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# Zipf rank approach and cross-country convergence of incomes

Jia Shao[1], Plamen Ch. Ivanov[1,2,3(a)], Branko Urošević[4,5], H. Eugene Stanley[1(b)] and Boris Podobnik[6,7,8(c)]

[1] *Center for Polymer Studies and Department of Physics, Boston University - Boston, MA 02215, USA*
[2] *Institute of Solid State Physics, Bulgarian Academy of Sciences - Sofia, Bulgaria, EU*
[3] *Harvard Medical School and Division of Sleep Medicine, Brigham and Women's Hospital Boston, MA 02115, USA*
[4] *Faculty of Economics, University of Belgrade - 11000 Belgrade, Serbia*
[5] *National Bank of Serbia*
[6] *Faculty of Civil Engineering, University of Rijeka - Rijeka, Croatia*
[7] *Zagreb School of Economics and Management - 10000 Zagreb, Croatia*
[8] *Faculty of Economics, University of Ljubljana - Slovenia, EU*

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**Abstract** – We employ a concept popular in physics —the Zipf rank approach— in order to estimate the number of years that EU members would need in order to achieve "convergence" of their *per capita* incomes. Assuming that trends in the past twenty years continue to hold in the future, we find that after $t \approx 30$ years both developing and developed EU countries indexed by $i$ will have comparable values of their *per capita* gross domestic product $\mathcal{G}_{i,t}$. Besides the traditional Zipf rank approach we also propose a weighted Zipf rank method. In contrast to the EU block, on the world level the Zipf rank approach shows that, between 1960 and 2009, cross-country income differences increased over time. For a brief period during the 2007–2008 global economic crisis, at world level the $\mathcal{G}_{i,t}$ of richer countries declined more rapidly than the $\mathcal{G}_{i,t}$ of poorer countries, in contrast to EU where the $\mathcal{G}_{i,t}$ of developing EU countries declined faster than the $\mathcal{G}_{i,t}$ of developed EU countries, indicating that the recession interrupted the convergence between EU members. We propose a simple model of GDP evolution that accounts for the scaling we observe in the data.



One of the main questions concerning economic growth and economic development is whether or not initially poor countries eventually catch up with rich countries. Two examples illustrate that either outcome is possible. During the period 1960–1996 Singapore had a 40% saving rate and an average annual GDP growth of 5–6% and its sustained growth transformed Singapore from a relatively poor country into one of the richest in the world. In contrast, during the same period Kenya had a 15% saving rate but its annual GDP growth rate was only 1%, and Kenya remains relatively poor.

There are two main approaches to economic growth, i) the neo-classical growth model [1], and ii) the endogenous growth theory model [2]. In the neoclassical growth model a) the long-run rate of growth is exogenously determined (outside of the model) and b) the income levels of a poorer country $i$ tend to equalize with or "converge" in time to the income levels of the richer countries as long as they have similar characteristics, *e.g.*, similar saving rates [1]. In the endogenous growth theory model, policy measures such as subsidies on research and development or education can have an important impact on the long-term growth rate of an economy [2].

In the empirical literature there are several methods used to establish (or disprove) the convergence hypothesis. In the absolute convergence hypothesis it is assumed that all economies tend to the same unique steady state [3]. In the conditional convergence hypothesis, each particular economy tends to its own unique equilibrium state [3]. Another complementary notion is the sigma convergence where one studies how cross-sectional dispersion of incomes changes over time [4]. In the club convergence

(a) E-mail: plamen@buphy.bu.edu
(b) E-mail: hes@buphy.bu.edu
(c) E-mail: bp@phy.hr

hypothesis, an economy which belongs to a given club (*e.g.*, EU member nations) tends to move from a non-equilibrium state to a final state where the growth rate is the same for all members of the club [5]. Yet another useful notion of convergence is advanced by Quah in a series of papers (see [6,7] and references therein) in which he studies the dynamics of cross-sectional income distributions. According to this approach, if over time income distributions are increasingly centered around a single peak convergence across all countries will result. If, in contrast, the limiting distribution has two or more peaks, it implies the existence of clubs of convergence. In this paper, we propose another notion of convergence based on the Zipf rank distribution [8,9].

