Quantum Percolation Theory: Wave Localization on Topological Structures

Xiangyi Meng

Department of Physics, Boston University

Outline

Outline

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- ☐ Infinite Cluster

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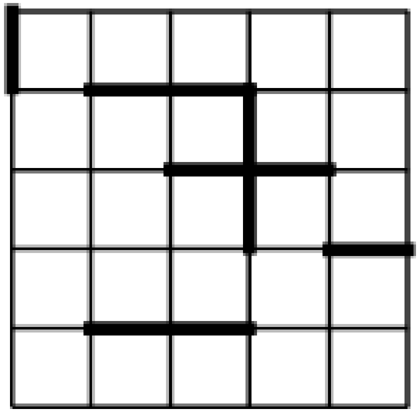
Introduction

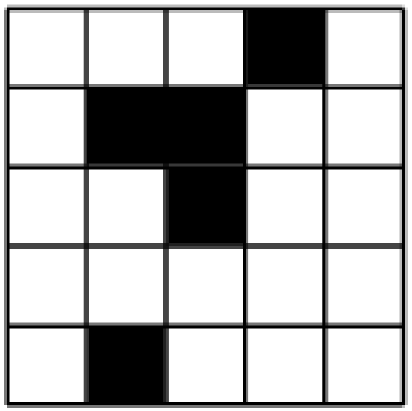
Percolation Theory

Coffee is poured on top of some porous material. Will the liquid be able to make its way from hole to hole and reach the bottom?

A site (bond) is occupied with probability p or empty with probability $1-p$.



bond percolation



site percolation

Introduction

Infinite Cluster

Thermodynamic Limit: Size -> ∞

Does an **infinite open** cluster exist? "Connectivity"

There is a critical probability $\boldsymbol{p_c}$, below which such an infinite cluster does **not** exist (with unitary probability) while above which such a cluster most surely exists.

Order parameter: $P_\infty(p)/\sum_s P_s(p)$.

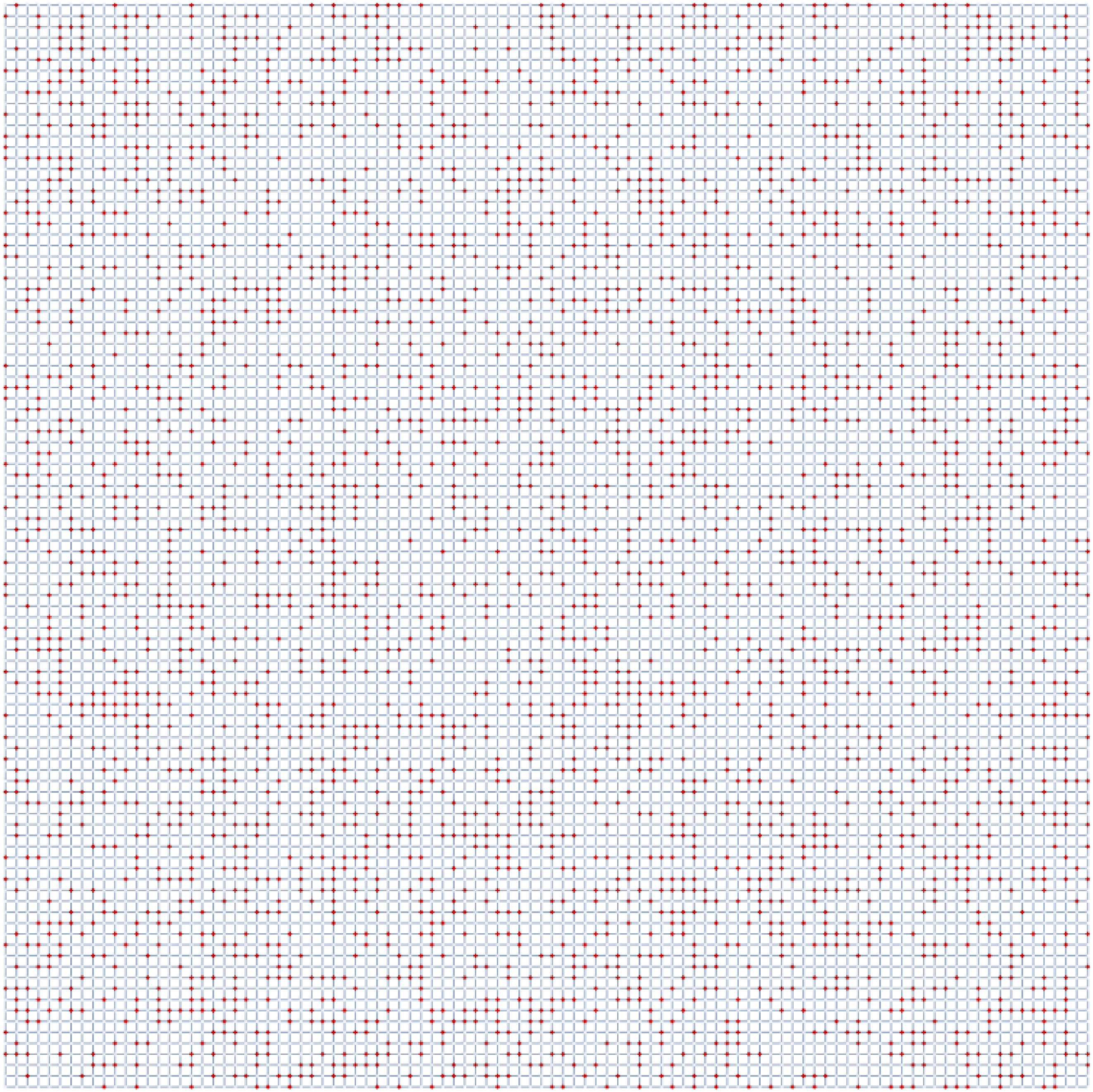
The fraction of occupied sites that belong to the infinite cluster; goes from 0 to 1 as p increases.

