The Rise and Fall of Business Firms

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Growth has been the main concern of economists since Adam Smith\(^1\). Some theories emphasize the importance of production factors accumulation (labor, human capital, physical capital), other emphasize the role of technological change, and still other, respectively, of institutions, capabilities, and culture. Recently, several authors have shown how endogenous technological change can shape processes of growth at different levels of aggregation, characterizing the impact of growth and decline of business firms on real business cycles and on economic growth at large (Acemoglu, Carvalho et al. 2012, Gabaix, 2013; Acemoglu and Cao, 2015).

We build on (Fu et al., 2005) to introduce a general statistical framework of proportional growth of business firms, from which we derive some fundamental economic relationships.

Our framework aims at reproducing all the fundamental regularities on size and growth of business firms Caves (1998); Klepper and Thompson (2006); Klette and Kortum (2004); Sutton (1997):

(I). The size distribution of firms is highly skewed. According to Gibrat, the size distribution of firms is approximately lognormal for a broad range of data (Gibrat, 1931; Sutton, 1997). Simon and co-workers, on the other hand, argued that real world size distributions are well approximated by a Pareto distribution, at least in the upper tail (Ijiri and Simon, 1977; Simon and Bonini, 1958). While the exact shape of the size distribution is still debated (Luttmer, 2010), the Pareto and lognormal distributions are retained as benchmarks (Axtell, 2001; Cabral and Mata, 2003; Growiec et al., 2008; Hall, 1987b; Luttmer, 2007; Marsili, 2005; Stanley, 1995).

(II). The growth rate distribution is not Gaussian but “tent-shaped” in the proximity of the mean growth rate (Bottazzi et al., 2001; Fu et al., 2005; \(^1\)see Acemoglu (2009); Aghion and Howitt (1998) for a discussion of modern economic growth.)
Stanley et al., 1996). When looking at the entire distribution, rare events involving extremely large positive and negative shocks are observed (Fu et al., 2005), which cause the distribution of firm growth rates to have power-law tails.

(III). \textit{Smaller firms have a lower probability of survival, but those that survive tend to grow faster than larger firms.} Among larger firms, growth rates are unrelated to past growth or to firm size (Dunne et al., 1989; Evans, 1987b; Hall, 1987a; Mansfield, 1962; Rossi-Hansberg and Wright, 2007a).

(IV). \textit{The variance of growth rates is systematically higher for smaller firms} (Evans, 1987b; Hymer and Pashigian, 1962; Mansfield, 1962). It was recently discovered that a firm’s growth rate variance decays as a power-law with size, with a power of approximately 1/5 (Bottazzi et al., 2001; Ibragimov and Gabaix, 2011; Riccaboni et al., 2008; Stanley et al., 1996; Sutton, 2002).

Against this rich set of regularities that hold across different industries and markets (Ijiri and Simon, 1977), we represent business firms as collections of elementary business units which operate in independent sub-markets\footnote{See also DeFabritiis et al. (2003); Ijiri and Simon (1977); Sutton (1997)}. A rule of proportional growth is then applied to both the sales of each existing business unit, and to the introduction of new ones (Sutton, 1998).

We introduce a Generalized Proportional Growth Model (GPGM), which is structured around two key assumptions:

- the number of units (business opportunities) in a firm grows in proportion to its number of units, while there is a positive constant probability that a new unit is captured by a new entrant (the Simon’s Growth Process);
- the size of each unit grows in proportion to its size, while the different units are operating in a set of independent sub-markets (the Gibrat’s Law of Proportionate Growth).

We combine Gibrat’s and Simon’s models, and decompose the growth of each business firm in the number and the size of its elementary components. Roughly speaking, the number of units in a firm evolves in proportion to the number of preexisting units, according to Simon’s model, whereas the growth of each unit is proportional to its size, following Gibrat’s Law.

Intuitively, our model hypothesize that, on average, large incumbent firms tend to generate more new products/business units and grab new opportunities originated by universities and other R&D institutions (Klette and Kortum, 2004). At the same time, a new opportunity can be introduced by a new firm, which then can grow and capture further opportunities. Moreover,
products and firms can die and exit (Kalecki, 1945; Luttmer, 2010), due to processes of "creative destruction". Analogously, Sutton (1998) generalizes the Simon’s model and assumes that the probability that next opportunity is filled by any currently active firm is nondecreasing in firm size.

