the financial status of firms and markets, but also to the overall economies of entire countries. Data obtained through the Harvard Institute for International Development on the economies of 152 countries have been analyzed using the same procedures, and virtually the same results as those obtained for business firms were produced [41]. The law has also been applied to data related to changes in the size of university research budgets [42], as well as to data recording the changing populations of various species of birds [43], and similar results were found in each case.

Buldyrev *et al.* models this firm structure as an approximate Cayley tree, in which each subunit of a firm reacts to its directives from above with a certain probability distribution [44, 45]. More recently, Amaral *et al.* [46] have proposed a microscopic model that reproduces both the exponent and the distribution function. Takayasu and Okuyama [40] extended the empirical results to a wide range of countries.

7. Open Questions

Econophysics is a field wherein questions are as difficult to pose as to answer. The empirical results shown above clearly beckon explanation. For example, in first two sections, we have looked mainly at two empirical results: (i) the distribution of fluctuations, which shows a power law behavior well outside the stable Lévy regime, and yet preserves its shape — scales — for a range of time scales and (ii) the long range correlations in the amplitude of price fluctuations. How are the two related?

Previous explanations of scaling relied on Lévy stable [4] and exponentially-truncated Lévy processes [6, 18]. However, the empirical data that we analyze are not consistent with either of these two processes. In order to confirm that the scaling is *not* due to a stable distribution, one can randomize the time series of 1 min returns, thereby creating a new time series which contains *statistically-independent* returns. By adding up n consecutive returns of the shuffled series, one can construct the n min returns. Both the distribution and its moments show a rapid convergence to Gaussian behavior with increasing n, showing that the time dependencies, specifically volatility correlations are intimately connected to the observed scaling behavior [33].

Using the statistical properties summarized above, can we attempt to deduce a statistical description of the process which gives rise to this output? For example, the standard ARCH model [28, 47] reproduces the power-law distribution of returns; however it assumes finite memory on past events and hence is not consistent with long-range correlations in volatility. On the other hand, the distribution of volatility and that of returns which have similar asymptotic behavior, however support the central ARCH hypothesis that $g(t) = \epsilon\, v(t)$, where ϵ is an *i.i.d.* Gaussian random variable independent of the volatility $v(t)$, and $g(t)$ denotes the returns. A consistent statistical description may involve extending the traditional ARCH model to include long-range volatility correlations [48].

A more fundamental question would be to understand the above results starting from a microscopic setting. Researchers have also studied microscopic models that might give rise to the empirically observed statistical properties of returns [5, 10]. For example, Lux and Marchesi [10] recently simulated a microscopic model of financial markets with two types of traders, what they refer to as "fundamentalist" and "noise" traders. Their results reproduce the power-law tail for the distribution of returns and also the long range correlations in volatility.

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