A Quantum-Mechanics Framework of Dynamic Economics

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Motivation & Outline

Motivation

Dynamic Economics [1]
Application of Quantum Mechanics in Finance [2]

<table>
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<tr>
<th>Business Cycles</th>
<th>Time Series</th>
<th>Path Integral and Futures</th>
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<td>Option Pricing</td>
<td>Probability Theory</td>
<td>Quantum Field Theory and Interest Rates</td>
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<td>Dynamic Systems</td>
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<td>Hamiltonian Systems</td>
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Motivation & Outline

Outline

Framework

Summary

- Dynamic Economics
  1. Equilibrium Price
  2. Damped Harmonic Oscillator
- Irrationality
  1. Quantum Harmonic Oscillator Model
  2. Quantum Brownian Motion Model (qBm(m))
- Data Analysis
  1. “Fat Tail” and Non-Markovianity
  2. Kurtosis and Autocorrelation of qBm(m)
  3. Additional Less-Colored Noise
The equilibrium price is determined by the demand and supply [1], but how? (what about the timescales?)

Dynamic problems in economics can be sorted into two classes: propagation and impulsion [2].

1) How does external information propagate in a given financial structure? (theory of dynamic systems)

2) What is the statistical behavior of impulsion? (theory of stochastic process)

Damped Harmonic Oscillator [1]

The propagation problem can be modeled by a damped harmonic oscillator (overdamped or underdamped). New equilibriums can be reached within the mechanism.

External (Noise) Reservoir [1]

External stochastic impulsion produces random patterns, which are similar to Brownian noise.

The classical model cannot explain why there exists persistent fluctuation of price [1,2].

The external (noise) reservoir would be too large if the persistent fluctuation was all produced by stochastic information [1].


Irrationality
Quantum Harmonic Oscillator Model

Quantum Harmonic Oscillator Model [1]

A microscopic explanation of **persistent fluctuation**:

If transactions are completely rational, the price should be determined with **certainty**; however, the irrationality of transaction will introduce additional fluctuations of the price and thus will lead to finite small but persistent **uncertainty** [2].

<table>
<thead>
<tr>
<th>Ground State (Gaussian wave packet)</th>
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<tr>
<td>(</td>
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<tr>
<td>( \sigma x \propto \hbar/2 m\omega, )</td>
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<tr>
<td>( E = \hbar \omega/2. )</td>
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<table>
<thead>
<tr>
<th>Quantum Closed System</th>
<th>Single Stock</th>
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<tbody>
<tr>
<td>Coordinate representation ( X\downarrow i )</td>
<td>(logarithmic) Stock price ( \ln s\downarrow i )</td>
</tr>
<tr>
<td>Momentum representation ( P\downarrow i )</td>
<td>Trend of stock price ( m\downarrow i d(\ln s\downarrow i)/dt )</td>
</tr>
<tr>
<td>Mass ( m\downarrow i )</td>
<td>Inertia of stock ( i )</td>
</tr>
<tr>
<td>Energy ( E\downarrow i )</td>
<td>Trading volume of stock ( i )</td>
</tr>
<tr>
<td>Wave function (amplitude) (</td>
<td>\psi_i(x,t)\rangle \propto )</td>
</tr>
<tr>
<td>Uncertainty relation ([X,P]=i\hbar)</td>
<td>Uncertainty of irrational transaction</td>
</tr>
</tbody>
</table>

[1] C. Ye and J. P. Huang, Ph X. Meng, J.-W. Zhang, H.
In physics, an **open quantum system** is a quantum-mechanical system which interacts with an external quantum system (with a large number of degrees of freedom), the **environment**.

In reality, every quantum system is open to some extent, causing **dissipation and fluctuation** in the quantum system.

For example: **spontaneous emission** in quantum optics (dissipation of photons).