The neo-classical growth model in economics [1] predicts that the lower the starting level of a country's per capita gross domestic product $\mathcal{G}_{i,t}$, the higher its expected average growth rate. In analogy with heat transfer in thermodynamics, one can imagine that globalization (free trade and capital flow) and the removal of barriers between countries would cause capital to transfer more efficiently across international borders, allowing richer countries to supply productive capital to poorer countries. If initially poorer countries eventually attain the economic level of initially richer countries, the equilibrium state as in thermodynamics will be reached. This theory raises the question: "When will this final state in which all $\mathcal{G}_{i,t}$ are approximately the same for all countries (or groups of countries) occur?"

To answer that question we assume that globalization and relative political stability will continue in the future. On the global level this would clearly be a big assumption. Thus to approach the question more realistically we focus our attention on only one trading block, the EU. We apply the Zipf scaling approach and find that, based on the past 20 years of convergence data, the 27 countries of the EU will experience "convergence", in which the $\mathcal{G}_{i,t}$'s of all countries converge to approximately the same value after $\approx$30 years. This kind of convergence does not occur at the world level; the rich countries, in contrast to the poor countries, are becoming increasingly rich. This conclusion assumes that all countries are put on the same footing (say, China and Luxemburg). On the other hand, the weighed Zipf approach, which takes into account country size, indicates that more recently there has been a convergence on the global level. This result is, most likely, driven by the remarkable growth of China and, to some extent, India and is consistent with findings reported, for example in [10].

A large number of studies have addressed the question of cross-country income convergence [4,6,11–21]. Reference [4], for example, raises the question: "Is the degree of income inequality across economies increasing or decreasing with time?" Reference [4] analyzed $\mathcal{G}_{i,t}$ for 110 countries over the 30-year period 1960–1990. In order to test whether poorer or richer countries grow faster with time, he performed a regression between the annualized growth rate of $\mathcal{G}_{i,t}$ of economy $i$, $\gamma_{i,t,t+\Delta T}$, between $t$ and $t+\Delta T$ and the logarithm of *per capita* $\mathcal{G}_{i,t}$ of economy $i$ at time $t$:

$$\gamma_{i,t,t+\Delta T} \equiv \frac{\ln(\mathcal{G}_{i,t+\Delta T}/\mathcal{G}_{i,t})}{\Delta T} = \beta_0 - \beta\, \ln(\mathcal{G}_{i,t}) + \epsilon_{i,t}. \quad (1)$$

For the initial and final years ref. [4] chose $t = 1960$ and $t + \Delta T = 1990$, and estimated a positive regression exponent $-\beta$ which he called the *speed of convergence* (the so-called absolute convergence). Reference [4] concluded that the absolute convergence hypothesis can be safely rejected when studied over the whole sample, *i.e.*, that *per capita* incomes *diverge*. Put another way, richer countries seem to grow more quickly than poorer countries [14]. On the other hand, ref. [4] and several other authors find evidence to support conditional beta convergence (convergence to a country-specific fixed point), with convergence speed of around 2% per annum on average. Note that in this approach the $\beta$ estimate obtained for the 30-year period is of the average behavior, and does not tell us whether the *speed of convergence* $\beta$ is constant or changes with time. Others have shown that if $\mathcal{G}_{i,t}$ are weighted by population size, international inequality appeared to decrease during the decades of the 1980s and 1990s [11]. For example, ref. [11] estimated eight different indices of income inequality, and all showed reductions in global inequality during the 1980s and 1990s, *i.e.*, a gradual move from divergence to convergence.

In this paper we use data on $\mathcal{G}_{i,t}$ and on population numbers over the 50-year period 1960–2010 contained in the World Bank database (`http://publications.worldbank.org`) on $\mathcal{G}_{i,t}$.

If there is one group of countries for which convergence should hold it is the EU. There, poorer EU members receive transfer payments designed to reduce the gap with the richer members. In addition, borders are relatively open to capital and, to a large extent, labor movements. If income convergence among the 27 EU countries were to hold, then it is natural to ask when will the final state occur in which all $\mathcal{G}_{i,t}$ in the EU are evenly distributed? The Zipf rank approach [22,23] suggests an answer. For the period of the past 20 years, we rank the $\mathcal{G}_{i,t}$ according to value, from the largest to the smallest value, and plot the data as a function of the rank. In this "Zipf rank" approach, income convergence (when all incomes become equal) would take place when the slope of the Zipf plot goes to zero. We show in fig. 1(a) that the Zipf rank plot of the values of $\mathcal{G}_{i,t}$ approximately follows an exponential function. More precisely, the Zipf rank plot of the values of $\mathcal{G}_{i,t}$ exhibits a crossover from an exponential function for developing countries to a power-law behavior for the $\approx$11 wealthiest countries. The same crossover was reported at the world level in ref. [24], but only for the years 2001–2005. The appearance of an exponential function in a class of economic problems is studied in refs. [25,26]. They explain it utilizing concepts from statistical physics: assuming that money is locally conserved in interactions between economic agents