In a nutshell, the entire book revolves around these hypotheses. In the next Chapter, we recap, with some detail, the motivating evidence that has inspired us. In Chapter 3, we develop our theoretical framework deriving its predictions across (I-IV). The statistical tests on the goodness of our predictions with real world data are presented in Chapter 4, while Chapter 5 outlines some plausible departures from the benchmark, illustrating why they indicate the advantage of adjusting it to encompass a series of additional economic factors, instead of abandoning it.

The model and the evidences presented in this book show how processes of industrial growth can be accounted for by a general and parsimonious representation of the stochastic environment in which they occur. More complex economic models, which postulate some profit maximizing firm behavior and strategic interactions among firms, should be confronted with the predictions of our statistical benchmark.

We characterize some universal laws, and we then derive a taxonomy of specific instantiations of the general framework. Readers will realize that we subject our model to a wide class of empirical tests, while the restriction of the model parameters given by all these tests is far from generating an empty set.

In general, the set of invariant laws is extremely reduced and the related mathematical results are fundamental cornerstones. The typical example is the Central Limit Theorem (Billingsley (2012), chapter 5 and Embrechts et al. (1997a) chapter 2), showing that the sum of independent random variables converges to the Gaussian distribution. Analogous results hold for the sum of fat tail random variables converging to alpha stable distributions (see Billingsley (2012), chapter 1) and maxima of random variables converging to extreme value distributions (see Embrechts et al. (1997b), chapter 3).

As clearly stated by Brock (1999), any universal law, like scaling in economic systems, is in fact an "unconditional object", i.e. a property of the long run equilibrium. However, one wants discern among the different possible underlying generative dynamics. The robustness of stable laws becomes then an issue, since a very large class of stochastic processes can have the same long-run behavior. As a consequence, accurate statistical tests are needed, to characterize the tails of marginal distributions and then to focus on joint distributions (see Chapter 4).

Our GPGM is based on the introduction/destruction of business units
and on their growth/decline. Three frequencies capture the creation of new firms (ν), innovation by existing firms (λ), and exit of business units (μ). Each new business unit is created with a size drawn from a probability distribution (Pξ) and its proportional growth is drawn in accord with (Pη).

As a matter of fact, all the parameters have a straightforward explanation, allowing a direct check of model consistency.

GPGM is defined as a stochastic framework. In particular, the Gibrat’s model can be seen as a geometric random walk, which is described by a linear dynamic equation and can reproduce both the exponential growth of a successful small firm, thanks to the geometrical effect, and the slow and painful death of another one, via the symmetric phenomenon known as Brownian trap. In the model, an urn scheme describes birth, innovation, and death of business units, while a geometric random walk reproduces the processes of growth a la Gibrat. In its simplest version, our framework assumes that all random variables used to describe the stochastic evolution are independent, while firms and their units are assumed, ex-ante, to be identical. As a first approximation, probability laws used to draw the initial conditions and the random evolution have no particular specification. Then, the model is extended in different directions.

We present all our results in discrete time. The perceptive reader will see that our mathematical proofs are generally based on limit arguments and expansions (see Section 6.2) and, in fact, a continuous time framework would be the natural setting for our stochastic benchmark. Without entering here a broader debate (Duffie and Protter, 1992; Lesne, 2007), we opted for a discrete time setting mainly for pedagogical reasons. First, fully continuous time models require a strong mathematical background. For instance, the extension of probability settings to the continuous time stochastic framework would require, at least, mastering Brownian motion, Wiener integrals and Ito calculus (Atkinson et al., 2005; Karatzas and Sherve, 1998; Stokey, 2008). Second, a continuous time setting would complicate the understanding of the nuts and bolts of the model, while readers might feel that the rationale of some effects is related more to some implicit properties of continuous time processes than to the framework itself. Our discrete time setting is then a choice of readability and transparency, at cost of elegance and mathematical beauty.
Empirical Laws Concerning Firm Growth

2.1 Background

In 1931, the French engineer Robert Gibrat proposed a simple model to explain the empirically observed size distribution of establishments (Gibrat, 1931). He made the following assumptions: (i) the number of producers is fixed; (ii) each firm faces the same distribution of growth rates, i.e. the growth rate $R$ of a company is independent of its size and other firm’s characteristics (this assumption is usually referred to by economists as the law of proportionate effect); (iii) the successive growth rates of a company are uncorrelated in time; and (iv) across firms, i.e. firms do not interact.

