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## Correlated randomness and switching phenomena

H.E. Stanley<sup>a,\*</sup>, S.V. Buldyrev<sup>b,a</sup>, G. Franzese<sup>c</sup>, S. Havlin<sup>d</sup>, F. Mallamace<sup>e</sup>, P. Kumar<sup>f</sup>,  
V. Plerou<sup>a</sup>, T. Preis<sup>g</sup>

<sup>a</sup> Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215, USA

<sup>b</sup> Department of Physics, Yeshiva University, 500 West 185th Street, New York, NY 10033, USA

<sup>c</sup> Departament de Física Fonamental, Universitat de Barcelona, Diagonal 647, 08028 Barcelona, Spain

<sup>d</sup> Minerva Center and Department of Physics, Bar-Ilan University, 52900 Ramat-Gan, Israel

<sup>e</sup> Dipartimento di Fisica, Univ. Messina Vill. S. Agata, C.P. 55, I-98166 Messina, Italy

<sup>f</sup> Center for Studies in Physics and Biology, Rockefeller University, New York, NY 10021, USA

<sup>g</sup> Institute of Physics, Johannes Gutenberg University Mainz, Staudingerweg 7, D-55128 Mainz, Germany

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### ABSTRACT

One challenge of biology, medicine, and economics is that the systems treated by these serious scientific disciplines have no perfect metronome in time and no perfect spatial architecture—crystalline or otherwise. Nonetheless, as if by magic, out of nothing but *randomness* one finds remarkably fine-tuned processes in time and remarkably fine-tuned structures in space. Further, many of these processes and structures have the remarkable feature of “switching” from one behavior to another as if by magic. The past century has, philosophically, been concerned with placing aside the human tendency to see the universe as a fine-tuned machine. Here we will address the challenge of uncovering how, through randomness (albeit, as we shall see, strongly correlated randomness), one can arrive at some of the many spatial and temporal patterns in biology, medicine, and economics and even begin to characterize the switching phenomena that enables a system to pass from one state to another. Inspired by principles developed by A. Nihat Berker and scores of other statistical physicists in recent years, we discuss some applications of correlated randomness to understand switching phenomena in various fields. Specifically, we present evidence from experiments and from computer simulations supporting the hypothesis that water’s anomalies are related to a switching point (which is not unlike the “tipping point” immortalized by Malcolm Gladwell), and that the bubbles in economic phenomena that occur on all scales are not “outliers” (another Gladwell immortalization). Though more speculative, we support the idea of disease as arising from some kind of yet-to-be-understood complex switching phenomenon, by discussing data on selected examples, including heart disease and Alzheimer disease.

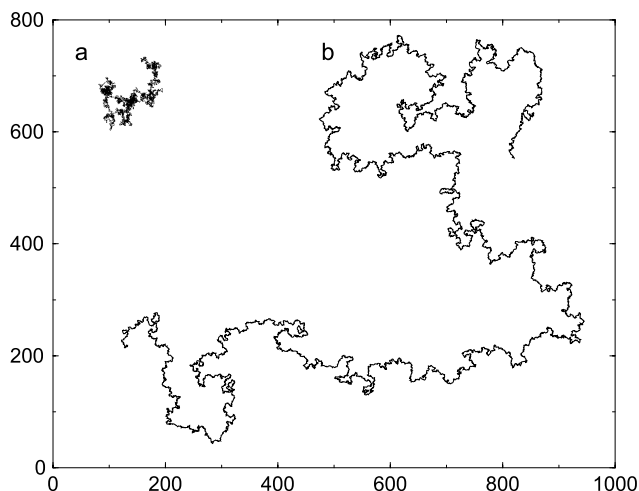
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## 1. Correlated randomness & switching phenomena: Introduction

The title I have given to this talk, “Correlated Randomness”, is owed in part to interactions with biological and medical researchers. Many of them believe that *randomness* means *uncorrelated randomness*. They learn that statistical physics deals with random phenomena, so they assume that our field cannot possibly yield any insights into the real world as they correctly know that no system in which they are interested corresponds to simple *uncorrelated randomness*. Hence we found

\* Corresponding author.

E-mail address: [hes@buphy.bu.edu](mailto:hes@buphy.bu.edu) (H.E. Stanley).



**Fig. 1.** (a) The trail of a random walk of  $10^4$  steps, compared with (b) the trail of a self-avoiding random walk of the same number of steps. The “correlated randomness” of the latter results in drastically different behavior. Specifically, the characteristic diameter jumps by a factor of 10, from approximately  $(10^4)^{1/2} = 100$  to approximately  $(10^4)^{3/4} = 1000$ , where we have used the fact that the fractal dimensions (defined as the exponent to which the length is raised to obtain the mass) are 2 and  $4/3$  respectively.

Source: This figure is courtesy of S.V. Buldyrev.

using the adjective “correlated” helped persuade our collaborators that what we do may possibly be applicable to systems in which they are interested.

To help educate our collaborators, as well as ourselves, we have learned to present simple visual examples of the concept of correlated randomness. One example we found useful was to visually compare a simple, unbiased random walk in two dimensions (“uncorrelated randomness”) and a simple, *self-avoiding* random walk in two dimensions (“correlated randomness”). In the case of the uncorrelated walk, the spread of a  $10^4$  step path is  $10^2$ . In the correlated random walk, the spread of a  $10^4$  step path is on the order of  $10^3$  steps, a full order of magnitude larger (Fig. 1). Thus a simple “microscopic correlation”, that a path cannot intersect itself, leads to a dramatic change in macroscopic behavior [1].

Indeed, 100 years ago most of our understanding of systems was based on the assumption that the constituents were basically uncorrelated, and so we understood well ideal gases, and ideal paramagnets. When correlations were introduced, gases developed a completely new phenomenon, the liquid state. And paramagnets developed a completely new magnetic phenomenon, the ferromagnetic state.

A second simple example of correlated randomness that people from all research fields can appreciate is critical opalescence, first discovered and interpreted – in terms of correlated randomness – by Thomas Andrews, M.D., in 1869 [2]. Critical points can occur in two-component fluids but also in one-component fluids—the so-called liquid–gas and liquid–liquid phase transitions [3]. In the more traditional two-component fluid, the concentrations of the two components and the temperature have been adjusted so that the system is near its consolute point. The correlated fluctuations observed at that consolute point are not uncorrelated, but rather are correlated over length scales comparable to the wavelength of light and over time scales comparable to the persistence time of human vision. One can see scattering of visible light (wavelength on the order of thousands of atomic spacings) in the form of an opalescent glow and a flickering in time.

