Physica A 428 (2015) 46–51

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Multifractal properties of price change and volume change of stock market indices

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ARTICLE INFO

Article history:
Received 23 December 2014
Available online 11 February 2015

Keywords:
Stock market index
Price change
Volume change
Multifractal detrended fluctuation analysis
Multifractal detrended cross-correlation analysis

ABSTRACT

We study auto-correlations and cross-correlations of daily price changes and daily volume changes of thirteen global stock market indices, using multifractal detrended fluctuation analysis (MF-DFA) and multifractal detrended cross-correlation analysis (MF-DXA). We find rather distinct multifractal behavior of price and volume changes. Our results indicate that the time series of price changes are more complex than those of volume changes, and that large fluctuations dominate multifractal behavior of price changes, while small fluctuations dominate multifractal behavior of volume changes. We also find that there is an absence of correlations in price changes, there are anti-persistent long-term correlations in volume changes, and there are anti-persistent long-term cross-correlations between price and volume changes. Shuffling the series reveals that multifractality of both price changes and volume changes arises from a broad probability density function.

1. Introduction

Price and trading volume are two key variables in finance. Understanding how changes in volume affect changes in price adds valuable insight on the structure (e.g. how the rate of information flow affects price dynamics) and complexity of financial markets [1]. The relationship between trading volume and security prices has been studied for over 40 years, since Osborne hypothesized in 1959 that security prices can be modeled as a diffusion process with its variance dependent on the number of transactions [2]. Ying analyzed six years of daily price and volume for the New York Stock Exchange (NYSE) and found that: (i) a small (large) volume is usually accompanied by a fall (rise) in price, and (ii) a large increase in volume is usually accompanied by a large rise or fall in price. This suggests there are positive correlations between volume and the magnitude of price change which was later confirmed in various studies [3–5].

Several techniques have been developed to study fractal and multifractal properties in time series [6–13], where multifractal detrended fluctuation analysis (MF-DFA) [10] and multifractal detrended cross-correlation analysis (MF-DXA) [12] have shown to be powerful tools in analyzing multifractal behavior of non-stationary time series. The stock market index is a good indicator of the overall market behavior and is frequently used by financial investors. The scale-invariant behavior of both the distribution of price changes and the long term correlations in the absolute values of price changes is a well known property of financial markets [14,15]. Similar behavior was observed for share volume [16,17], and

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http://dx.doi.org/10.1016/j.physa.2015.02.046
0378-4371/

it was also found that magnitudes of volume changes display long-term correlations and power-law cross-correlations with magnitudes of price changes [18]. Most work on the multifractality of stock market indices have focused on correlations in price changes [19–21]. However, a more thorough study on the multifractality of price and volume changes across many stock market indices is still lacking.

In this paper we study the multifractal properties of daily price changes and daily volume changes for 13 stock market indices. We apply the MF-DFA method to examine correlations in price changes and volume changes, and look for common behavior across all market indices. We also use the MF-DXA method to analyze cross-correlations between the two market variables, which may provide additional insight into the market's dynamics. The paper is organized as follows. Section 2 introduces a brief overview of the MF-DFA and MF-DXA methods. Section 3 describes our dataset and presents the empirical results. Section 4 draws a conclusion on our analysis.

2. Methodology

2.1. Multifractal detrended fluctuation analysis

Multifractal processes are characterized by different scaling behavior of segments with small and large fluctuations, and their description requires a hierarchy of scaling exponents [10]. We employ the MF-DFA method which quantifies multifractality of non-stationary time series [10], and has been applied to physiological signals [22], geophysical [23], hydrological [24] and financial time series [25,26]. The MF-DFA procedure is briefly described as follows [10]: (i) Integrate the original time series $x(i),\ i = 1, \ldots, N$ to produce $X(k) = \sum_{i=1}^{k}[x(i) - \langle x\rangle]$, where $\langle x\rangle$ is the average. (ii) Divide the integrated series $X(k)$ into N_n non-overlapping segments of equal length n and estimate the local trend $X_i(k)$ in each segment from a mth order polynomial regression. (iii) Detrend the integrated series in each segment (by subtracting the local trend) to calculate the detrended variance and average over all segments to find the qth order fluctuation function

$$F_q(n) = \left\{ \frac{1}{N_n} \sum_{i=1}^{N_n} \left[\frac{1}{n} \sum_{k=(i-1)\,n+1}^{in} [X(k) - X_i(k)]^2 \right]^{q/2} \right\}^{1/q} \tag{1}$$

where q can take any real value except zero. (iv) Repeat this calculation to find the fluctuation function $F_q(n)$ for all box sizes n. If long-term correlations are present, $F_q(n)$ will increase with n as a power law $F_q(n) \sim n^{h(q)}$, where the scaling exponent $h(q)$ is calculated as the slope of the linear regression of $\log F_q(n)$ versus $\log n$. Since the scaling exponent $h(2)$ is identical to the well-known Hurst exponent in a stationary time series, $h(q)$ is called the generalized Hurst exponent.

