Comment on "Dynamic Opinion Model and Invasion Percolation"

In [1], Shao *et al.* claim, based on low statistics simulations, that a model with majority rule coarsening exhibits in $d = 2$ a percolation transition in the universality class of invasion percolation *with trapping* (IPT). They report also that the system reaches its final state rapidly with no diverging time scale. Since the original configurations are random and thus in the ordinary percolation (OP) universality class, it seems unlikely that long range correlations could develop in a finite time that would change this. Indeed, it was proved rigorously [2] that similar 2D models (called "dependent percolation" in [2]) belong to the OP universality class.

Here, we present high statistics (up to $L = 2^{14}$, $>10^4$ realizations) on $L \times L$ square lattices and confirm that the phase transition is in the OP universality class, thus refuting a central tenet of [1]. Initially, each site i is randomly assigned one of two opinions (or spins): $\sigma_i = +1$, with probability f; otherwise, $\sigma_i = -1$. At each time step, all sites are updated in parallel. If at least three of their four neighbors disagree with them, they change their opinion; otherwise, they keep it. As noted in [1], this leads quickly [within $O(10)$ time steps] to a static state, except for sites that flip permanently with period 2. The critical probability f_c where a "+1" cluster percolates depends slightly on how these flicker sites are treated (we treat them as "+," if $\sigma_i = +1$ at even times), but the universal properties do not.

We first determine f_c by measuring the chance that a cluster in the final state percolates through lattices with open boundary conditions. Using finite size scaling [3], we obtain $f_c = 0.506\,425(20)$, in agreement with the less precise estimate of [1]. After that, we measure the distribution of cluster sizes with $\sigma = +1$ in final states obtained with helical boundary conditions for $f \approx f_c$. Figure 1 confirms the above estimate of f_c and shows that the data are excellently described by a power law $P(s) \sim s^{-\tau}$ with the OP critical exponent $\tau = 187/91 \approx 2.055$, ruling out the IPT exponent $\tau \approx 1.89$. We see also deviations from this power law at small masses s, as small clusters are eliminated by the coarsening. This, together with using open boundary conditions and neglecting finite size corrections, explains why τ was underestimated in [1]. A data collapse of the right-hand side peaks in Fig. 1 gives $D_f = 1.895(15)$ as for OP, but in disagreement with IPT. Notice that the exponents obtained in [1] strongly violate the hyperscaling relation $\tau = d/D_f + 1$.

Shao *et al.* claim that IPT is relevant because local clusters get trapped. The difference between OP and IPT is that clusters can grow both outwards and inwards (into empty holes) in OP, while they can only grow outward in IPT. In this respect, the model of [1] is exactly as OP.

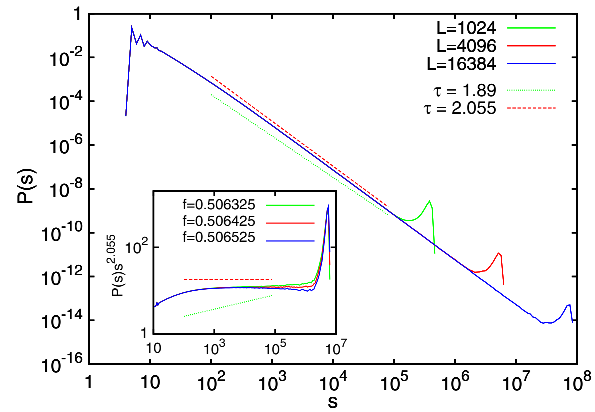

FIG. 1 (color online). Probability distribution of cluster sizes at $f = 0.506\,425$ for different L on a log-log scale. The straight lines represent power laws with exponent $\tau = 2.055$ (dashed red line) and $\tau = 1.89$ (dotted green line), corresponding to OP and IPT, respectively. Inset: data for $L = 4096$ at different values of f, after multiplication with $s^{2.055}$. Small changes of f give rise to deviations from power law behavior. At our estimate of f_c obtained independently from the spanning probability, power law scaling with the OP exponent $\tau = 2.055$ extends over 3 orders of magnitude.

We also simulated the process on random Erdös-Renyi networks. For small average degrees, we confirm the claim of [1] that the percolation transition is in the OP class. But for large average degrees we find an unexpected first-order transition [4].

As a model for opinion dynamics, the model is of limited interest, since the dynamics leaves essentially unchanged all large clusters present in the initial state—except for clusters with hubs, in the case of scale-free networks, which immediately adopt the majority opinion.

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