Introduction

Infinite Cluster

Size -> 100×100 sites

$p = 0.3$

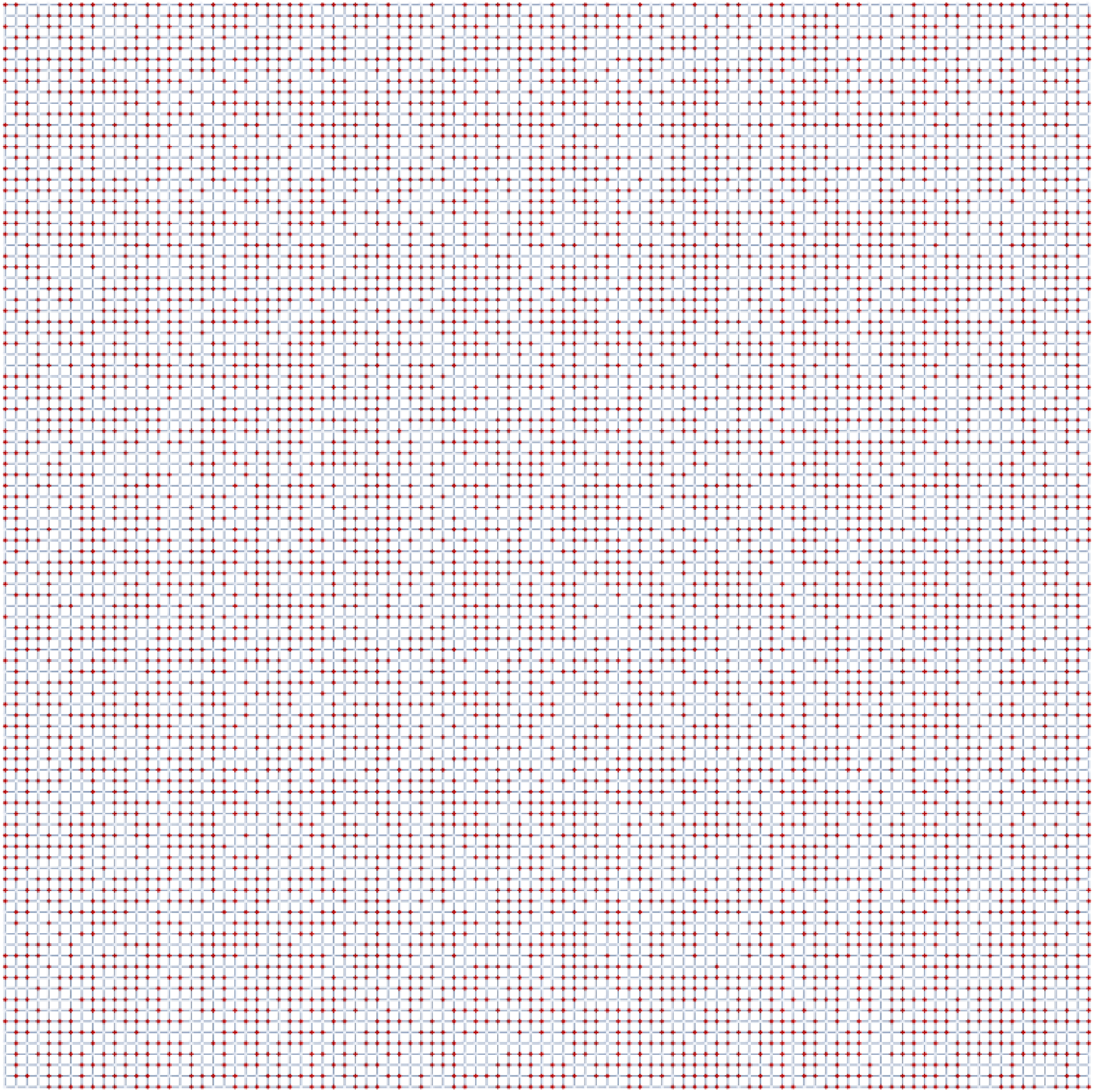


Introduction

Infinite Cluster

Size -> 100×100 sites

$p = 0.6$

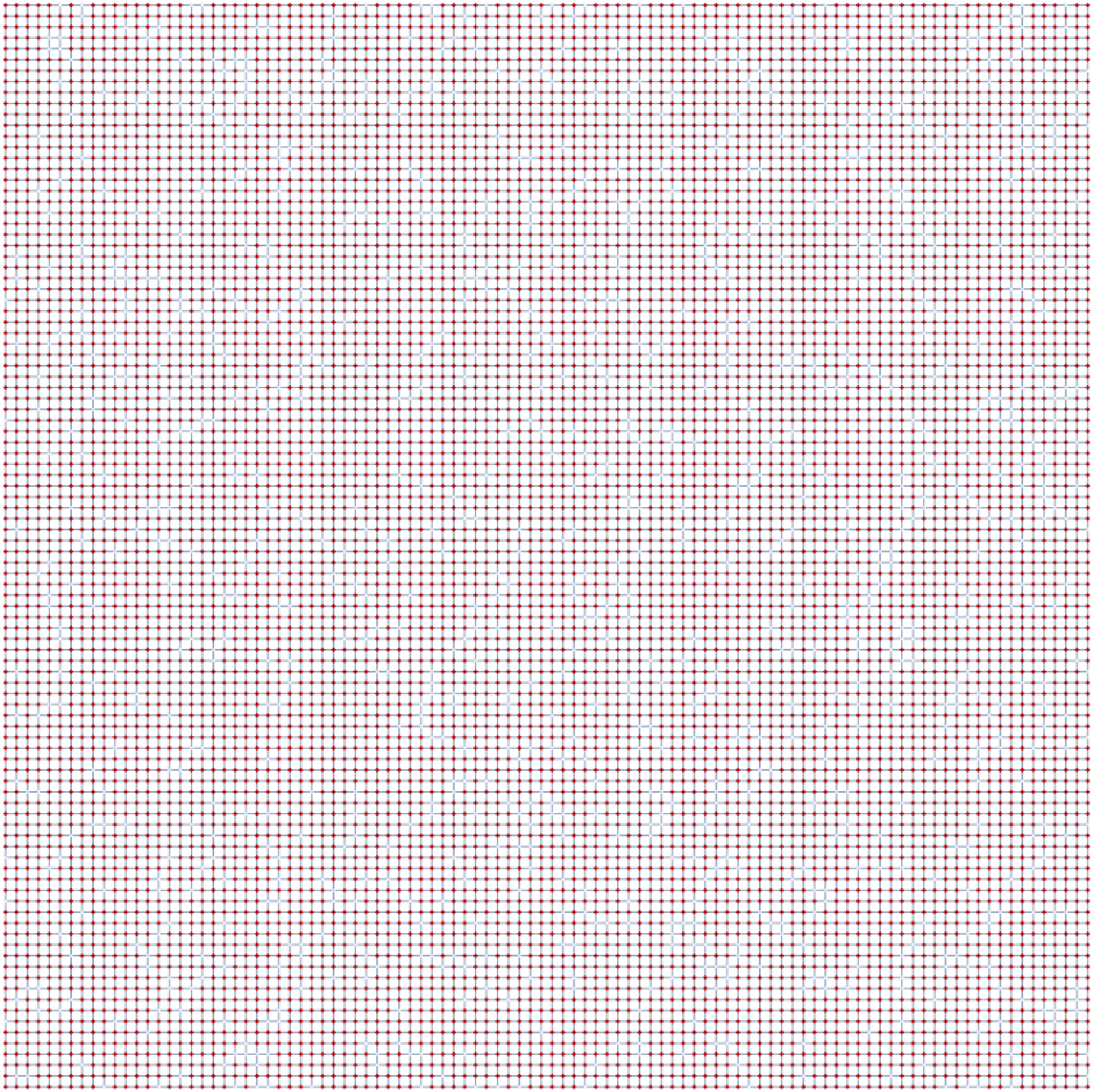


Introduction

Infinite Cluster

Size -> 100×100 sites

$p = 0.9$



Introduction

Quantum Percolation

Quantum Percolation Theory

Main difference: Each site j is not only "0" or "1", but has a complex amplitude a_j!

Hamiltonian (tight-band model)

Two different sites A and B with energies E_A and E_B are placed randomly on the grid with fractions p and $1 - p$, respectively.

Interaction between nearest neighbors is V.

If E_B is much larger than E_A, then eigen-functions with non-zero amplitude on B sites should have much larger eigen-energies which belong to a higher energy band (B subband); while we are only interested in the A subband.

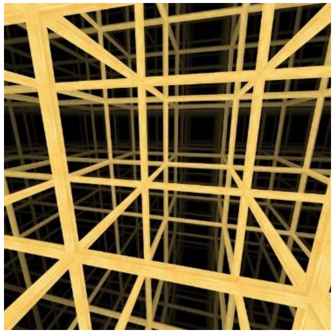
Set $E_A = 0$ and $V = 1$, the Hamiltonian can be written as

$H = \sum_{j,k} |j >< k|$, j, k are nearest neighbors only on A sites.

Now we see H is the adjacent matrix of the grid (network)!

Introduction

Quantum Percolation



Classical Percolation Theory

A "static" system with a_j constants.

Statistical Physics

Epidemiology

Material Science

Quantum Percolation Theory

$i\,\dot{a}_j = \sum_k a_k$, j, k are nearest neighbors (on A sites); determine how wave (information) transmits.

A complex feedback dynamic system.