For multifractal processes $h(q)$ is a decreasing function of q and describes scaling behavior of large (small) fluctuations for positive (negative) values of q. These exponents are directly related to the classical multifractal exponents $\tau(q)$ defined in the standard partition function multifractal formalism: $\tau(q) = qh(q) - 1$, where $\tau(q)$ is a linear function for monofractal signals and a nonlinear function for multifractal signals. A multifractal series can also be described by the singularity spectrum $f(\alpha)$ from the Legendre transform $\alpha(q) = \mathrm{d}\tau(q)/\mathrm{d}q, f(\alpha) = q\alpha - \tau(q)$, where $f(\alpha)$ denotes the fractal dimension of the series' subset that is characterized by the Hölder exponent α [10].

In order to measure the complexity of the series, we fit the singularity spectra to a fourth-degree polynomial

$$f(\alpha) = A + B(\alpha - \alpha_0) + C(\alpha - \alpha_0)^2 + D(\alpha - \alpha_0)^3 + E(\alpha - \alpha_0)^4 \tag{2}$$

and calculate a set of multifractal spectrum parameters: the position of maximum α_0; the width of the spectrum $W = \alpha_{\max} - \alpha_{\min}$ obtained from extrapolating the fitted curve to zero; and the skew parameter $r = (\alpha_{\max} - \alpha_0)/(\alpha_0 - \alpha_{\min})$ where $r = 1$ for symmetric shapes, $r > 1$ for right-skewed shapes, and $r < 1$ for left-skewed shapes. Roughly speaking, a small value of α_0 suggests the series is correlated and more regular in appearance. The width W measures the degree of multifractality in the series, a wider range of Hölder exponents α results in a "richer" structure and a higher degree of multifractality. The skew parameter r indicates that the scaling behavior of small fluctuations dominates the multifractal behavior if the spectrum is right-skewed, and the scaling behavior of large fluctuations dominates if the spectrum is left-skewed. These three parameters (α_0, W, r) lead to a measure of complexity where a series with a high value of α_0, a wide range W of scaling exponents, and a right-skewed shape can be considered more complex than one with the opposite characteristics [27].

2.2. Multifractal detrended cross-correlation analysis

We use the multifractal detrended cross-correlation analysis (MF-DXA) [12] to analyze the relationship between daily changes in price and volume. This method is a generalization of the MF-DFA and is designed to measure long-term cross-correlations between two simultaneously recorded non-stationary time series. It has been successfully applied in climatic [28], geophysical [29] and financial [30] data. The MF-DXA procedure follows that of MF-DFA in steps (i) and (ii): integrate the time series $x(i), y(i),\ i = 1, \ldots, N$ to produce $X(k) = \sum_{i=1}^{k}[x(i) - \langle x\rangle]$, $Y(k) = \sum_{i=1}^{k}[y(i) - \langle y\rangle]$ and calculate the local trends $X_i(k), Y_i(k)$ in each segment from an mth order polynomial regression. (iii) Detrend the integrated series in each segment (by subtracting the local trend) to calculate the detrended covariance and average over all segments to get the qth order detrended covariance

$$F_q^{xy}(n) = \left\{ \frac{1}{N_n} \sum_{i=1}^{N_n} \left[\frac{1}{n} \sum_{k=(i-1)\,n+1}^{in} |X(k) - X_i(k)|\ |Y(k) - Y_i(k)| \right]^{q/2} \right\}^{1/q} \tag{3}$$

(iv) Repeating steps (i) to (iii) for all box sizes provides the relationship between the fluctuation function $F_q^{xy}(n)$ and the box size n. If the original series $x(i)$, $y(i)$ have power-law cross-correlations, then $F_q^{xy}(n) \sim n^{h_{xy}(q)}$ where the scaling exponent $h_{xy}(q)$ is determined from a linear regression of $\log F_q^{xy}(n)$ versus $\log n$. Long-term cross-correlations between two series imply that each series has long memory of its own previous values and long memory of the previous values of the other series [12]. Calculations of the multifractal exponents $\tau(q)$, multifractal spectrum $f(\alpha)$, and measure of complexity (α_0, W, r) follow from the MF-DFA procedure.