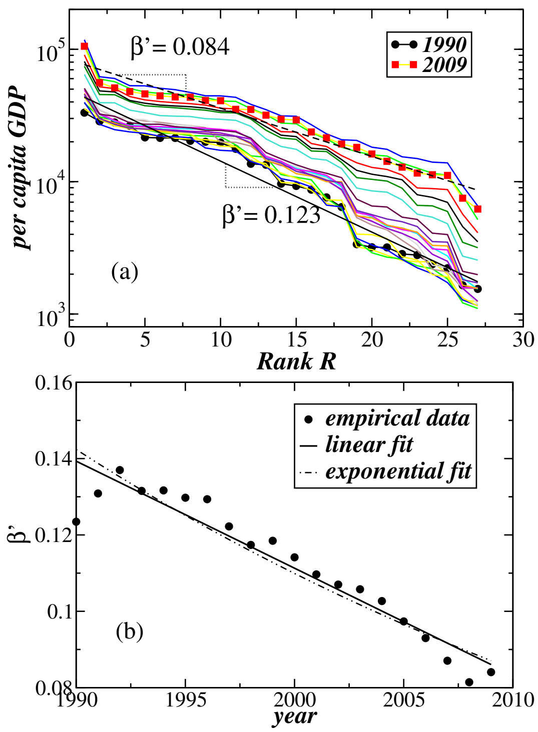


Fig. 1: (Colour on-line) Decrease in the EU income inequality measured by the Zipf rank plot of $\mathcal{G}_{i,t}$ *vs.* rank for different years. (a) Linear-log Zipf plot of $\mathcal{G}_{i,t}$ for each year follows an exponential function with a slope decreasing in time. The smaller the slope (exponential parameter $\beta'$), the larger the income equality. (b) Exponential parameter $\beta'$ calculated from panel (a) *vs.* year. From the regression we estimate the year when $\beta'$ becomes negligible, implying the existence of EU income equality. Besides the best linear fit, we also show the best exponential fit.

yields Boltzmann-Gibbs exponential dependence for pdf of money. Note that in our analysis we treat all 27 countries on the same footing, *i.e.*, do not distinguish between the so-called old and new EU members. Neglecting to distinguish among the two groups of EU members is reasonable since new EU members have been on the membership track for the past twenty years. Thus, they benefited from open borders with the old EU members and had access to EU funds. On the other hand, neglecting to distinguish between the old and new EU members may, in fact, bias the results against us, since pre-accession funds may not have been quite the same as post-accession funds would have been.

The Zipf rank approach shown in fig. 1(a) reveals that the Zipf exponent $\beta'$ calculated for each year exhibits a decreasing functional dependence with time. This result suggests a decrease in EU income differences since the smaller the slope (parameter $\beta'$), the larger the cross-country income equality. Income convergence between 25 EU countries (excluding Bulgaria and Romania) was reported for the period 1995–2005 in ref. [27]. However, ref. [27] does not study the convergence dynamics, *i.e.*, whether and how convergence changes over time.

In order to address that issue and to estimate when the final state of capital flow equilibrium, where inflow equals outflow, will occur, *i.e.* the state in which all EU $\mathcal{G}_{i,t}$'s are evenly distributed, we show in fig. 1(b) how the parameter $\beta'$ calculated for each year between 1990 and 2009 changes. We fit $\beta'$ *vs.* year with a linear fit in which the slope $\alpha'$ quantifies the annual convergence (or *deceleration of divergence*). From the regression line between $\beta'$ and year —$\beta' = 5.719 - 0.0028$ (year), we obtain the estimate

$$\alpha' = -0.0028 \pm 0.0002. \qquad (2)$$

For the sake of comparison, in addition to the best linear fit we also show the best exponential fit.