Optical Fibers

Optical Matrix Operations

Quantum Information

Automation Control

Introduction

Quantum Percolation

Anderson Localization

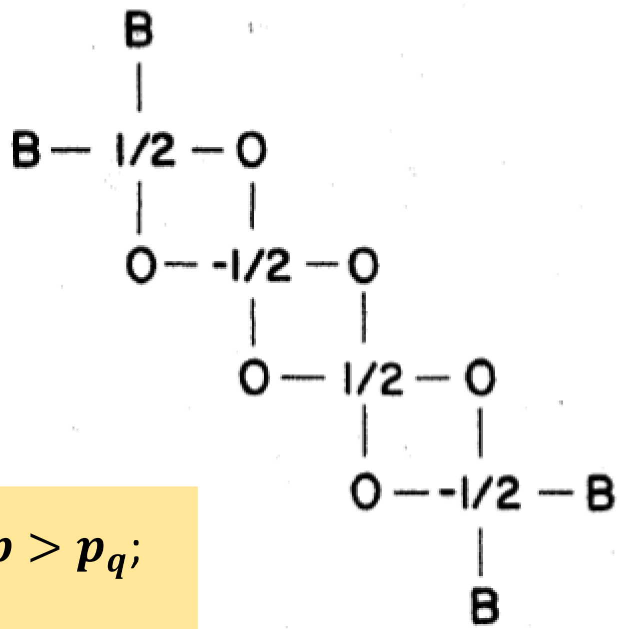
Given $i\dot{a}_j = \sum_k a_k$. What will happen to $a_0(t)$ if we put $a_0(0) = 1$ at $t = 0$? Can the amplitude spread infinitely long?

Essential condition**: there exists an infinite open cluster!**

But that's **not enough**…

There must exist **an eigenstate which is not localized!**

Localized states exist even in the infinite cluster.



Quantum phase transition: extended states appear when $\boldsymbol{p} > \boldsymbol{p_q}$;

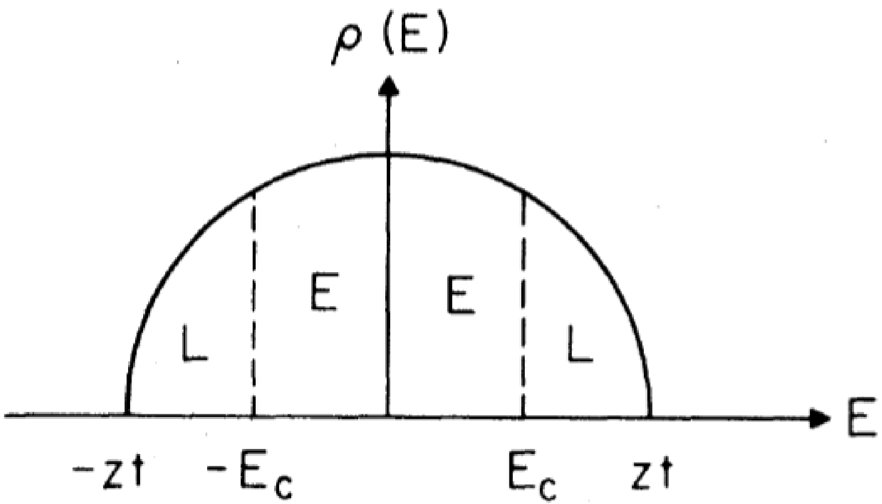
Quantum critical probability: $\boldsymbol{p_q}$ (one must have $\boldsymbol{p_q} > \boldsymbol{p_c}$).

Introduction

Quantum Percolation

Anderson Localization

In 1958, Anderson proved that **all eigenstates are localized in disordered grids when $\boldsymbol{d \leq 2}$, which means $\boldsymbol{p_q = 1}$.**



Density of states of a $d > 2$ disordered system.

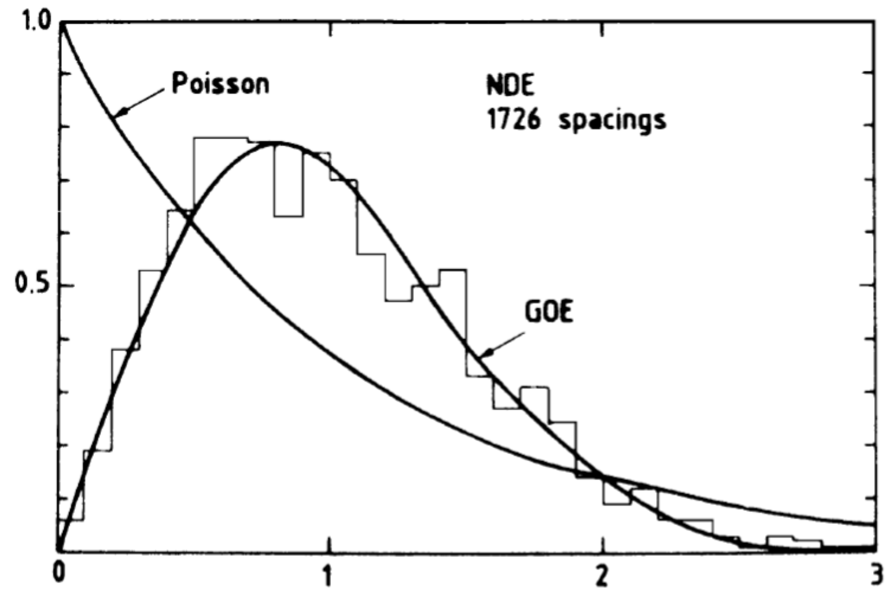
Introduction

Quantum Percolation

Anderson Localization

$\gamma = \langle max\{E_i - E_{i-1}, E_{i+1} - E_i\}/min\{E_i - E_{i-1}, E_{i+1} - E_i\}\rangle_i$,
E_{i-1}, E_i, E_{i+1} **are three consecutive energy levels.**