2.3. Types of multifractality

A time series can exhibit two different types of multifractal behavior: (i) multifractality from a broad probability density function of the series, and (ii) multifractality from different long-term correlations of small and large fluctuations. The type of multifractal can be found by randomly shuffling the series and analyzing its behavior. For multifractals of type (ii) the shuffled series exhibits simple random behavior (since long-term correlations are destroyed) and the width of the $f(\alpha)$ spectrum is significantly reduced, for multifractals of type (i) the width of the $f(\alpha)$ spectrum of the shuffled series remains the same (since probability density cannot be removed), and for multifractals of type (i) and (ii) the shuffled series shows weaker multifractality than the original series [10].

3. Data and analysis

We analyze daily closing price and daily trading volume of 13 stock market indices over the eight year period August 30, 2006 to March 1, 2014, selecting indices with the longest available overlapping time series: AEX, BFX, BSESN, BVSP, FCHI, FTSE, GSPC, HSI, KS11, MXX, N225, SSMI, TWII (http://finance.yahoo.com/). For each market index we calculate the logarithmic change in price $R_i(t) = \ln S_i(t + \Delta t) - \ln S_i(t)$ and the logarithmic change in volume $R_i'(t) = \ln Q_i(t + \Delta t) - \ln Q_i(t)$, where $S_i(t)$ is the daily closing price at time t, $Q_i(t)$ is the daily trading volume at time t, i represents the index of the time series, and $\Delta t = 1$ day. The time series for both price changes and volume changes for the Bovespa (BVSP) and the S&P 500 (GSPC) market indices are shown in Fig. 1.

We apply the MF-DFA and MF-DXA methods to price changes and volume changes of all 13 stock market indices, where the local trends are fitted with a polynomial of order $m = 2$. Next, we perform a fourth-degree polynomial regression of the singularity spectra $f(\alpha)$ to determine α_0, $\alpha_{\max}$, and $\alpha_{\min}$ and then estimate the three measures of complexity of a multifractal series: the position of maximum α_0, the width of the spectrum $W = \alpha_{\max} - \alpha_{\min}$, and the skew parameter $r = (\alpha_{\max} - \alpha_0) / (\alpha_0 - \alpha_{\min})$. The multifractal spectra of the Bovespa and the S&P 500 market indices are shown in Fig. 2. We observe from these plots that multifractal spectra of volume changes are left-shifted from $\alpha_0 = 0.5$ (indicating anti persistent behavior) with shorter ranges W. The multifractal spectra from the MF-DXA method are also left-shifted, indicating anti-persistent cross-correlations between the two market variables, and have shorter ranges. We also observe that for price changes large fluctuations (described by the left side of the spectrum) show anti persistent behavior, while small fluctuations (described by the right side of the spectrum) show persistent behavior.

The complexity measure parameters (α_0, W, r) displayed in Table 1 indicate a common behavior among stock market indices: multifractal spectra of price changes are left-skewed $r < 1$ with a maximum $\alpha_0 \approx 0.6$ and broader width W, of volume changes are right-skewed $r > 1$ with a maximum $\alpha_0 \approx 0.2$ and shorter width, and of the relationship between price and volume changes are symmetric $r \approx 1$ with a maximum $\alpha_0 \approx 0.4$ and shorter width. This suggests that the time series of price changes display high complexity and an absence of correlations, while volume changes exhibit reduction in complexity and anti-persistent long-term correlations. We also find that large fluctuations dictate the multifractal behavior in price changes (left-skewed spectrum) and small fluctuations dominate in volume changes (right-skewed spectrum). Both small and large fluctuations contribute equally to the multifractality in cross-correlations between price change and volume change as indicated by the symmetric $f(\alpha)$ spectrum. The relationship between price change and volume change displays reduced complexity and anti-persistent long-term cross correlations, meaning that large (small) changes in one variable are most likely followed by small (large) changes in the other variable. We compare our results and find them consistent with previous work on correlations in price changes and cross-correlations between price changes and volume changes [19–21,31].

We next shuffle the time series of price and volume changes for all market indices and then apply the MF-DFA analysis; the shuffling procedure performed $1000 \times N$ transpositions on each series and was repeated 100 times with different random number generator seeds, in order to find the mean shuffled series spectrum, together with standard deviation. The multifractal spectra of original and shuffled series of the Bovespa and the S&P 500 market indices are shown in Fig. 3. We find that for both price and volume changes the width of the multifractal spectrum is unaffected by shuffling, indicating that the multifractality arises from a broad probability density function. We obtain similar results for other market indices.