In mathematical form, Gibrat’s model is expressed by a simple stochastic process,

$$S_{t+\Delta t} = S_t \eta_{t+\Delta t},$$

(2.1)

in which $S_{t+\Delta t}$ and $S_t$ are, respectively, the size of the firms at times $(t+\Delta t)$, and $\eta_{t+\Delta t}$ is an uncorrelated positive random number with some distribution satisfying the hypothesis of central limit theorem. Hence $\log S_t$ follows a simple random walk, and for sufficiently large time intervals $u \gg \Delta t$, the growth rates,

$$R_u \equiv \frac{S_{t+u} - S_t}{S_t},$$

(2.2)

are log-normally distributed. If we assume that all companies are born at approximately the same time and have approximately the same initial size, (i.e. are differentiated only by the random sequence $\eta$), then the distribution of company sizes is also log-normal. Indeed, taking the logarithm of Eq. 2.1, we can express the distribution of $\log S_t$ as the convolution of $t$ distributions of $\log R_u$.

The law of proportionate effect (LPE) has become quite popular among


industrial economists for two reasons. First, the stochastic properties of the LPE are broadly consistent with the dynamic patterns of firm growth and decline observable in most countries and industrial sectors over time. Second and more importantly, the Gibrat’s Law postulates one of the simplest processes to generate a highly skewed distribution with a bulk of small- and medium-sized firms and few larger ones, which is approximately consistent with the observed size distribution of firms (Hart and Oulton, 1996; Prais, 1976; Stanley et al., 1995; Steindl, 1965). Even though the above derived lognormal distribution of firm sizes is not a steady-state solution, for its parameters going to infinity when time goes on (Sornette and Cont, 1997; Sutton, 1997), Kalecki (1945) examined a modified process, in which the expected rates of growth increase slightly less than proportionately, leading to a steady-state lognormal distribution with a constant variance. This approach is theoretically grounded in the presence of constraints to firm growth (such as finite assets, limited resources or bounded market sizes) that prevents firms from growing indefinitely. This argument has been used even recently to justify the tendency of firms to evolve toward an “optimal” firm size (the so-called mean reversion effect) (Rossi-Hansberg and Wright, 2007b). Alternatively, instead of adding frictions to impede the growth of too large firms, to ensure that the steady-state distribution exists, one can add some friction to prevent firms from becoming too small, i.e., a lower bound for sizes enforced by a reflecting barrier. Such a stochastic process, known as Kesten process, generates a Power Law distribution of firm sizes (Gabaix, 2009). Thus, we should keep in mind that there is no unique relation between a given size distribution and firm dynamics (Brock, 1999), that is to say multiple dynamic processes can lead to the same or very similar distributions and similar growth processes can generate different size distributions: A Gibrat’s growth process with frictions to prevent firms from becoming too large à la Kalecki generate a stable Log-normal distribution whereas adding to Gibrat some frictions to prevent firms from becoming too small à la Kesten generate a stable Power Law distribution.

On the theoretical ground, the Gibrat’s Law has been widely criticized. One of the main critiques is that the Gibrat’s Law is purely stochastic with no room for rational decision making by economic agents. To overcome this criticism, the random growth process has been replaced by one in which firms that differ in various attributes make different profit maximizing choices. However, the new firm growth models remain stochastic since the source of randomness is still present into a description of firms’ attributes and random

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1 Similar processes have been popularized in general science under different names such as preferential attachment, the rich get richer, and, less correctly, the Matthew effect.
outcomes (Sutton, 1997). Moreover, the proponents of a stochastic approach to the growth of firms maintain that similar properties are present in the dynamics of complex natural and artificial systems where rational decision-making does not play any role. On the one hand, a general and parsimonious approach is called for, to account for empirical regularities across different empirical settings. On the other hand, as stated by Sutton (1997) (pag. 42): “there is no obvious rationale for positing any general relationship between a firm’s size and its expected growth rate, nor is there any reason to expect the size distribution of firms to take any particular form for the general run of industries”. Independently of the benchmark model for the evolution of firm sizes, the main focus in economics is to rationalize departures from the benchmark, also for policy implications.