$\gamma \approx 0.53$**: extended states exist;**
$\gamma < 0.5$**: only localized states;**



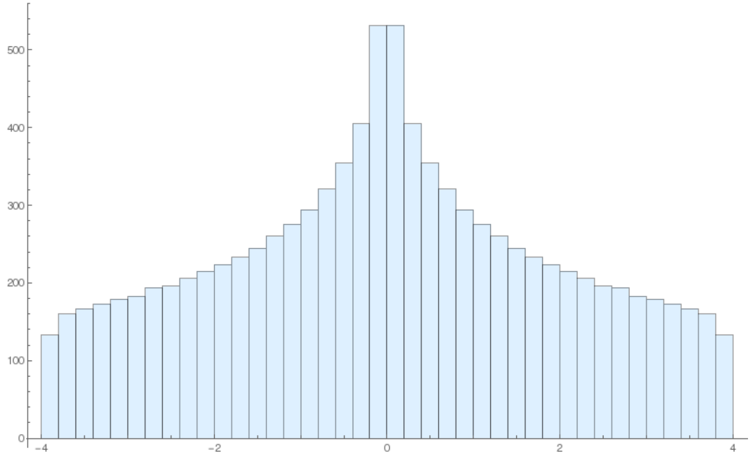
Energy level spacing: Wigner-Dyson distribution vs. Poisson distribution