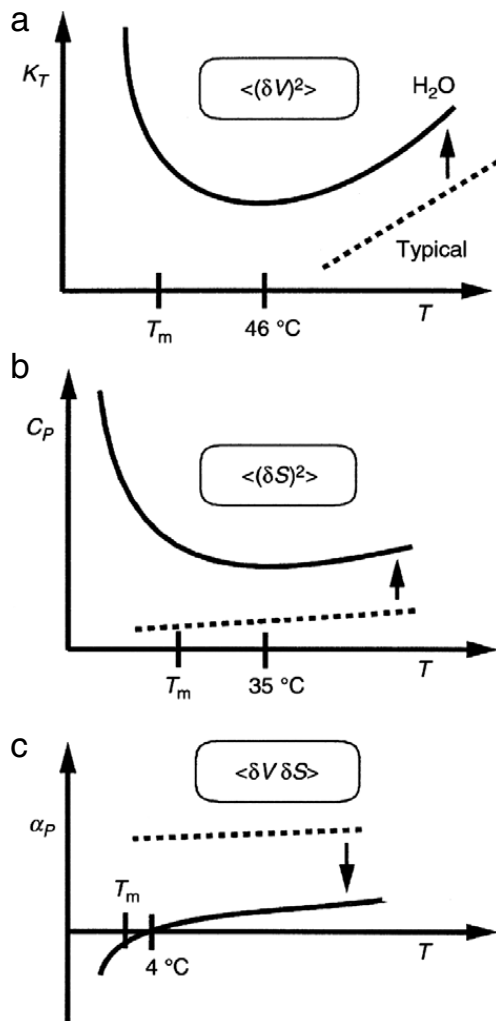
In this talk, I will discuss recent applications of correlated randomness to three areas of science for which statistical physics is becoming useful: liquid water, economics, and physiology/medicine. I organize each of these seemingly unrelated topics around the same three questions: (i) what is the question or problem that has emerged from the area of inquiry? (ii) why should we (practically and scientifically) care about this question or problem? and (iii) what have we actually done in response to the question or problem?

Our overall “take-home” message today sounds pretty general. In general, systems that display correlated randomness cannot be solved exactly. Not even the simplest two-dimensional self-avoiding random walk can be solved, and the Ising model can be solved only in the limiting case of zero magnetic field and spatial dimension of two. Nonetheless, there are two unifying principles that have organized many of the results we will be presenting today—scale invariance and universality [4].

## 2. Correlated randomness & switching phenomena in water

### 2.1. What is the phenomenon?

We start with three thermodynamic functions. The first is the compressibility—the response of the volume to an infinitesimal change in pressure. In a typical liquid, this response function decreases when we lower the temperature.



**Fig. 2.** Schematic dependence on temperature of (a) the isothermal compressibility  $K_T$ , (b) the constant-pressure specific heat  $C_p$ , and (c) the thermal expansivity  $\alpha_p$ . The behavior of a typical liquid is indicated by the dashed line, which, very roughly, is an extrapolation of the high-temperature behavior of liquid water. Note that while the anomalies displayed by liquid water are apparent above the melting temperature  $T_m$ , they become more striking as one supercools below  $T_m$ .

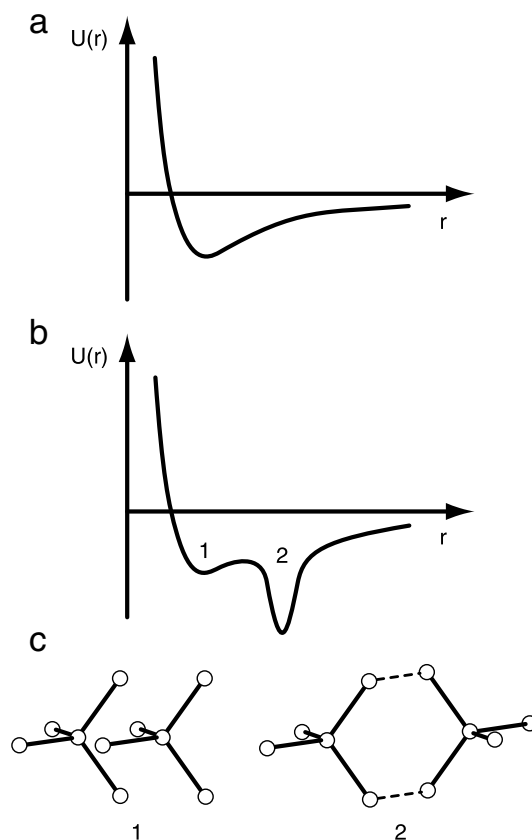
I understand this decrease via statistical physics. This thermodynamic response function is proportional to the thermal average of all the fluctuations in specific volume in the system. As we lower the temperature, we imagine that fluctuations of necessity decrease, thus the compressibility decreases.

Water is unusual in three respects. First, the average compressibility of water is twice as large as what one would expect were water a typical fluid and were one to plug all the prefactors into the formulas that give compressibility in terms of volume fluctuations. Second, the magnitude of that factor of two actually increases as one lowers the temperature. That being the case, there is ultimately a minimum—which occurs at 46 °C. Below that temperature, the compressibility increases dramatically. At the lowest attainable temperature (−40 °C) the compressibility takes on a value that is twice of that at the minimum. This is not a tiny effect; it is huge (Fig. 2).

The second thermodynamic function is the specific heat, and we observe three similar anomalies: it is twice as large as that of a typical liquid, the discrepancy gets bigger as the temperature is lowered, and a minimum occurs at 35 °C.

The third thermodynamic function is the coefficient of thermal expansion, the response of the volume to an infinitesimal change in temperature. This quantity we assume to always be positive because if there is a local region of the liquid in which the specific volume is larger than the average, then there will be more arrangements of the molecules and hence the entropy will be larger than the average. This is true of almost all liquids, but the magnitude of this cross-fluctuation of volume and entropy in water is approximately three times smaller than we would expect, and at 4 °C the coefficient of thermal expansion passes through zero and actually becomes negative.

In addition to anomalies in thermodynamic functions, there are anomalies in dynamic behavior. For example, the viscosity and inverse diffusion constant of a typical liquid decrease when pressure is applied, while for water at low temperatures the reverse occurs (see, e.g., the work of M. Barbosa and collaborators [5]).



**Fig. 3.** Physical arguments relating to the plausibility of the existence of the known liquid–gas critical point  $C$  and the hypothesized LDL–HDL critical point  $C'$ . (a) Idealized system characterized by a pair interaction potential with a single attractive well. At low enough  $T$  ( $T < T_c$ ) and high enough  $P$  ( $P > P_c$ ), the system condenses into the “liquid” well shown. (b) Idealized system characterized by a pair interaction potential whose attractive well has two sub-wells, the outer of which is deeper and narrower. For low enough  $T$  ( $T < T_c'$ ) and low enough  $P$  ( $P < P_c'$ ), the one-phase liquid can “condense” into the narrow outer “LDL” sub-well, thereby giving rise to a LDL phase, and leaving behind the high-density liquid phase occupying predominantly the inner subwell. (c) Two idealized interaction clusters of water molecules (“Walrafen pentamers”) in configurations that may correspond to the two sub-wells of (b).  
 Source: This figure is courtesy of Dr. Osamu Mishima.