A second line of attack to the Gibrat’s Law has scrutinized the empirical validity of the assumptions and predictions of the model. First, some of the main assumptions of the Gibrat’s Law are clearly violated since: (a) the number of firms varies in time due to entry, exit, mergers, acquisitions and spin-offs; (b) firms do have some idiosyncratic features such as the quality of management, competences and resources which motivate the stability of differential performances; (c) firms do strategically interact with each other when they compete in the market and they are connected in production, financial and strategic collaboration networks. Moreover, despite the popularity of the Gibrat’s Law, after more than fifty years of research, there is now considerable empirical evidence that challenges its overall validity. The LPE assumes no relation between expected percentage growth and size, and no relation between variance of percentage growth rates and size. However, the growth rate does depend on firm sizes, or it is scale dependent. First, conditional on firm survival, average firm growth declines with firm size holding firm age constant, and declines with firm age holding size constant (Dunne et al., 1989; Evans, 1987a; Hall, 1987a; Sutton, 2007). Second, the variance of firm growth declines with firm size holding firm age constant (Dunne et al., 1989; Hymer and Pashigian, 1962; Mansfield, 1962; Stanley et al., 1996; Sutton, 2002) and declines with firm age holding firm size constant (Dunne et al., 1989). At a first glance, the negative relationship between growth fluctuations and size is not surprising because large and established firms are likely to be more diversified. Singh and Whittington (1975) state that the decline of the standard deviation with size is not as rapid as it would be if the firms consisted of independently operating subsidiary divisions (Hymer and Pashigian, 1962). The latter would imply that the relative standard deviation decays as $\sigma(S) \sim S^{-1/2}$. In other words the relative variance, is inversely proportional to the size. This confirms the
common-sense view that the performance of different parts of a firm are related to each other as stated by Mansfield (1962) (pag. 1034): “The growth rate of a large firm can be viewed as the mean of the growth rates of its smaller ‘components’ (e.g., plants). Note that, if the growth rates of the components (plants or otherwise) were independent, the standard deviation would be inversely proportional to the square-root of a firm’s size. But, since they tend to be located in the same region and have other similarities, one would expect the growth rate of such components to be positively correlated. Thus, the standard deviation would not be expected to decrease as rapidly with increases in size as the square-root formula suggests”. This tradition of considering a firm as a portfolio of almost independent units dates back to Simon (1955) and has been recently revitalized by considering firms as aggregation of products sold in almost independent submarkets (see for instance Sutton’s island model (Sutton, 1998) and the quality ladder model by Klette and Kortum (2002)).

The situation for the mean growth rate is less clear. Singh and Whittington (1975) consider the assets of firms and observe that the mean growth rate increases slightly with size. However, the work of Evans (1987) and Hall (1987a), using the number of employees to define the company’s size, suggests that the mean growth rate declines slightly with size. Dunne et al. (1989) emphasize the effect of the failure rate of firms and the effect of the ownership status (single- or multi-unit firms) on the relation between size and mean growth rate. They conclude that the mean growth rate is always negatively related with size for single-unit firms, but for multi-unit firms the growth rate increases modestly with size because the reduction in their failure rates overwhelms a reduction in the growth of nonfailing firms (Dunne et al., 1989).

Another testable implication of Gibrat’s law is that the growth rate of a firm is uncorrelated in time. However the empirical results in the literature are not conclusive. Singh and Whittington (1975) observe positive first-order correlations in the one-year growth rate of a company (persistence of growth) whereas Hall (1987a) finds no such correlations. The possibility of negative correlations (regression towards the mean) has also been suggested (Friedman, 1992; Leonard, 1986).

These patterns have been challenging to explain, suggesting that they should be revealing about the fundamental determinants of firm growth. On the theoretical ground, most of the recent attempts to capture the departures from the Law found by the empirical research use models where the basic Gibrat process has been heavily modified (Fu et al., 2005; Sutton, 2007). On the empirical ground, even though the Gibrat’s Law has been rejected in its
general formulation, it is still defended as a long-run regularity from large
and mature firms (Lotti et al., 2009).

In this chapter we describe the main empirical regularities, or “stylized
facts”, about the size and growth of firms. The next chapter presents a
theoretical framework able to account for regularly observed patterns of
firm dynamics.