The usual procedure in economics is to base predictions of the future on relationships established in the past. In making a prediction we extrapolate the regression outside the range of values used to obtain the regression, and the further we extrapolate beyond the existing data, the less reliable will be our predictions. Keeping this caveat in mind, we assume the EU countries will continue to pursue their open borders policy and successfully avoid political turmoil. We also assume that $\beta'$ will continue to decrease in time so that the fit in fig. 1 will continue to hold. Our resulting extrapolation of the regression line into the future yields the estimate that cross-country *per capita* GDP ($\mathcal{G}_{i,t}$) of the current EU members will become approximately equal after ≈30 years. Clearly, this estimate will be more accurate in its quantification of the rate of EU convergence in the past than in its prediction of what that rate will be in the future. Great care should be taken when attempting to draw conclusions about the future on the basis of a relatively short period of time in the past.

Note that a positive $\beta$ from a growth-initial level regression of eq. (1) does not imply a reduction in cross-sectional variation of income over time [4,28] (*i.e.*, sigma convergence does not follow from beta convergence). When using the Zipf rank approach a decrease in variance of the log of *per capita* GDP ($\mathcal{G}_{i,t}$), follows a decrease in $\beta$. In the final state where all $\mathcal{G}_{i,t}$'s are equal, the slope $\beta'$ would be zero and the variance of the log of ($\mathcal{G}_{i,t}$) would also be zero.

In our considerations thus far we have disregarded country size. As stated above, such an approach may be misleading. Sala-i-Martin [11] and other researchers [10,29] found that if one is concerned about human welfare, disregarding country size when carrying out a regression is misleading because different countries have differing populations. Clearly the regression in eq. (1) can estimate convergence between nations (different countries) but not convergence between individuals since, *e.g.*, Germany and Luxemburg are given the same weight. Thus we repeat the previous analysis of $\mathcal{G}$, but this time weight according to population size, expressing the population of each country by rounding off to the nearest million and, to further simplify, assuming that every citizen lives equally well (has approximately the same $\mathcal{G}$). In fig. 2 we show the

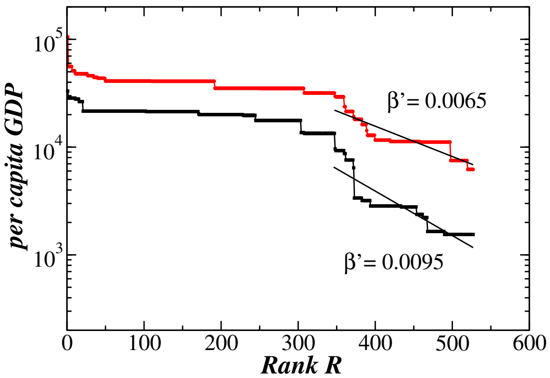


Fig. 2: (Colour on-line) Decrease in the EU income inequality measured by the weighted Zipf plot of $\mathcal{G}_{i,t}$ *vs.* rank for different years. Linear-log Zipf plot of $\mathcal{G}_{i,t}$ for each year follows an exponential function with a slope decreasing in time.

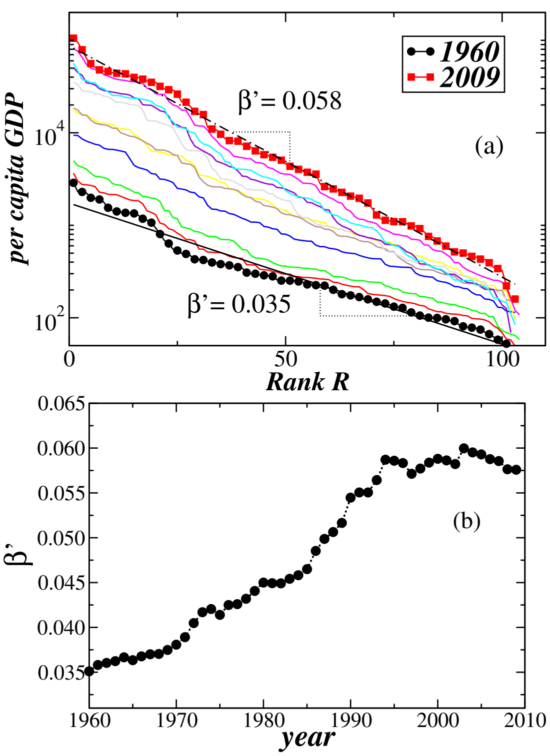


Fig. 3: (Colour on-line) Increase in the world income inequality. (a) Zipf plot of $\mathcal{G}_{i,t}$ *vs.* rank for different years. The number of countries is fixed for each year. $\beta'$ is calculated from exponential fit. The slope measured by $\beta'$ is gradually increasing implying increase in the world income inequality. (b) Exponential parameter $\beta'$ calculated from fig. 2 *vs.* year. In the past few years, the growth of inequality seem to have stopped and turn to decrease in the world income inequality.