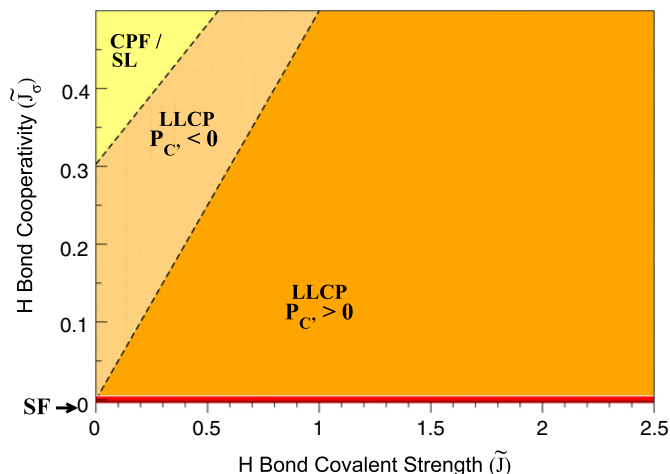
## 2.2. Why do we care about this anomalous behavior?

It is widely acknowledged that if we do not understand water we cannot understand biology. Scientifically, water is the prototype complex fluid. It is not a simple, “bag-of-marbles” liquid, but a “bag of tetrahedra”–charged tetrahedra interacting with long-range Coulomb forces.

## 2.3. What do we do?

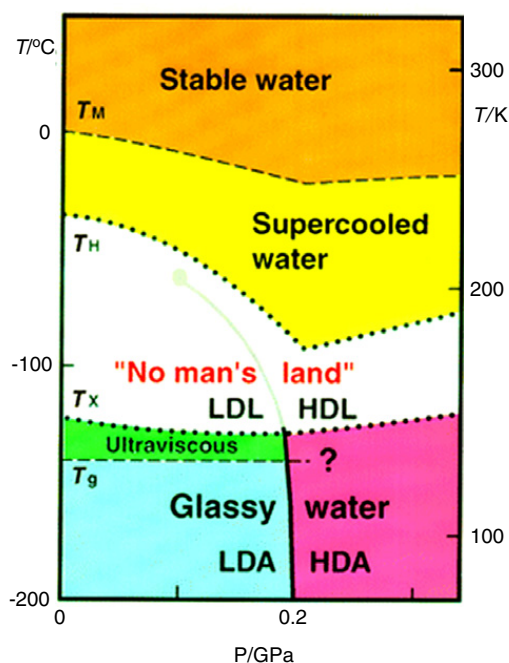
Our approach is based on the fact that water has a tetrahedral local geometry. In this sense water shares features with other liquids such as silicon, silica, and carbon. Because water is both tetrahedral and charged means that a simple Lennard-Jones potential is not sufficient to describe its complexity. One way to modify the Lennard-Jones potential to provide at least a simplified description is to bifurcate the single minimum into two minima. The first minimum, at a closer distance, corresponds to two pentamers (a water molecule and its four neighbors) of water interacting with each other in a rotated configuration. The second minimum, at a greater distance, occurs in the unrotated position (Fig. 3). This second position is a deeper minimum because although the pentamers are farther apart there is the potential for hydrogen bonding between the molecules and we can see the beginnings of an ice-like hexagonal structure [6–8].

The important point is that there are two minima with the outer one corresponding to a larger specific volume – because the distance is larger – and a lower entropy. The possibility is that liquid water could at low temperatures condense not into a single phase – as we anticipate when a gas with a simple interaction like a Lennard-Jones potential condenses into a fluid – but into two different phases. This possibility was first raised by Takahashi 60 years ago and various elaborations of this model have been made by a number of people [9–11]. Hence at low temperatures there will be enhanced entropy and volume fluctuations, and enhanced anticorrelations between entropy and volume (since the deeper outer well corresponds to lower entropy but higher specific volume). The consequences of this fact qualitatively explain the phenomenon we were describing at the beginning—volume fluctuations are increased, entropy fluctuations are increased, and cross-fluctuations of volume and entropy are decreased. This singularity-free picture has been actively studied for the past 31 years [12–16]. Additional scenarios are discussed in Ref. [17] (see Fig. 4).



**Fig. 4.** Possible scenarios for water for different values of H bond energies  $\tilde{J}$  of the covalent (or directional) component, and  $\tilde{J}_\sigma$  of the cooperative (or three-body) component, obtained from MF calculations [17]. (i) If  $\tilde{J}_\sigma = 0$  (red line along  $x$ -axis), the singularity free (SF) scenario is realized, independent of  $\tilde{J}$ . (ii) For large enough  $\tilde{J}_\sigma$ , water would possess a first-order liquid–liquid phase transition line terminating at the liquid–gas spinodal—the critical point free (CPF) scenario; the liquid spinodal would retrace at negative pressure, as in the stability limit (SL) scenario (yellow region in top left). (iii) For other combinations of  $\tilde{J}$  and  $\tilde{J}_\sigma$ , water would be described by the liquid–liquid critical point (LLCP) scenario. For larger  $\tilde{J}_\sigma$ , the LLCP is at negative pressure (ochre region between dashed lines). For smaller  $\tilde{J}_\sigma$ , the LLCP is at positive pressure (orange region in bottom right). Dashed lines separating the three different regions correspond to MF results of the microscopic cell model. The  $P$ – $T$  phase diagram evolves continuously as  $\tilde{J}$  and  $\tilde{J}_\sigma$  change. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Source: Courtesy of Kevin Stokely and Marco G. Mazza.

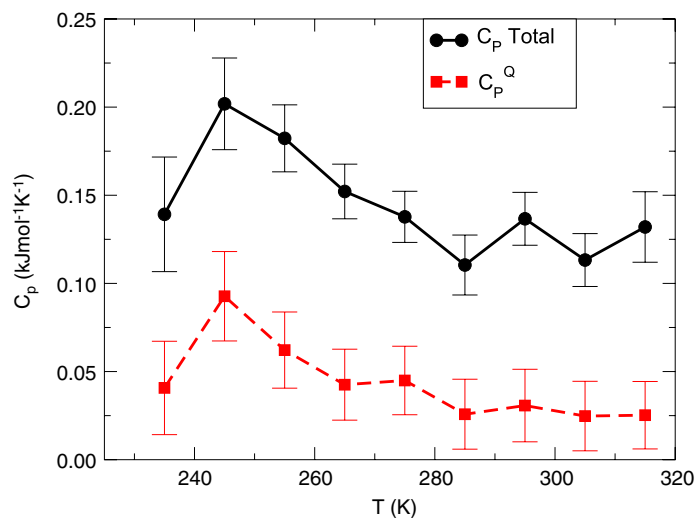


**Fig. 5.** Schematic illustration indicating the various phases of liquid water (color-coded).

Source: This figure is courtesy of Dr. Osamu Mishima.