2.2 Size Distribution of Firms

To investigate the distribution of company sizes we must first define and
measure firm size. If all companies produce the same product (e.g. steel),
we can use a physical measure of output (e.g. tons). However, when firms
producing different goods for which there is no common physical measure
of output is available, one solution is to use the dollar value of output, i.e.
firms’ total sales. An alternative to measuring the size of outputs would be
to measure inputs. Again, when firms produce different goods and services,
they use different production factors. Because virtually all companies have
employees, some economists have used the number of employees as a measure
of firm size. Other possibilities involve the dollar value of specific inputs, such
as the “cost of goods sold”, “property, plant, and equipment” or “assets”. Though
we might in principle expect systematic differences between several
alternative measures, our analysis in the following sections shows similar
results for all of them. Therefore in this book we will focus on sales as a
measure of firm sizes, as prevalent in the literature.\footnote{\textsuperscript{2}}

Since the LPE implies a multiplicative process for the growth of companies, it is natural to study the logarithm of sales. We thus define

\[ s_t \equiv \ln S_t \]  

(2.3)

and the corresponding growth rate

\[ r \equiv \ln R = \ln \frac{S_{t+1}}{S_t}, \]  

(2.4)

that corresponds to Eq. 2.2 when \( r \) is the one year growth rate of firms
between year \( t \) and \( t + 1 \).\footnote{\textsuperscript{3}} The firm data are taken from the Compustat files
for U.S. publicly traded firms from 1950 to 2010. These data cover approxi-
mately one half of the employment in the US manufacturing sector, although

\footnote{\textsuperscript{2} To make the value of sales in different years comparable, we adjust all values to 1950 dollars
using the GNP price deflator.}

\footnote{\textsuperscript{3} Since by convention firm growth rates are measured on yearly bases, we drop the subscripts
numbering the length of the time span between two consecutive size measures whenever we refer to yearly growth rates.}
Empirical Laws Concerning Firm Growth

Figure 2.1 Number of publicly-traded manufacturing companies in the US for the period 1950–2010 (figure (a)) and numbers of companies entering and exiting the market (figure (b)).

they account for less than one percent of the firms in this sector. Thus, as argued by Axtell (2001), Compustat does not provide a representative sample of U.S. firms across all sizes and should not be used to determine the shape of the size distribution of firms. To overcome the limits of Compustat data, we also analyze two data sets covering firms of all sizes in a specific country (i.e. France) and sector (i.e. the worldwide pharmaceutical industry). Our goal in analyzing Compustat data is not to ascertain the exact shape of the size distribution but to highlight some preliminary findings about the
relationship of growth and size across firms that have already reached a certain minimum size and importance to be publicly traded in the U.S. stock market.

Figure 2.1 shows the total number of firms present in the database each year. We also plot the number of new companies and of “dying” companies (i.e., companies that leave the database because of merger, change of name or bankruptcy). Clearly, the population of firms is not fixed but it undergoes a significant market turnover. In particular, a significant market “shakeout” (Klepper, 1996) happened at the turn of the millennium, when the number of firms first grew to a peak, and later fell to a lower level.

Figure 2.2(a) shows the distribution of firm size every four years from 1950–2010.

The distribution is approximately stable over the sixty years of observation and especially for the last twenty years, despite the sizable increase in market turnover. This is surprising because there is no theoretical reason to expect that the size distribution of firms would remain stable at a value slightly smaller than the average of all companies, although one might expect so if the economy is growing, the composition of output changing, and factors that economists expect to affect firm size (such as technological progress) evolving. Conversely, a remarkable feature of most industries and markets is that at any given moment there is great variation in the size of producers: the bulk of the firms are concentrated at smaller sizes, but there are also a number of larger firms whose size distribution is skewed or heavy-tailed.

This fat-tailed stable distribution is also important because it contradicts the predictions of the Gibrat model. As already noticed, Eq. 2.1 implies that the distribution of sizes of companies should get broader over time with a variance of the distribution increasing linearly in time. We thus must conclude that other factors not included among Gibrat’s assumptions have important roles, such as, for instance, firms’ entry, exit and some frictions preventing firms to become too large or too small.

At a closer inspection, we can detect another departure from the Gibrat’s assumptions, that is the size distribution is not lognormal. As shown in fig. 2.2 (a) the body of the distribution could be approximate by a lognormal distribution while the tails have different behaviors. This fact could be a consequence of the selection bias of the Compustat data. In fact, since in Compustat only the information about publicly-trade company are collected, the sample is not representative of the whole economy (Axtell, 2001; Cabral and Mata, 2003).