population-weighted Zipf rank plot of $\mathcal{G}_{i,t}$'s. Compared with the unweighted Zipf rank plot of $\mathcal{G}_{i,t}$'s there is a clearer distinction between old EU members and new EU members. After extrapolating we find that the *per capita* GDP ($\mathcal{G}_{i,t}$) will become approximately the same after $\approx 40$ years, *i.e.*, 10 years longer than when estimated using the unweighted Zipf rank plot.

Next we use an unweighted Zipf rank approach to study income inequality globally. Among the 104 countries for which data was available beginning in 1960, we rank the $\mathcal{G}_{i,t}$ according to value, from largest to smallest, for the period 1960–2009. Not surprisingly, and consistent with findings that use different methods, the values of parameter $\beta'$ of the Zipf plot obtained from an exponential fit in fig. 3 increase over time. This implies that cross-country income inequality has been growing over time. However, the process is not homogeneous across all countries. In fig. 3(a), for the the years 1960–2009, we see that the poorest countries in the right-hand tail of Zipf plots seem to be unaffected by the process of globalization. This may be because these countries were more frequently involved in wars and civil wars ref. [10].

Figure 3(b) shows the parameter $\beta'$ calculated for each year in the period 1960–2009. They support the previous result that cross-country income inequality for these countries has been growing during that period. In recent years this growth in inequality seems to have stopped, the most likely reason being the influence of China.

We next employ the Zipf rank approach in another study of the 1960–2009 period. This time we vary the number of countries for each year. In contrast to the result shown in fig. 3, we observe a decrease in $\beta'$ over time. This decrease in $\beta'$ is not due to a decrease in cross-country income inequality, but due to an increase in the number of countries during this period (*e.g.*, the Soviet Union and Yugoslavia were subdivided into 21 new countries). Thus the empirical results are dependent on the methods employed and on the sample size of countries that are analyzed, and great care is necessary when employing different methods [29].

Next we ask which economic events are predominant in their contribution to world income convergence. In order to test the impact of recessions on world income inequality, for shorter time intervals $\Delta T$ we study regressions of eq. (1) and the regression

$$\ln(\mathcal{G}_{i,t+\Delta T}) = \Delta T\beta_0 + (1-\Delta T\beta)\ln(\mathcal{G}_{i,t}) + \Delta T\epsilon_{i,t}, \quad (3)$$

which we obtain from eq. (1). We choose subsequent years, $\Delta T = 1$, and study the year-on-year convergence during times of recession. This is appropriate because in the recent past recessions have lasted for relatively short time periods, say 0.5–3.0 years. When $\Delta T$ is short, we expect that $\beta$ will behave erratically, flipping between positive and negative $\beta$ values, with positive values indicating convergence.

In order to quantify how globalization and global economics crises affect world income inequality, in fig. 4 for 2006–2010 we estimate the $\beta$ parameter calculated between the logarithm of the initial $\mathcal{G}_{i,t}$, ln(GDP), and the logarithm of the final $\mathcal{G}_{i,t}$ of eq. (3): (2006–2007) $\beta = 0.0072 \pm 0.0035$, (2007–2008) $\beta = 0.013 \pm 0.0044$, (2008–2009) $\beta = 0.043 \pm 0.0056$, and (2009–2010) $\beta = 0.0055 \pm 0.0025$. We find that for each year the

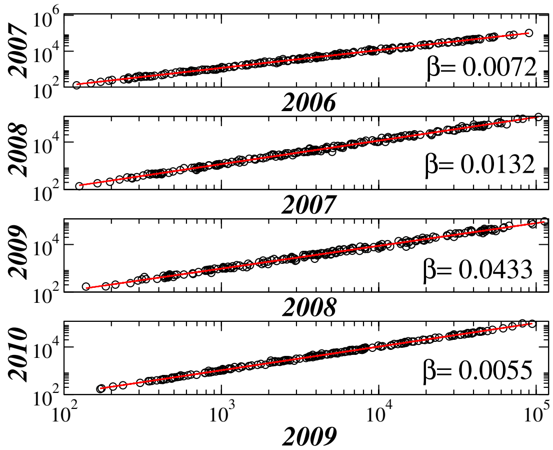


Fig. 4: (Colour on-line) Effect of the 2007–2009 worldwide recession on world income inequality. $\ln \mathcal{G}_{i,t}$ *vs.* $\ln \mathcal{G}_{i,t}$ calculated for two subsequent years in the period 2006–2010. Note the significant decrease in world income differences during global economic crises. The $\beta$ parameter of eq. (3) dramatically increased during the 2008–2009 global recession, implying that the $\mathcal{G}_{i,t}$ of richer countries declined even more than the $\mathcal{G}_{i,t}$ of poorer countries.