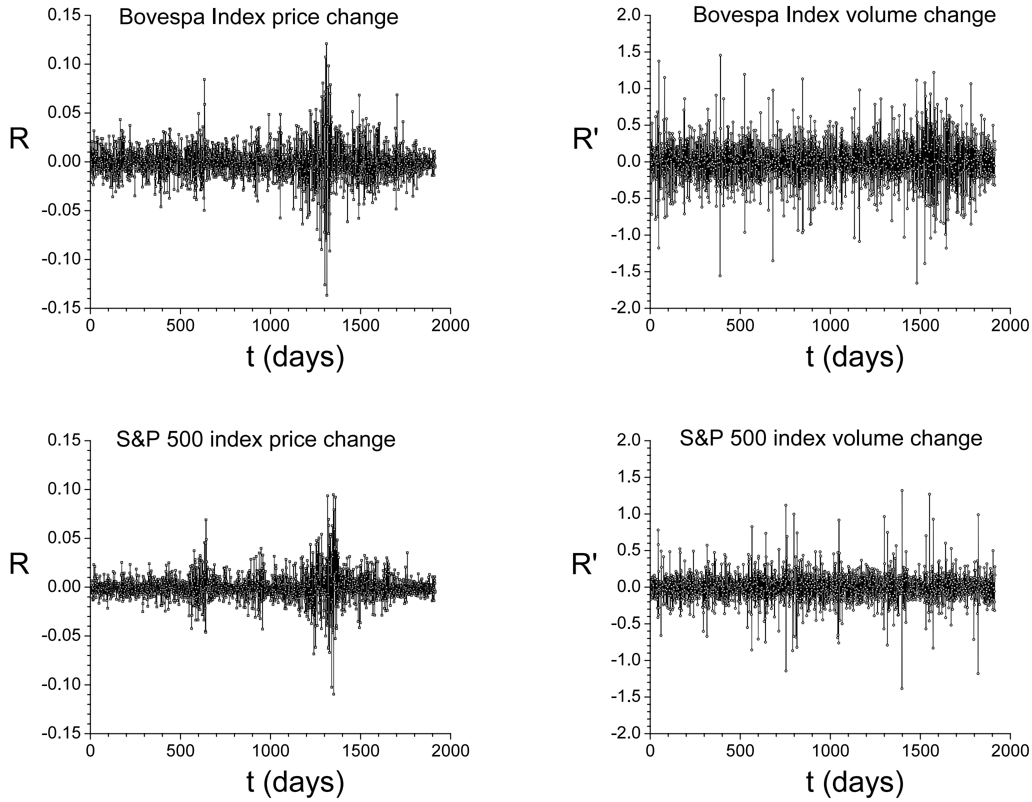

Fig. 1. Price changes R and volume changes R' of the Bovespa and the S&P 500 market indices, for the period August 30, 2006 to March 1, 2014.

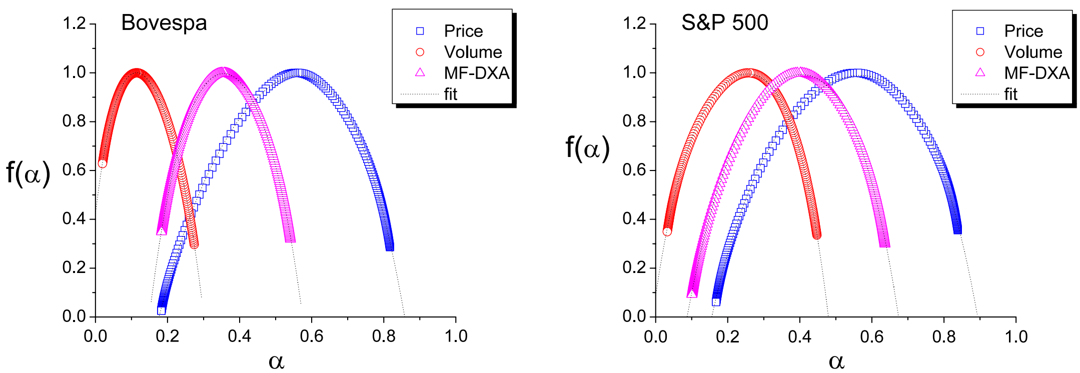

Fig. 2. Multifractal spectrum $f(\alpha)$ resulting from the MF-DFA and MF-DXA analysis of price changes R and volume changes R' of the Bovespa and the S&P 500 market indices, for the period August 30, 2006 to March 1, 2014. The dotted lines represent regression curves to the fourth order polynomial form.