It has been speculated that at low temperatures there will be a genuine phase transition in which the single component liquid actually separates into two different phases. The implications of this when applied to real water molecules produce a phase diagram of liquid water (Fig. 5). This liquid–liquid phase transition hypothesis was first proposed on the basis of computer simulations [18]. A first test of this hypothesis is that at very low temperature we find a glassy phase, not unlike that of any other liquid except that at high pressure this glassy phase suddenly switches from a low-density form to a high-density form. These two forms correspond to the two different local arrangements characteristic of water tetrahedra. The order parameter jump between these two phases is not a trivial amount, but on the order of 30% [19].

Between the liquid and glassy phases of water we have a region in which water does not exist as a liquid—a “No Man’s Land”. The hypothesis that follows from the reasoning we have just described is that this first order phase transition line known to separate the two amorphous forms of solid water extends into this No Man’s Land and ultimately terminates at



**Fig. 6.** Constant pressure specific heat  $C_p$  and specific heat associated with tetrahedral entropy  $C_p^Q$  of water at atmospheric pressure for TIP5P model of water.

Source: Courtesy of Pradeep Kumar.

a critical point. Just as the glassy water first-order transition line separates a low-density amorphous from a high-density amorphous phase of water, so also this extension of the line into the liquid region separates a low-density liquid from a high-density liquid. The power-law behavior uncovered over the years by Angell, Anisimov and collaborators corresponds to the fact that the extension of this first-order line beyond the critical point—the “Widom line”, defined to be the locus of maximum correlation length [20–23]. Experiments on a path approaching the Widom line display phenomena that initially look as though there will be a divergence, as the correlations increase. However since the correlation length itself is not infinite, there will ultimately be rounding.

The degree of ordering and disordering of local tetrahedrality of water upon changing thermodynamic variables, such as pressure or temperature, offers a simple measure of order and disorder in the case of liquid water. Kumar and his collaborators derive a relation for the subset of the structural entropy which is associated with the degree of local tetrahedral ordering [21]. They find that the most relevant part of the total entropy fluctuations (or the specific heat) of water comes from the tetrahedral fluctuations — suggesting that the fluctuations of local tetrahedrality of water contributes the most to the total specific heat [21]. Moreover, the specific heat associated with the tetrahedral ordering  $C_p^Q$  behaves identically, within error bars, to the total specific heat  $C_p^{total}$  (Fig. 6), consistent with the qualitative idea that the principal contributor to temperature dependence of the entropy is the angular variables, not the translational variables. In particular, both functions display clear maxima at the Widom line, which is equivalent to saying that at the Widom line the entropy fluctuations are a maximum (and the dependence of entropy on temperature is a maximum).

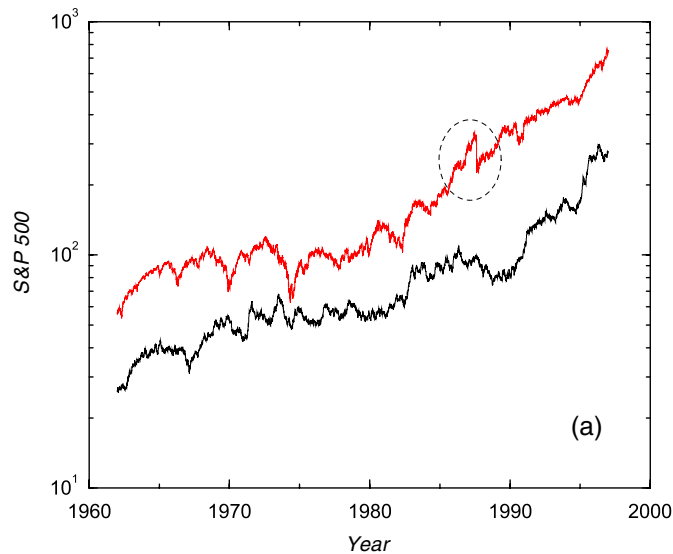
This phase diagram is hypothesized, but it has not been proved. What has been proved is that computer simulations using tried and tested models of liquid water confirm the broad features of this phase diagram (see, e.g., Refs. [24–27] and refs. therein). But computer models of water (like computer models of anything) are subject to the charge “garbage-in, garbage-out”—you get out what you put in. All computer models of complex systems such as liquid water are of necessity simplifications.

Here we are guided by exact solutions of simplified models. Simplified models are designed less for the purpose of matching experimental detail, but rather to capture the essential physical features of a real system to explore the effect of these features. A family of models introduced first by Jagla seems to reveal the fact that in order for a system to display a liquid–liquid phase transition, the essential feature of an interaction potential must be that it has not one but rather two characteristic length scales (see Ref. [28] and references therein).

Current experiments on this problem are of two sorts. The first is a set of experiments inspired by Mishima that involves probing the No Man’s Land by studying the metastable extensions of the melting lines of the various high-pressure polymorphs of ice: ice III, ice V, ice IV, and ice XII [29,30]. Two of these lines clearly display “kinks”. Since the slope of any melting line is the difference of the volume change divided by the entropy change of the two phases that coexist at that line, if there is a change in slope there must be a change in these quantities. Since there is no change in the crystal part, there must be a change in the liquid part. This means the liquid must undergo a jump in either its volume or its entropy or both. That is the definition of a first-order phase transition.

The second sort of experiments avoid the existence of a no-man’s land by using water confined either in nanotubes or near the surface of a macromolecule. These experiments, carried out largely in the research groups of S.-H. Chen and F. Mallamace, are consistent with the possibility of a liquid–liquid phase transition [31–46]. Other novel collective behavior of water is under active investigation, e.g., S. Han and collaborators have found evidence for a solid–liquid critical point in confined water films [47].





**Fig. 7.** The S&P 500 index is the sum of the market capitalizations of 500 companies. The sharp jump seen in 1987 is the market crash of October 19. Comparison of the time evolution of the S&P 500 for the 35-year period 1962–96 (top line) and a biased Gaussian random walk (bottom line). The random walk has the same bias as the S&P 500—approximately 7% per year for the period considered.

Source: This figure is courtesy of Vasiliki Plerou and Parameswaran Gopikrishnan.