To avoid the selection bias of the Compustat data, we exploit a comprehensive dataset covering the universe of French firms similar to the one
Figure 2.2 (a) Probability density of the logarithm of the sales for publicly-traded manufacturing companies (with standard industrial classification index of 2000-3999) in the US for each of the years in the 1950–2010 period. All the values for sales were adjusted to 1950 dollars by the GNP price deflator. Also shown (solid circles) is the average over the 60 years and lognormal fitting (black line). It is visually apparent that the distribution is approximately stable over the period. (b) Normalized probability density of the logarithm of sales for all the manufacturing companies, for the companies entering the market, and for the companies leaving the market, averaged over the 1950–2010 period. (c) Plot of the fraction of “dying” companies by size. We define this probability as the ratio of dying companies of a given size over the total number of companies of that size.
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originally analyzed by Gibrat. The data on total revenues that we use to measure firm size are taken from the FICUS (Fichier complet de Système Unifié de Statistique d’Entreprises) database maintained by the French National Statistical Office (INSEE). We focus on the year 2003 (although the choice of year is actually irrelevant in terms of the results), and have information on more than two million firms, excluding the very few cases in which a firm reports total revenues equal to zero.

Figure 2.3 displays the histogram of the logarithms of the data along with the truncated normal (logarithm of the truncated lognormal), the exponential (logarithm of Pareto) and the best fitting Maximum Entropy (ME) distribution. The optimal ME distribution has $k = 4$; hence, neither the Pareto ($k = 1$) nor the lognormal ($k = 2$) distributions are good approximations, although it is clear from the graph that the lognormal one is a better approximation to the true distribution than the Pareto distribution. Noticeably, the size distribution of firms displays positive skewness. This feature is consistent with the evidence gathered by Cabral and Mata (2003) who found that skewness declines and the variance increases as the cohort of firms ages.

Figure 2.4 shows the size distribution for all pharmaceutical companies worldwide. As rationalized by Cabral and Mata (2003), the size distribution is approximatively lognormal for long-lived companies. Conversely, the size distribution of young companies is skewed to the left with a smaller mean and variance. We also find evidence of the “shadow of death”: firms become smaller in the years prior to exit (Griliches and Regev, 1995).

One factor not taken into account by Gibrat is the appearance of new firms. Fig. 2.2(b) shows that the size distribution of new publicly-traded firms is very similar to the distribution of existing firms. However, as argued by Hall (1987a), new firms in Compustat means new publicly traded companies. Thus they must be large enough to require outside capitalization. When we look at all firms in the pharmaceutical sector (see Fig. 2.4) new firms are born smaller than established companies.

Another factor not included in Gibrat’s assumptions is the “death” of firms. As shown in Fig. 2.2(b), this distribution has the left tail heavier than the distribution for all firms. It thus suggests that the probability that a company will leave the market seems to be negatively dependent on its size. [Fig. 2.2(c)]. There are two possible interpretations of this negative relationship: (a) when hit by similar negative shocks, large firms have a

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4 We use observations larger than 14 000 euros, as below this threshold the distribution is very irregular. Only the smallest 3.7% of the observations are discarded in this way. The Maximum Entropy distribution is described in Appendix 1.
higher chance of survival than smaller companies; (b) firms decline in size before exit (the so called “shadow of death” effect). The second mechanism of exit seems to be at work for pharmaceutical data, as shown in Figure 5.14 in Chapter 5.\(^5\)

### 2.3 The Distribution of Firm Growth Rates

Firm dynamics and the distribution of firm sizes are clearly related. For sure, the size distribution of firms depends on the evolution of firm sizes in time. On the other hand, the Gibrat’s Law postulates that the growth-of-firm does not depend on size or, to put it differently, firm growth is scale independent. However, since the 1960s, the empirical literature has been piling up sound evidence on the dependency of firm growth rates on the size and age of firms, conditional on firm survival. The growing availability

\(^5\) There are other mechanisms for firm’s entry and exit apart from birth and death dynamics. Firms can be created/dissolved through the merger of two existing firms, however. New firms are also created when very large firms divest themselves of divisions that are, by themselves, large firms. An example is AT&T’s divestiture of its manufacturing division (Lucent) and its computer division (NCR).
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Figure 2.4 The distribution of pharmaceutical firm sizes. The size distribution of all firms includes all pharmaceutical firms active in years 1998-2008. Stable firms are long-lived firms which have been active for at least 10 years. The distribution of stable firms approaches a log-normal shape (a parabola in double-log scale). New firms are new-born companies in their first year of life. The size distribution of new firms is shifted to the left with smaller mean and variance and larger skewness as compared to long-lived firms. Finally, old firms are companies in the year preceding their exit. Just before exit companies are considerably smaller than long-lived firms.