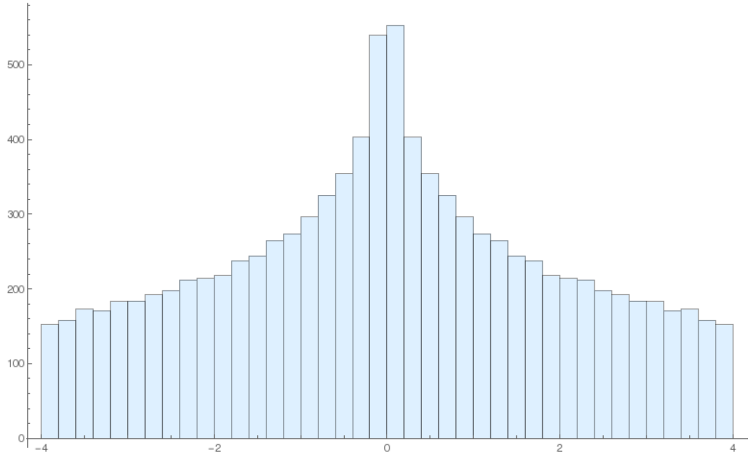
Results

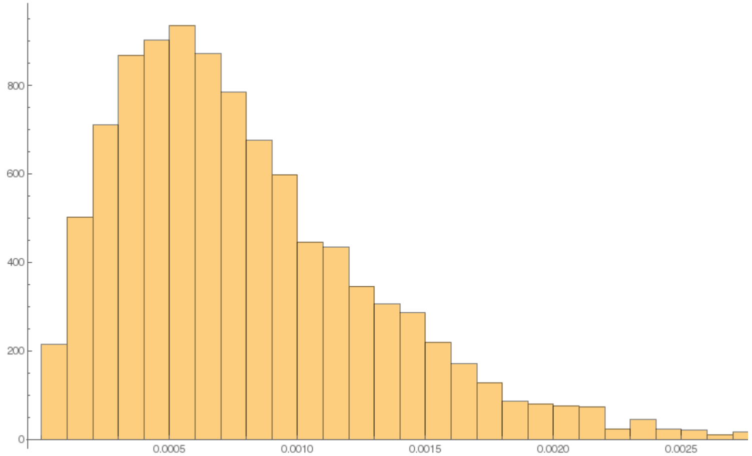
2D Grid

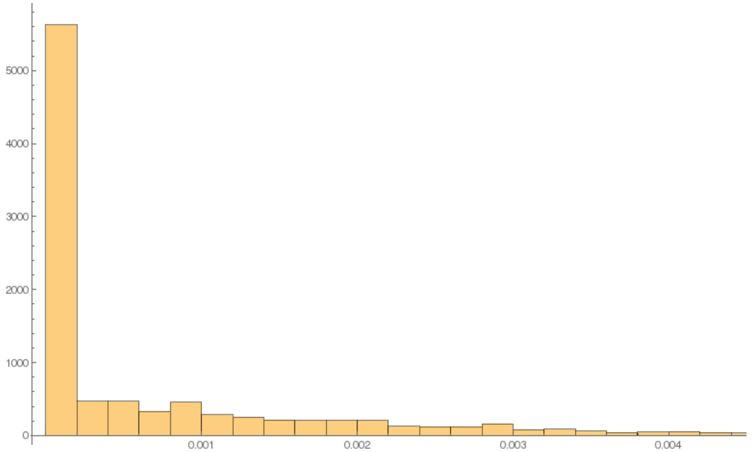
$p = 0.99$

$p = 1.00$





