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Econophysics

Topic: Understanding Options and other Derivatives
- The solution of the Black-Scholes for options
- How options behave

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Derivatives market

![OTC Derivative Growth](image-url)
Notional Outstanding, As of December 2011, US$ in trillions

Interest rates
$504
78%

FX
$63
10%

CDS
$29
4%

Equity
$6
1%

Commodities
$3
0%

Unallocated
$43
7%

Notional amount outstanding:
$648 trillion

Source: BIS
Why would you use derivatives?

• Example 1: You are an owner of an aluminum smelter in Europe.

• Aluminum is made from:
  • Electricity
  • Bauxite ore
  • Work(ers) (😊)
  • Other (caustic soda etc.)

• Aluminum is sold on:
  • London Metal Exchange
  • Price is in: dollars (exchange rate risk)
Black–Scholes–(Merton) equation

\[ \frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf \]

Solution for a call option:

\[ C(S, t) = N(d_1)S - N(d_2)Ke^{-r(T-t)} \]

where

\[ d_1 = \frac{1}{\sigma \sqrt{T-t}} \left[ \ln \left( \frac{S}{K} \right) + \left( r + \frac{\sigma^2}{2} \right)(T-t) \right] \]

\[ d_2 = \frac{1}{\sigma \sqrt{T-t}} \left[ \ln \left( \frac{S}{K} \right) + \left( r - \frac{\sigma^2}{2} \right)(T-t) \right] \]

\[ = d_1 - \sigma \sqrt{T-t} \]
Changing $S_0$, keeping else constant.

Figure 9.1 Effect of changes in stock price, strike price, and expiration date on option prices when $S_0 = 50$, $K = 50$, $r = 5\%$, $\sigma = 30\%$, and $T = 1$.

Book: John C. Hull.
Changing $K$, keeping else constant.

Figure 9.1  Effect of changes in stock price, strike price, and expiration date on option prices when $S_0 = 50$, $K = 50$, $r = 5\%$, $\sigma = 30\%$, and $T = 1$.
Changing $T$, keeping else constant

**Figure 9.1**  Effect of changes in stock price, strike price, and expiration date on option prices when $S_0 = 50$, $K = 50$, $r = 5\%$, $\sigma = 30\%$, and $T = 1$.

Book: John C. Hull.
Changing **volatility**, keeping else constant

**Figure 9.2** Effect of changes in volatility and risk-free interest rate on option prices when $S_0 = 50$, $K = 50$, $r = 5\%$, $\sigma = 30\%$, and $T = 1$. 

Book: John C. Hull.
Changing $r$, keeping else constant

Figure 9.2 Effect of changes in volatility and risk-free interest rate on option prices when $S_0 = 50$, $K = 50$, $r = 5\%$, $\sigma = 30\%$, and $T = 1$. 

Book: John C. Hull.
Option price VS volatility
Example of a derivative

- Does it make sense to buy both a call and a put?

Its price is positive! This derivative is called STRADDLE.
Long Straddle Initially with 1 year to expiry

Profit vs. Asset Price graph for different time periods:
- Initiation
- 75% to Expiry
- 50% to Expiry
- 25% to Expiry
- Expiry
Round 10

• Choose a secret integer $X$ between 0 and 15.

Prize = $-20 + 8 \cdot |\text{distance to your closest neighbor}|$

• You can choose to **fold** (not play this game).
  • In that case, write “FOLD” on your paper.
  • This is the only round in which you can **freely communicate**.