speed of convergence $\beta$ is positive, implying that convergence exists for each year analyzed. We also find that each year during the period 2007–2009, a time characterized by global recessions and market crashes, the $\mathcal{G}_{i,t}$ of the richer countries declined even more than the $\mathcal{G}_{i,t}$ of the poorer countries (the $\mathcal{G}_{i,t}$ of some developing countries actually grew during the crisis). One way to interpret this finding is that richer countries are more vulnerable to financial crashes than poorer countries.

Repeating the same analysis for the 27 EU countries, we find a new and surprising result. For 2006–2010 we estimate the $\beta$ parameter and find: (2006–2007) $\beta = 0.067 \pm 0.011$, (2007–2008) $\beta = 0.062 \pm 0.012$, (2008–2009) $\beta = -0.028 \pm 0.017$, and (2009–2010) $\beta = -0.004 \pm 0.013$. Thus we find that during this period of global recession and market crashes, the convergence between developing and developed EU member countries stopped. Since the EU is only comprised of countries that are either developed or developing, and the world outside the EU is comprised of undeveloped countries as well, these convergence results reveal the complexity of the vulnerability hierarchy among countries of differing levels of wealth during a recession.

In order to reproduce qualitatively the scaling feature we find in $\mathcal{G}_{i,t}$ data, we propose a simple model comprising approximately the same number of countries (100) as those found in our empirical data when analyzing divergence at a world level. At the initial year ($t = 1960$) each country is assigned the initial $\mathcal{G}_{i,t}$ supplied in the data. Since GDP can be calculated as the sum of four macroeconomics variables —one of which is export minus import— we assume that the change of $\mathcal{G}$ of a country $i$ is the sum of two terms: a) the aggregate of exports and imports between country $i$ and all other countries, and b) all other GDP constituents of country $i$ apart from export and imports.

It is widely believed that convergence across countries is primarily due to free trade and increased globalization [30–32]. In our model, when considering the aggregate result of trading between country $i$ and all other countries, we note that any aggregate result of trading $R_{i,j}$ between a pair of countries $(i,j)$ that contributes to the growth rates of both $i$ and $j$ is size dependent,

$$R_{i,j}(t) \equiv -\tilde{\beta} \ln \mathcal{G}((i,t)) + \epsilon_i. \tag{4}$$

We assume $j \geqslant i$, and we take $\epsilon$ from a Gaussian distribution. It is reasonable to assume that any interaction is strongly affected by the size of smaller country because the size of the trade between two countries depends on the size of the smaller country. Recalling that $R_{i,j} = \ln(\mathcal{G}_{i,t+\Delta T}/\mathcal{G}_{i,t})$, we note that eq. (4) resembles eq. (3). In agreement with eq. (3), the sign of $\tilde{\beta}$ controls whether we obtain convergence ($\tilde{\beta} > 0$) or divergence ($\tilde{\beta} < 0$) in $\mathcal{G}_{i,t}$.

Since trades in goods (imports and exports) between all countries are not publicly available, we study only imports $I$ and exports $E$ between the US and its trading partners. The data we use for the period 1985–2005 are from the Census Bureau (`www.census.gov/foreign-trade/balance/c4810.html`). For each of the 71 countries trading with the US, we calculate $(\mathcal{G}_{i,1985} + \Delta(E-I))/\mathcal{G}_{i,1985}$. Clearly, this represents the increase in a country's wealth due to trade. From the regression $(\mathcal{G}_{i,1985} + \Delta(E-I)_i)/(\mathcal{G}_{i,1985}\Delta T) = \beta_{\ell s} \ln(\mathcal{G}_{i,t}) + \epsilon_i$, we obtain annualized $\beta_{\ell s} = 0.002 \pm 0.0014$, statistically insignificant, but surprisingly close to the value obtained for speed of convergence in ref. [4]. It is clear that changing the period of time studied would yield different estimates for $\beta_{\ell s}$.