in recent years of detailed information about the evolution of the size of all firms in specific countries and industries has enabled a more complete understanding of the empirics of firm growth. It is now clear that among surviving firms, both the mean and variance of firm growth decline with firm size and also with firm age, even after controlling for other factors influencing firm survival and performances (Klepper and Thompson, 2006; Sutton, 2007). Figure 2.5 shows the relationship between firm size ($S$) and firm growth ($r$) for US manufacturing firms. According to Gibrat, firm growth rates should be evenly distributed above and below the horizontal axes (zero growth rate) with the same width along all sizes. This is clearly not the case. First, Figure 2.5 suggests that the variance of growth rates is size dependent. In statistics, the circumstance in which the variability of a variable is unequal across the range of values of a second variable that predicts it is known as heteroskedasticity. More precisely, the variance of

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6 See also Hall (1987b) for a similar plot (Figure 1) and an in-depth analysis of the relationship between firm size and firm growth.
growth rates declines with size. Second, we notice a prevalence of positive growth observations for small firms which vanishes for medium and large firms. This suggests that also the average growth rate could be negatively dependent on size. Third, Figure 2.5 shows growth performance conditional upon survival, that is to say we need firms to be alive in two consecutive years to measure the growth rate, \( r \). However, Figure 2.2(c) shows that the exit probability negatively depends on size, i.e. small firms are more likely to exit from the sample. Should small firms be more likely to exit when they experience negative growth shocks, this will explain the prevalence of positive growth rates for small firms in Figure 2.5. This issue, know as sample censoring, combined with the heteroskedasticity, has been a major focus of empirical research since the 1980s to understand if the failure of the Gibrat’s Law can depend on any of these effects.

Figure 2.5 Yearly firm growth rate minus the average firm growth rate of about 0.11 \( (r - \langle r \rangle) \) versus log-size \( (log(S)) \), US manufacturing firms, years 2009-2010; USD, thousands. Source: Compustat.

Another prediction of Gibrat is that the logarithmic growth rate of firms
2.3 The Distribution of Firm Growth Rates

The distribution of firm growth rates should be normally distributed. The seminal contribution of Stanley and co-workers (Stanley et al., 1996) brought to the attention of the scientific community the fact that the growth rate distribution is not Gaussian but “tent-shaped”. Figure 2.3 shows the distribution \( p(r|s) \) of the growth rates of Compustat firms from 1950 to 2010 for small, medium and large firms. The definition of firm size is based upon the number of employees, a small firm having fewer than 100 employees, a medium firm having between 100 and 500 employees, and a large firm having more than 500 employees. Note that all distributions in Fig. 2.3 are far from being Gaussian, as would be expected in the Gibrat approach (Gibrat, 1931). The growth rate distributions have sharper peak around the mean and heavier tails than a normal distribution. Therefore, extreme (positive and negative) events of growth are much more likely than predicted by the Gibrat’s Law, especially for small firms. In a single word, the growth distribution is leptokurtic, i.e. it has excess positive kurtosis as compared to the normal. A simple leptokurtic distribution which has been proposed in the literature to fit the data well is the Laplace distribution (Bottazzi and Secchi, 2006; Stanley et al., 1996):

\[
p(r|s) = \frac{1}{\sqrt{2\sigma(s)}} \exp\left(-\frac{\sqrt{2}|r - \bar{r}(s)|}{\sigma(s)}\right). \tag{2.5}
\]

The straight lines shown in Fig. 2.3(b) are calculated from the average growth rate \( \bar{r}(s) \) and the standard deviation \( \sigma(s) \) obtained by fitting the data by the Laplace distribution. The tails of the distribution in Fig. 2.3 are somewhat fatter than Eq. 2.5 predicts. This deviation is the opposite of what one would find if the distribution were Gaussian. Thus the body of the distribution for large companies can be described by a tent shape distribution while the growth rate distribution for the small companies shows fatter tails. In the next chapter we will introduce a theoretical model to take into account the shape of the empirical growth rate distribution. In Chapter 4 we will further investigate the exact shape of the growth distribution.