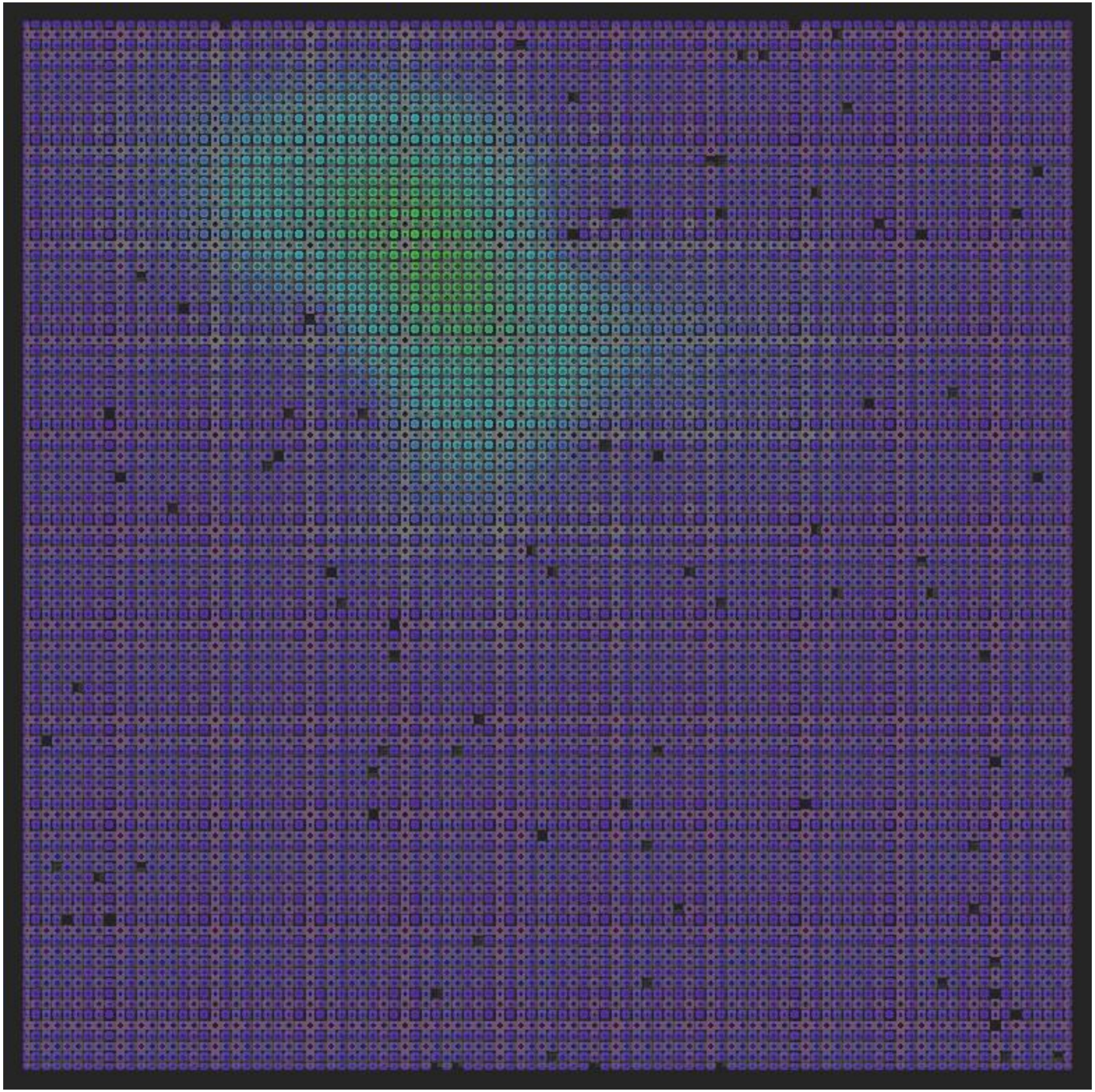




Results

2D Grid

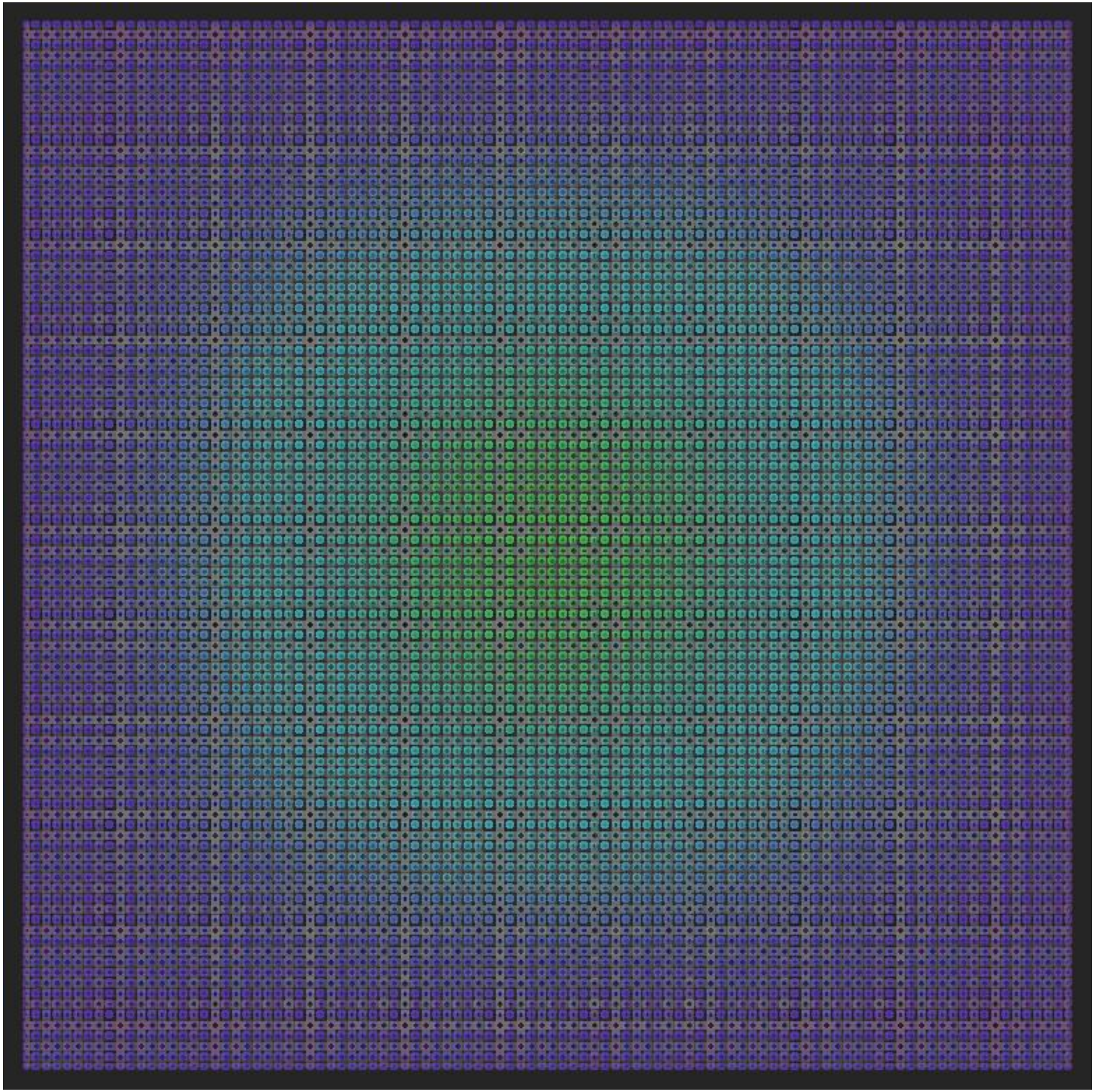
$p = 0.99, E_g \approx -4.0$



Results

2D Grid