### 3. Correlated randomness & switching phenomena in economics

#### 3.1. What is the phenomenon?

One quarter of any newspaper with a financial section is filled with economic fluctuation data. Most economic graphs look approximately like the one we get when we plot the S&P 500 stock index as a function of time over 40 years (Fig. 7). We can compare this empirical data with that generated by a simple uncorrelated biased random walk, a model first used over 100 years ago by Bachelier. At first it seems that there is little difference, but looking more closely we see events in the real data that do not have counterparts in the random walk. Black Monday in October 1987 is reflected in the real data, which shows a loss of about 30% of the total value of the market in just one day. In the random walk we do not see fluctuations anywhere near this magnitude because the probability of taking  $n$  steps in the same direction of a random walk is  $(1/2)^n$ —it decreases exponentially with  $n$ .

Economists nevertheless have traditionally used this uncorrelated biased Gaussian random walk to describe real economic data, relegating events such as Black Monday to the dustbin category of “outliers”.

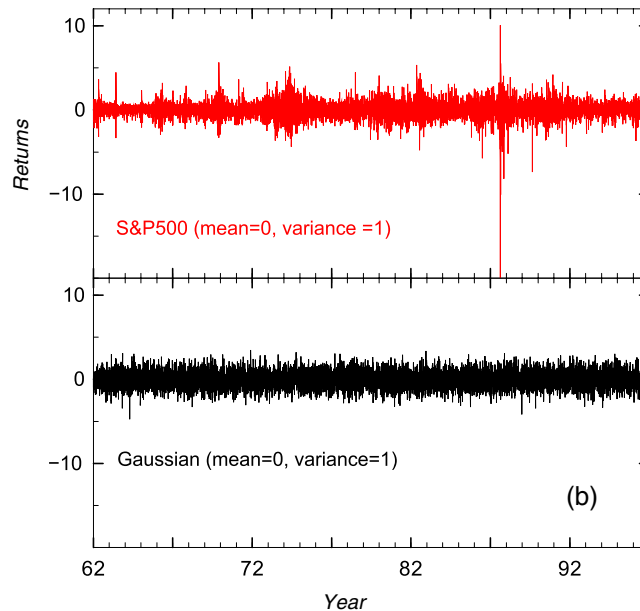
#### 3.2. Why do we care?

We physicists do not like to do things this way. We do not take Newton’s law seriously part of the time, and then – if we suddenly see an example of what appears to be levitation – simply call it an “outlier”. We like to find laws that describe all examples of a phenomenon. Economists themselves, in a journal called *The Economist*, have admitted failure. This is a strong motivation for we physicists to step in and try our hand—we sense a delicious scientific challenge. Also, practically speaking, catastrophic economic events such as Black Monday have extreme societal impacts. Widespread suffering is the usual outcome, especially among the poor. The ability to predict economic crashes (and other large-scale risks) would have an obvious utility.

#### 3.3. What do we do?

We return to our two graphs, the S&P 500 stock index as a function of time over 40 years and the simple uncorrelated biased random walk, and plot not the absolute value of the index but instead the *change* in the index (the numerical derivative, the “return”). We normalize that by the standard deviation. We look in Fig. 8 over a 13-year period rather than our original 40-year period of Fig. 7 and see, e.g., that on Black Monday the fluctuations were more than 30 standard deviations (both positive and negative) for the day. We also note obvious time correlations in the volatility. Perhaps more striking is to look at the other curve, the uncorrelated random walk, and see the Gaussian distribution for the fluctuations—with no fluctuations greater than five standard deviations. The “outliers” that the economists are content to live with are any fluctuations of the actual data that are greater than five standard deviations. In this 13-year period there are exactly 64, i.e.,  $2^6$ . If we count only those fluctuations of the actual data that are greater than ten standard deviations, we get exactly 8,





**Fig. 8.** Sequence of 10-min returns for the S&P 500, normalized to unit variance, compared with sequence of i.i.d. Gaussian random variables with unit variance, which was proposed by Bachelier as a model for stock returns. Note that, in contrast to the top curve, there are no “extreme” events in the bottom curve.

Source: This figure is courtesy of Vasiliki Plerou and Parameswaran Gopikrishnan.

i.e.,  $2^3$ . If we count only those that are greater than 20, we get one, i.e.,  $2^0$ : Black Monday. Each time we double the  $x$ -axis we change the  $y$ -axis by a power of  $2^3$ . This behavior is consistent with a functional equation of the form

$$f(\lambda x) \sim \lambda^p f(x). \quad (1)$$

Functional equations have as their solutions functional forms, and the solution to the homogeneous functional equation is a power-law form. From visual inspection of the raw data in Fig. 8, we see a power law of the “inverse cubic” form

$$f(x) = x^{-3} \quad (2)$$

which corresponds to a functional equation, a scaling equation, with  $p = -3$ .

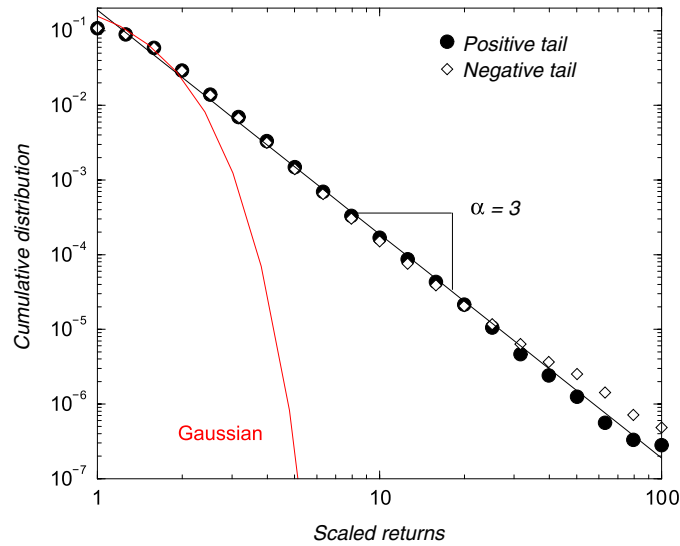
If we replace our visual examination of these two graphs with a close computer analysis of not just the the S&P 500 stock index but every stock transaction over an extended time period (approximately one GB of data), we find [48,49] that the actual graph giving the number of times a fluctuation exceeds a given amount as a function of that amount is perfectly straight on log-log paper out to 100 standard deviations (Fig. 9). The slope of the line,  $\alpha$ , is indistinguishable from the value  $\alpha = 3$  that we guessed from visual inspection. Note also that our slope is outside the Lévy stable regime [50].

This is how we find laws in statistical physics, but finding new empirical laws in economics is useful. It is useful for practical reasons, one can quantify risk—produce an actual number for the chance some fluctuating quantity changes by a given amount. It is also useful for conceptual reasons, since knowing the empirical laws places constraints on the range of acceptable theoretical idea. A mathematical economist has been heard to say “if you know the answer, then that is cheating because the theory already knows what answer to get”.