4. Conclusion

In this work we investigate the auto-correlations and cross-correlations of daily price changes and daily volume changes for thirteen stock market indices. We find distinct multifractal properties for price changes and volume changes which indicate that the two market variables follow different dynamics. The time series of price changes are uncorrelated and exhibit multifractality driven by large fluctuations. On the other hand, volume changes have strongly anti-persistent correlations where the scaling exponents describing the behavior of small fluctuations are dominant in the multifractal spectrum. The two series also display anti-persistent cross-correlations, suggesting their complex dynamics arise from intrinsic properties of the series' as well as mutual interactions. For both price and volume changes the multifractality is due to a broad probability density function. Future studies should be directed at investigating the multifractal dynamics and correlations of price and volume changes. We believe such studies will lead to a better understanding of the common behavior in stock market indices.

Table 1
Multifractal parameters α_0, W and r for price changes R and volume changes R'.

Index	Price MF-DFA			Volume MF-DFA			Price and volume MF-DXA		
	α_0	W	r	α_0	W	r	α_0	W	r
AEX	0.575	0.603	0.745	0.238	0.466	0.965	0.399	0.429	1.165
BFX	0.557	0.564	0.805	0.249	0.422	1.347	0.371	0.475	1.286
BSESN	0.570	0.584	0.805	0.260	0.435	1.211	0.429	0.399	1.300
BVSP	0.567	0.682	0.746	0.114	0.329	1.316	0.355	0.426	1.065
FCHI	0.555	0.678	0.808	0.281	0.546	1.144	0.376	0.423	1.031
FTSE	0.539	0.691	0.756	0.268	0.579	1.067	0.374	0.417	1.175
GSPC	0.567	0.738	0.793	0.256	0.488	0.845	0.405	0.587	0.848
HSI	0.577	0.664	0.726	0.204	0.456	1.539	0.383	0.453	1.029
KS11	0.594	0.703	0.760	0.230	0.394	1.038	0.401	0.449	0.975
MXX	0.550	0.642	1.075	0.295	0.577	1.407	0.423	0.499	1.026
N225	0.584	0.752	0.772	0.232	0.548	1.460	0.407	0.495	0.959
SSMI	0.550	0.705	0.668	0.180	0.348	2.021	0.373	0.459	0.976
TWII	0.589	0.437	0.872	0.271	0.498	1.117	0.407	0.328	0.670
Mean	0.567	0.649	0.795	0.237	0.468	1.267	0.393	0.449	1.039

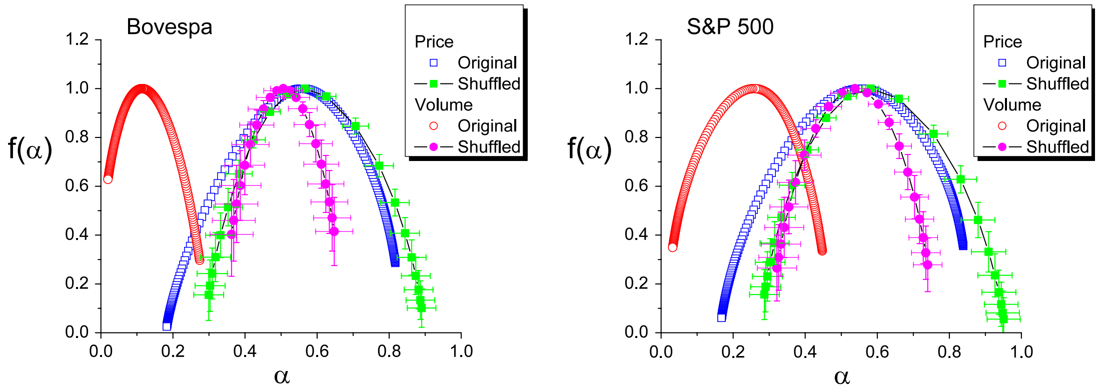

Fig. 3. Multifractal spectrum $f(\alpha)$ for price changes R and volume changes R' of the original and shuffled series of the Bovespa and the S&P 500 market indices, for the period August 30, 2006 to March 1, 2014.

Acknowledgments

This work is supported by research grants from CNPq, CAPES and FACEPE (Brazilian research agencies).

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