Mean growth rate

Figure 2.7 displays the average growth rate \( \bar{r} \) as a function of initial size \( s \) for several years. Although the data are quite noisy, they suggest that there is a negative dependence of the mean growth rate on size \( s \). The figure also suggests that the results do not change when we consider alternative measures of firm size. In the next chapter we will present an explanation,
Figure 2.6  (a) Probability density $p(r|s)$ of the growth rate $r \equiv \ln(S_{t+1}/S_t)$ for all publicly-traded US manufacturing firms in the Compustat database with Standard Industrial Classification index of 2000–3999. The distribution represents all annual growth rates observed in the 60-year period 1950–2010. We show the data for three different groups of firms: small, medium and large firms. (b) The solid lines are Laplace fits to the empirical data close to the peak. We can see that the wings are somewhat “fatter” than what is predicted by Eq. 2.5.

Based on a theoretical model, for the negative relationship between size and mean growth rate. In Chapter 4 we will perform a regression analysis to pin down the determinants of scale dependence.
2.3 The Distribution of Firm Growth Rates

Figure 2.7 Average growth rate for the 60 years of \( \bar{r} \) for different measures of firm size: sales, assets, cost of goods sold and, employees; plant property and equipment.

**Standard deviation of the growth rate**

We next study the dependence of the standard deviation of growth \( \sigma(r) \) on size \( (s) \). Figures 2.3(a) and 2.3 (b) clearly show that the width of the distribution of growth rates decreases with increasing \( s \). The relationship between \( \sigma(r) \) and firm size \( (s) \) can be approximated by the scaling law (Stanley et al., 1996; Sutton, 2002):

\[
\sigma(r) \sim S^{-\beta}. \tag{2.6}
\]

that in double log scale implies a linear dependency with slope \( \beta \):

\[
\log(\sigma(r)) \sim -\beta s, \tag{2.7}
\]

where, for sales, \( \beta = 0.17 \pm 0.07 \) (see Figure 2.8). Figure 2.8 displays \( \sigma(r) \) vs. \( s \), and we can see that Eq. (2.6) is indeed verified by the data.

**Other Measures of Size**

In order to test further the robustness of our findings, we perform a parallel analysis for additional indicators of firms size: (i) assets; (ii) cost of goods sold; (iii) property, plant, and equipment; (iv) number of employees. We find that the analogs of \( p(r|s) \) and \( \sigma(r) \) behave similarly. For example, Fig. 2.8 shows the standard deviation of different measures of size: sales, assets, cost
Empirical Laws Concerning Firm Growth

Figure 2.8 Standard deviation of the 1-year growth rates for different definitions of firm size as a function of the initial size. Least squares power law fits were made for all quantities leading to the estimates of $\beta$: 0.18 $\pm$ 0.06 for “assets,” 0.17 $\pm$ 0.07 for “sales,” 0.17 $\pm$ 0.03 for “number of employees,” 0.16 $\pm$ 0.06 for “cost of goods sold,” and 0.17 $\pm$ 0.03 for “plant, property & equipment.” The straight lines are guides for the eye and have slopes 0.17.

of goods sold (C.O.G.S.), property, plant, and equipment (P.P.E) and of the number of employees. Considering for instance the sales and the number of employees, we see that the number of employees data are linear over roughly five orders of magnitude, from firms with less than 10 employees to firms with almost $10^5$ employees. The slope $\beta = 0.16 \pm 0.06$ is the same, within the error bars, as found for the sales.

Figure 2.8 shows that Eqs. (2.5) and (2.6) provide an approximate description of three additional indicators of firm size: (i) assets, (ii) C.O.G.S., and (iii) P.P.E.

As a robustness check, we divide firms by sectors: agriculture, manufacturing and services. It is visually apparent in Fig. 2.9 that the same behavior holds for the different sectors of the economy.

The evidence about firm growth and size is remarkable as it highlights a set of empirical regularities (or stylized facts) that hold for diverse firms that range in size and industrial sectors. The conventional economic theory of the firm is based on production technology, which varies from product to product. Conventional theory does not suggest that the processes governing the growth rate of car companies should be the same as those governing, e.g., pharmaceutical or paper firms.
2.3 The Distribution of Firm Growth Rates

Our results support the possibility that the scaling laws used to describe complex but inanimate systems comprised of many interacting particles (such as it occurs in many physical systems) may be extended to describe complex economic systems comprised of many interacting subsystems. The empirical regularities about firm growth and size ask for a general and parsimonious theory about the self-organization of the economy (Krugman, 1996). In the next Chapter we take on the challenge to develop a fully fledged theoretical model to account for the regularities which have been repeatedly encountered in multiple and different empirical settings.
References

References