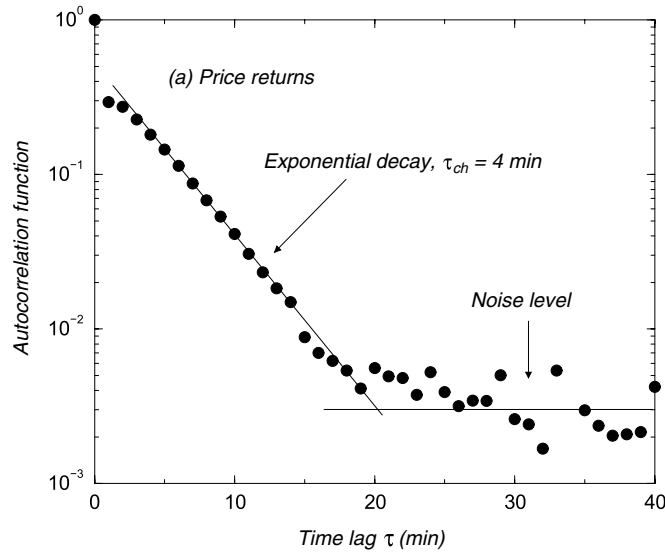
When we studied critical phenomena 40 years ago (when Berker was an MIT student), the empirical part was a very important contributor toward our ultimate understanding of phase transitions and critical phenomena. Uncovering empirical facts led to the recognition of regularities to which certain approaches could be applied, e.g., the Widom scaling hypothesis and the Wilson renormalization group. So also in economics we can perhaps first discover empirical regularities – e.g., the inverse cubic law – that will prove useful in ultimately understanding the economy. We have the beginnings of an explanation, but it is only the beginning since the current theory explains the inverse cubic law of price changes, as well as the “half cubic law” of trade volume [51] but does not explain the strange nature of the temporal correlations. The autocorrelation function of price changes decays exponentially in time so rapidly that after 20 min it is in the level of “noise” (Fig. 10). However the autocorrelation function of changes in the absolute value of the price (called the “volatility”) decays with a power law of exponent approximately 0.3 (Fig. 11) [52].

However, how can we characterize trend switching processes in financial markets? At first glance, price fluctuations on financial markets appear to be completely random. In order to uncover the patterns formed by correlated randomness, one has to find an appropriate “metronome” that is able to “synchronize” these sudden trend reversals occurring on time scales from very large down to very small. One leading candidate as a metronome for financial markets is the local extreme value in the price time series.

Such an analysis can provide insight into switching processes in complex systems in general and financial systems in particular. The study of dramatic crash events is limited by the fortunately rare number of such events. Increasingly, one



**Fig. 9.** Cumulative distributions of the positive and negative tails of the normalized returns of the 1000 largest companies in the TAQ database for the 2-year period 1994–1995. The solid line is a power-law regression fit in the region  $2 \leq x \leq 80$ . Shown for comparison is the behavior of a Gaussian function with the same value of the standard deviation. Note the striking fact that traditional theories based upon assumed Gaussian functional forms for this distribution predict that the probability of a fluctuation larger than 5 standard deviations is approximately 8 orders of magnitude smaller than the probability of an everyday event. In contrast, the empirical data demonstrate that the probability of a fluctuation larger than 100 standard deviations is approximately 8 orders of magnitude smaller than the probability of an everyday event. Source: This figure is courtesy of Vasiliki Plerou and Parameswaran Gopikrishnan.

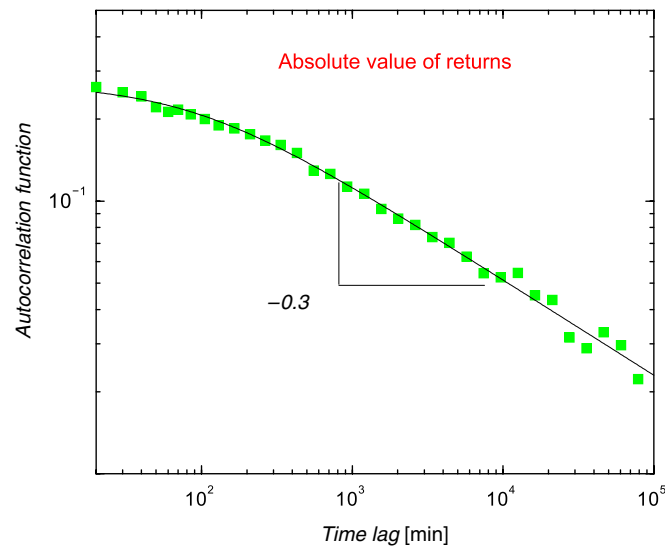


**Fig. 10.** Semilog plot of the autocorrelation function for the S&P 500 returns  $G_{\Delta t}(t)$  sampled at a  $\Delta t = 1$  min time scale,  $C_{\Delta t}(\tau) \equiv [(\langle G_{\Delta t}(t) G_{\Delta t}(t + \tau) \rangle - \langle G_{\Delta t}(t) \rangle^2) / (\langle G_{\Delta t}(t)^2 \rangle - \langle G_{\Delta t}(t) \rangle^2)]$ . The straight line corresponds to an exponential decay with a characteristic decay time  $\tau_{ch} = 4$  min. Note that after 20 min the correlations are at the noise level. Source: This figure is courtesy of Vasiliki Plerou and Parameswaran Gopikrishnan.

seeks to understand the current financial crisis by comparisons with the depression of the 1930's. Here we ask if the smaller financial crises – trend switching processes on all time scales – also provide information of relevance for large crises. If this is so, then the large abundance of data on smaller crises should provide quantifiable statistical laws for *bubbles on all scales*.

To analyze switching processes of financial fluctuations, we first propose how a switching process can be quantitatively analyzed. Let  $p(t)$  be the transaction price of trade  $t$ , which is a discrete variable  $t = 1, \dots, T$ . A transaction price  $p(t)$  is defined to be a *local maximum* of order  $\Delta t$  if there is no higher transaction price in the interval  $t - \Delta t \leq t \leq t + \Delta t$ , and is defined to be a *local minimum* of order  $\Delta t$  if there is no lower transaction price in this interval (Fig. 12) [53].

However, the time series of transaction prices provides only the metronome, with which we analyze volume fluctuations  $v(t)$  from one beat of the metronome to the next. We use 500 price time series of all 500 S&P500 constituents (2.6 million data points) and a price time series of the German DAX Future (14 million data points) [54]. The volume is the number of



**Fig. 11.** Log–log plot of the autocorrelation function of the absolute returns. The solid line is a power-law regression fit over the entire range, which gives an estimate of the power-law exponent,  $\eta \approx 0.3$ . Better estimates of the exponent  $\eta$  can be obtained from the power spectrum or from other more sophisticated methods.