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**Irrationality**

Quantum Brownian Motion Model (qBm(m))

**Quantum Open System [1]**

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Irrationality
Quantum Brownian Motion Model (qBm(m))

Quantum Brownian Motion Model

The Hamiltonian of qBm:
\[
H = H_{\downarrow A} + H_{\downarrow E} + H_{\downarrow I} = 1/2M P \uparrow 2 + V(X) + \Sigma i\uparrow \sum (1/2m\downarrow i \ p\downarrow i\uparrow 2 + 1/2m\downarrow i \omega\downarrow i\uparrow 2 x\downarrow i\uparrow 2) -X\Sigma i\uparrow \kappa\downarrow i x\downarrow i.
\]

The Caldeira-Legget master equation [1]:
\[
\frac{d}{dt} \rho_{\downarrow A}(t) = -i/\hbar [H_{\downarrow A}, \rho_{\downarrow A}(t)] - i\gamma/\hbar [X,\{P,\rho_{\downarrow A}(t)\}] - 2M\gamma kT/\hbar \uparrow 2 [X, [X, \rho_{\downarrow A}(t)]].
\]

Mapping between quantum open system and stock index [2]

<table>
<thead>
<tr>
<th><strong>Quantum Open System</strong></th>
<th><strong>Stock Index</strong></th>
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<tr>
<td>Density operator (population) ( \rho_{\downarrow A}(x,x,t) )</td>
<td>Probability density distribution of stock index</td>
</tr>
<tr>
<td>Potential well ( V(x) )</td>
<td>Macroscopic external influence on stock index</td>
</tr>
<tr>
<td>Thermal reservoir ( \rho_{\downarrow E} )</td>
<td>Large numbers of stocks</td>
</tr>
<tr>
<td>Temperature ( kT )</td>
<td>Strength of fluctuation</td>
</tr>
<tr>
<td>Dissipation coefficient ( \gamma ) of thermal reservoir</td>
<td>Strength of dissipation</td>
</tr>
<tr>
<td>Spectral density ( J(\omega) ) of thermal reservoir</td>
<td>Autocorrelation features</td>
</tr>
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Data Analysis
“Fat Tail” and Non-Markovianity

a) Non-Gaussian distribution of log return $\Delta \ln S(\tau) \rightarrow$ the kurtosis is larger than 0.

b) Non-zero autocorrelation (non-Markovianity) $R(\tau)$, not a Brownian motion
$\rightarrow$ business cycles exist.
The kurtosis $\kappa_{x^4}(t)$ of qBm has an exponential decrease as the actual kurtosis of the stock index (Shanghai Composite Index) does.
Fitting autocorrelation function: 
\[ R(\tau) = (\xi e^{\tau - \nu \tau/2} \cos \omega \tau)^2 = \frac{1}{2} \xi^2 e^{\tau - \nu (\cos 2\omega \tau + 1)}, \]

Non-Markovian master equation of qBm:
\[
d/dt \rho \downarrow A(t) = -i/\hbar [H \downarrow A, \rho \downarrow A(t)] + \mathcal{K}(t) \rho \downarrow A(t),
\]
\[
\mathcal{K}(t) \rho \downarrow A(t) = -i\gamma/\hbar [X, \{P, \rho \downarrow A(t)\}] - \Delta(t)[X,[X,\rho \downarrow A(t)]] + \Lambda(t)[X,[P,\rho \downarrow A(t)]].
\]
The propagating noise is less-colored than Brownian noise.

There exists additional less-colored intrinsic noise (simplest example: additional white noise).

\[ F(L) = \left( \frac{1}{L} \sum_{j=1}^{L} (Y_j - ja - b)^2 \right)^{\frac{1}{2}} \]

\[ F(L) \propto L^{\alpha} \]

\( \alpha = 1/2 \): white noise;
\( \alpha = 1 \): pink noise;
\( \alpha = 3/2 \): Brownian noise;

Take-home messages

- Irrationality: additional persistent fluctuations from quantum uncertainty relation.

- Quantum open system dynamics: fat-tail phenomena and non-Markovian behaviors.

- Detrended fluctuation analysis: less-colored intrinsic noise in propagation.
Special thanks to...

- H. E. Stanley, Boston University
- J.-W. Zhang, Peking University
- H. Guo, Peking University
- B. E. Baaquie, National University of Singapore
- J. P. Huang, Fudan University
- P. Chen, Peking University

... and many others