In numerical simulations, in order to simulate 50 years of growth, we perform 50 steps for each country $i$, where for each step, given by a year $t$, we model the growth rate as $\gamma(i,t) \equiv \ln(G(i,t)/G(i,t-1))$. For each step (one year), the we model the growth rate of all none-trading GPD according to a Gaussian distribution $R_i = N(\mu, \sigma)$ with mean $\mu = 0.005$ and $\sigma = 0.0025$. The particular values for $\mu$ and $\sigma$ are not essential to the scaling properties we analyze. Note that, according to the annual US data for the period 1954–2004, the growth rate of real *per capita* GDP (which includes both trading exchange and other contributions) has a mean of 0.021 and a standard deviation of 0.022 [33]. For each step (one year) in calculating the growth rate due to trading between countries we assume that each country $i$ interacts with any other country $m$. We thus have many temporary variables $\mathcal{G}_{i,t}^{(m)}$ for each country $\mathcal{G}_{i,t}$ inside year $t$, where $m$ stands for an iteration $m$. After each possible couple interaction of eq. (4), we calculate a new temporary variable $\mathcal{G}_{i,t}^{(m+1)}$ from the previous one $\mathcal{G}_{i,t}^{(m)}$ as, $\mathcal{G}_{i,t}^{(m+1)} = \mathcal{G}_{i,t}^{(m)} \exp[R_{i,m}(t)]$, and after all possible interactions the last temporary variable we assign as a new $\mathcal{G}$ value, $\mathcal{G}_{i,t+1}$, that holds for the beginning of the next year. Then we start with a new step.

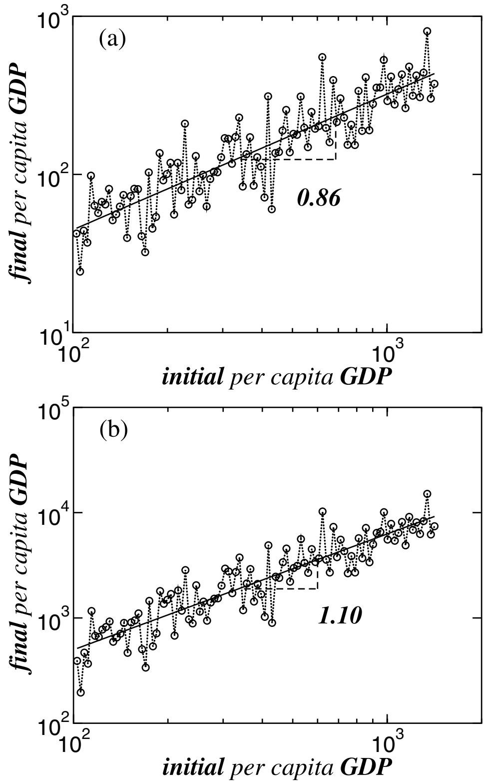


Fig. 5: (Colour on-line) Example of (a) decrease (convergence) and (b) increase (divergence) in world income differences. Results of numerical simulations. "Final" $\mathcal{G}_{i,t}$ after 50 steps *vs.* initial $\mathcal{G}_{i,t}$, and a power-law fit with the exponent (a) 0.86 (convergence) and (b) 1.1 (divergence). We take $\epsilon$ from a Gaussian distribution. We take (a) $\tilde{\beta}=0.005$ and (b) $\tilde{\beta}=-0.005$.

As stated above, in our model of eq. (4) $\tilde{\beta}>0$ yields convergence whereas $\tilde{\beta}<0$ yields divergence in $\mathcal{G}_{i,t}$. Thus, $\tilde{\beta}>0$ is appropriate for modeling convergence among EU countries (fig. 1), while $\tilde{\beta}<0$ is more appropriate for modeling divergence at a world level (fig. 3). In fig. 5(a) using $\tilde{\beta}=0.005$ from eq. (4) we plot $\mathcal{G}_{i,t}$ after 50 steps *vs.* the initial $\mathcal{G}_{i,1}$, and find a nice fit by a power law with exponent 0.86 (convergence). In fig. 5(b) using $\tilde{\beta}=-0.005$ we find a nice fit by a power law with exponent 1.1 (divergence). Note that the values of the power-law exponent depend on both $\tilde{\beta}$ in eq. (4) and the number of steps performed in numerical simulations.