Source: This figure is courtesy of Vasiliki Plerou and Parameswaran Gopikrishnan.

contracts traded in each individual transaction in case of the German market and the number of traded stocks per day in case of the US market. For the analysis, we introduce a renormalized time scale  $\varepsilon$  between successive extrema—successive beats of the metronome. Thus,  $\varepsilon = 0$  corresponds to the beginning of a trend and  $\varepsilon = 1$  indicates the end of a trend (Fig. 12(a)). We analyze a range of  $\varepsilon$  for the interval  $0 \leq \varepsilon \leq 2$ , so we can analyze trend switching processes both before as well as after the critical value  $\varepsilon = 1$ . We find that new price extrema in the price are linked with peaks in the volume time series (Fig. 12(b) and (c)) and, more surprisingly, that the volume obeys power law scaling both before as well as after the critical point  $\varepsilon = 1$  (Fig. 12(d) and (e)).

This switching mechanism has no scale, for time scales varying over 9 orders of magnitude down to the smallest possible time scale—the scale of single transactions measured in units of 10 ms. Thus, the well-known catastrophic bubbles occurring on large time scales may not be outliers but in fact single dramatic events caused by the inherent, scale-free behavior related to the formation of increasing and decreasing trends on time scales from very large down to very small.

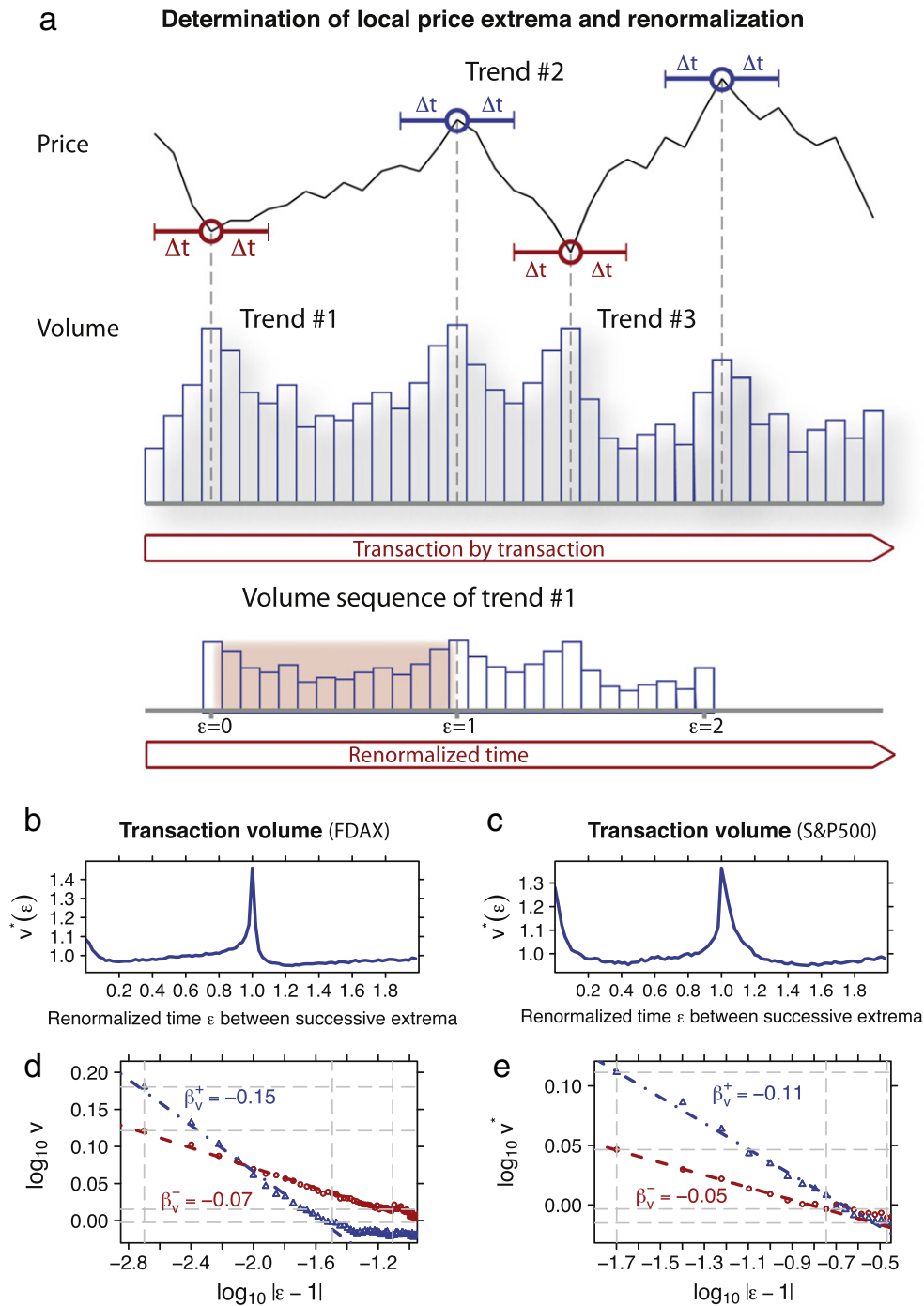
One reason the economy is of interest to statistical physicists is that, like an Ising model, it is a system made up of many subunits. The subunits in an Ising model are called spins, and the subunits in the economy are buyers and sellers. During any unit of time these subunits of the economy may be either positive or negative as regards perceived market opportunities. People interact with each other, and this fact often produces what economists call “the herd effect”. The orientation of whether we buy or sell is influenced not only by our neighbors but also by news. If we hear bad news, we may be tempted to sell. So the state of any subunit is a function of the states of all the other subunits and of a field parameter.

#### 4. Correlated randomness & switching phenomena in physiology & medicine

##### 4.1. What is the problem?

The most magical switching phenomena come to light when one examines biology and medicine. A developing embryo suddenly switches from one state to another, almost as if there were a master hand guiding it. A healthy individual suddenly develops a systemic and fatal disease, almost as if “fate” has decreed that the time is up for living. Or genetic diseases like cancer—the gene is present yet the patient remains healthy until – suddenly – tumor growth switches on. And, tragically, molecules suddenly begin to aggregate in the brains when one contracts Alzheimer disease. Why? On the microscopic level, chain molecules switch from an unfolded state to a reproducible folded state for reasons not at all understood. Even everyday phenomena involve switching—from wake to sleep, and then from sleep to wake. Actually there are countless sleep-related switchings occurring “while sleeping”—from one sleep stage to another, and then to a brief wake state, followed by more sleep [58].