In the previous simulations we assumed, for the sake of simplicity, that $\tilde{\beta}$ is i) constant and hence, does not change over time, and ii) is $\mathcal{G}$ independent. We next investigate these two cases. We first perform numerical simulations in which $\tilde{\beta}$ of eq. (4) gradually changes from a negative to a positive value, yielding a gradual move from divergence to convergence, as seen at the world level in fig. 3(b) for 100 countries existing in 1960. To model the size $\mathcal{G}$-dependence of $\tilde{\beta}$ in eq. (4), we subdivide all countries into two groups: A) countries participating in global trade and B) countries not participating in global trade. We model the size dependence of $\tilde{\beta}$ such that $\tilde{\beta}$ is much larger if trade is carried out between two countries belonging to group A) than if at least one country is from group B). Note that in figs. 1(a) and 2 the richest countries in EU (apart from Luxembourg) belong to a club with a Zipf plot slope that is smaller than the slope for the remainder of the EU countries. Size-dependent scalings have recently also been found in *per capita* public debt [34].

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REFERENCES


[1] Solow R. M., *Q. J. Econ.*, **70** (1956) 65.
[2] Romer P. M., *J. Polit. Econ.*, **94** (1986) 1002.
[3] Barro R. J. and Sala-i-Martin X. X., *Econ. Growth* (MIT Press, London) 1995.
[4] Sala-i-Martin X. X., *Econ. J.*, **106** (1996) 1019.
[5] Durlauf S. N., *J. Econom.*, **100** (2001) 65.
[6] Quah D., *Eur. Econ. Rev.*, **37** (1993) 426.
[7] Quah D., *Scand. J. Econ.*, **95** (1993) 427.
[8] Axtell R. L., *Science*, **293** (2001) 1818.
[9] Podobnik B. *et al.*, *Proc. Natl. Acad. Sci. U.S.A.*, **107** (2010) 1832.
[10] Milanovic B., *Carnegie Pap.*, **62** (2005).
[11] Sala-i-Martin X. X., *Q. J. Econ.*, **121** (2006) 351.
[12] Atkinson A. B., *J. Econ. Theory*, **2** (1970) 244.
[13] Baumol W., *Am. Econ. Rev.*, **76** (1986) 1072.
[14] Barro R. J. and Sala-i-Martin X., *J. Polit. Econ.*, **100** (1992) 223.
[15] Mankiw N. G., Romer D. and Weil D., *Q. J. Econ.*, **107** (1992) 407.
[16] Barro R. J. and Sala-i-Martin X. X., *Economic Growth* (MIT Press, Cambridge, Mass.) 1994.
[17] Durlauf S. N. and Johnson P. A., *J. App. Econom.*, **10** (1995) 365.
[18] Durlauf S. N., *Econ. J.*, **106** (1996) 1016.
[19] Pritchett L., *J. Econ. Perspect.*, **11** (1997) 3.
[20] Atkinson A. B. and Brandolini A., *J. Econ. Lit.*, **XXXIX** (2001) 771.
[21] Milanovic B., *Econ. J.*, **112** (2002) 51.
[22] Zipf G. K., *Human Behavior and the Principle of Least Effort* (Addison-Wesley) 1949.
[23] Stanley M. H. R. *et al.*, *Econ. Lett.*, **49** (1995) 453.
[24] Shao J. *et al.*, *Eur. Phys. J. B*, **56** (2007) 157.
[25] Dragulescu A. A. and Yakovenko V. M., *Physica A*, **299** (2001) 213.
[26] Dragulescu A. A. and Yakovenko V. M., *Eur. Phys. J. B*, **20** (2001) 585.
[27] Paas T. *et al.*, SSRN-id1078863, No. 60-2007 (2007).
[28] Bernard A. B. and Durlauf S. N., *J. Econom.*, **71** (161) 1996.
[29] Milanovic B., *EconWPA, HEW Ser.*, paper No. 0305003 (2003).
[30] Edwards S., *J. Econ. Lit.*, **31** (1993) 1358.
[31] Hidalgo C. A. *et al.*, *Science*, **317** (2007) 482.
[32] Ausloos M. and Lambiotte R., *Physica A*, **382** (2007) 16.
[33] Barro R. J., *Q. J. Econ.*, **121** (2006) 823.
[34] Petersen A. M. *et al.*, *EPL*, **90** (2010) 38006.