Not only switchings occur, but fractal forms abound. We now know, e.g., that nerve cells adopt patterns with well-defined fractal dimensions [59] but we do not know why. We know that the noncoding DNA has long-range power law (fractal) correlations among its constituent base pairs [60,61]. We are only beginning to understand the relation between the various fractal forms that abound and the various forms of switchings. We do know that the two phenomena appear sometimes to be related; e.g., we now know that sleep-wake switching interacts with the fractal scaling behavior of heartbeat intervals [62].



**Fig. 12.** (a) Schematic visualization of trend segregation. Positive trends start at local price minima (red circles) and end at local maxima (blue circle)—and vice versa. A transaction price  $p(t)$  is a local maximum if there is no higher transaction price in the interval  $t - \Delta t \leq t \leq t + \Delta t$ . Analogously,  $p(t)$  is a local minimum if there is no lower transaction price in the interval  $t - \Delta t \leq t \leq t + \Delta t$ . We assign  $\varepsilon = 0$  to the start of each trend, and  $\varepsilon = 1$  to the end of each trend as shown for the first trend. In order to study trend switching processes – both before as well as after the end of a trend – we consider additionally the subsequent sequence of identical length. In the region around  $\varepsilon = 1$ , we find scale-free behavior of related quantities—volume and inter-trade time [55,54,53]. This behavior is consistent with “self-organized” macroscopic interactions among many traders [56], not unlike “tension” in a pedestrian crowd [57]. (b) Averaged volume sequence  $v^*(\varepsilon)$  of the German DAX Future time series.  $\Delta t$  ranges from 50 to 100 transactions (ticks). Extreme values of the price coincide with sharp peaks in the volume time series. (c) A very similar behavior is obtained for the averaged volume sequence  $v^*(\varepsilon)$  of S&P500 stocks. Here,  $\Delta t$  ranges from 10 days to 100 days. (d) Log–log plot of the FDAX transaction volumes ( $50 \text{ ticks} \leq \Delta t \leq 1000 \text{ ticks}$ ) before reaching an extreme price value ( $\varepsilon < 1$ , circles) and after reaching an extreme price value ( $\varepsilon > 1$ , triangles). The straight lines correspond to power-law scaling with exponents  $\beta_v^+ = -0.155 \pm 0.004$  and  $\beta_v^- = -0.068 \pm 0.001$ . (e) Log–log plot of the transaction volumes shown in (c) indicates a power-law behavior with exponents  $\beta_v^+ = -0.109 \pm 0.003$  and  $\beta_v^- = -0.052 \pm 0.001$  which are similar to our results on short time scales. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Source: This figure is courtesy of T. Preis.

#### 4.2. Why do we care?

Diseases appear to fall into different “universality classes”. For example, Alzheimer’s disease is only one form of aggregation disease. Others may include ALS (amyotrophic lateral sclerosis) and Parkinson’s disease, and many think they all involve amyloid mediated aggregation.

Since we do not really know for sure what AD is, our group’s approach is to try to discover what it is [63–67]. We are not the only ones trying to do this, but our focus is on the “first three minutes” of the disease, a phrase we like to use because it is the analog for AD of Weinberg’s *The First Three Minutes of the Universe*. What initially triggers AD? One of the current hypotheses is that it is triggered by a phase transition. In a liquid-to-crystal phase transition, the liquid nucleates to a lower free-energy state called a crystal. In AD, it looks as if the analogous event is a two-step process—first of protein folding and then of protein aggregation. So one starts with a protein, specifically a peptide, in a correlated random configuration, which first folds itself and then aggregates into a form called an amyloid fibril. These amyloid fibrils aggregate and form microscopic objects in the brain.

#### 4.3. What do we do?

The protein that aggregates in Alzheimer disease is actually a peptide, a fragment of a protein, and it comes in two forms: one with 40 amino acids and the other with 42 amino acids. The extra two amino acids are hydrophobic. The “first three minutes” of Alzheimer’s disease involves the aggregation of the entire peptide. The first step seems to be the formation of what are called paranuclei, the joining together of a small number—e.g., 6 or 12-peptides, and then the aggregation of the paranuclei into larger objects. The time scale here is slow, so the study of this phenomenon by molecular dynamics is problematic. A typical timescale for a molecular dynamics simulation is on the order of nanoseconds, and here we are talking about minutes. We need to do something to speed up the simulation, and thus we draw on the concept of universality.

One thing we learn from universality is that completely different fluids behave the same way near phase transitions. They even have identical critical exponents, regardless of the details. The critical factor does not seem to reside in the details of the molecule but in the fact that they have an attractive part. Similarly, in the spirit of universality, one finds the same generic collective behavior when one coarse grains the actual peptide by replacing each amino acid group by only four balls: three in the peptide backbone and one representing the side group.

A typical simulation might start with 28 of these coarse-grained peptides at time zero [68]. Then using the algorithm [69, 70] that speeds up the simulation by ten orders of magnitude, one can achieve aggregation of these peptides in a reasonable amount of computer time (on the order of days of computer time). The structure of what we found is of considerable interest to those studying Alzheimer’s disease. In particular, we find aggregation, with remarkably reproducible microscopic detail concerning exactly what sticks to what. This is terribly important because if we know what sticks to what, we can imagine covering the sticky spots and hence offering some potentially useful hints concerning possible therapies for Alzheimer’s disease. That is, if we know what is sticking to what we can begin to think about how to block that sticking process.

### 5. Outlook

When one of the authors was fortunate, 40 years ago in 1969, to have Professor Berker as his first thesis student, the concept of correlated randomness existed. We all knew a few limited results, such as the exact solution of the two-dimensional Ising model for the special case of zero magnetic field. However knowing a solution does not tell us “what matters”, and only gradually did the key concepts of scaling, universality, and renormalization group develop sufficiently that one could really understand the various phenomena.

The realization that a host of geometric phenomena share analogous features of scaling, universality, and renormalization was not fully appreciated in 1969 [71]. Later still, the interplay between geometric and thermal phenomena revealed new complexity such as higher order critical points [72]. Today we are still struggling to understand the basis of seemingly ubiquitous phenomena, such as the Zipf law, which appear to work not only for word usage but for describing the size distribution of firms [73]. Finally, the *dynamics* of firm growth (and firm shrinkage) is a problem with little understanding [74–77]. Also, the *cross-correlations* between price changes of different stocks can be analyzed using the techniques of random matrix theory [78].

Price growth (and price collapse) has traditionally been viewed as a complex form of random walk, while now we are beginning to understand this ubiquitous economic phenomenon as a sequence of switches (or “bubbles”) of all sizes ranging in time scales over 10 orders of magnitude from msec to days [79,80]. Perhaps most dramatic are the various novel forms of switching phenomena (a catastrophic cascade of failures) that can occur when one damages a single node of one of the several coupled networks [81].

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