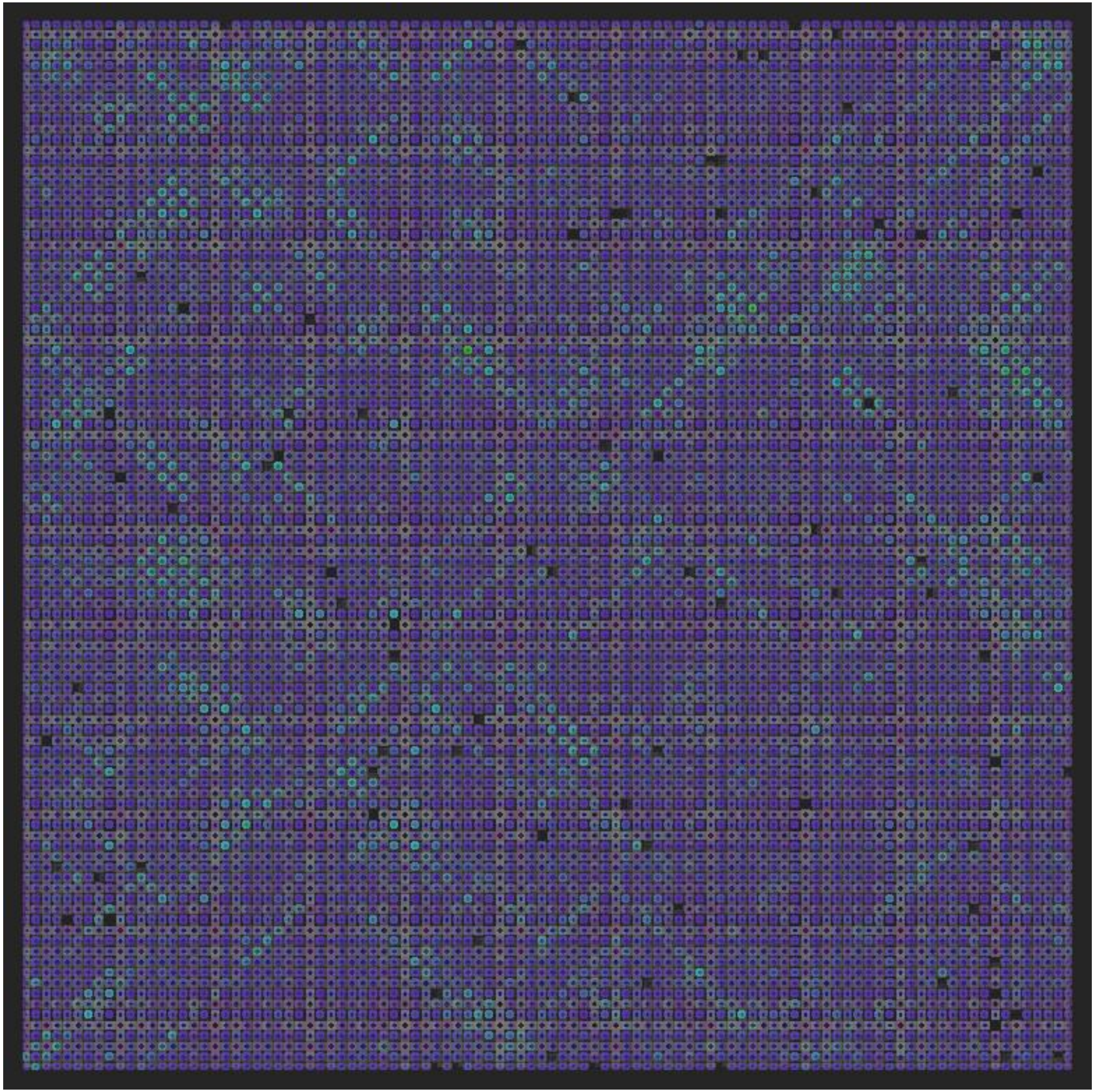
$p = 1.00, E_g \approx -4.0$



Results

2D Grid

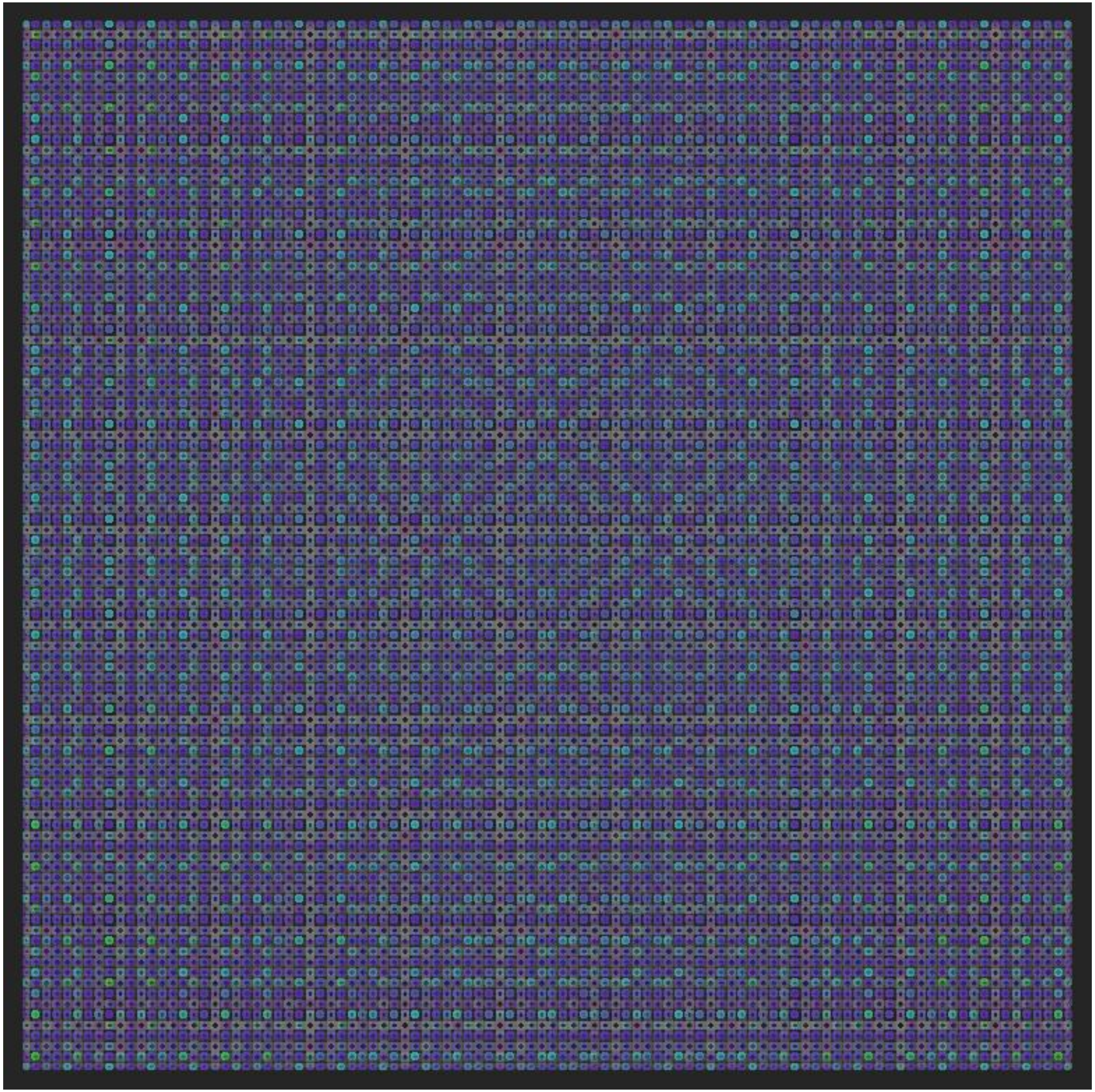
$p = 0.99, E_e \approx 4.0$



Results

2D Grid

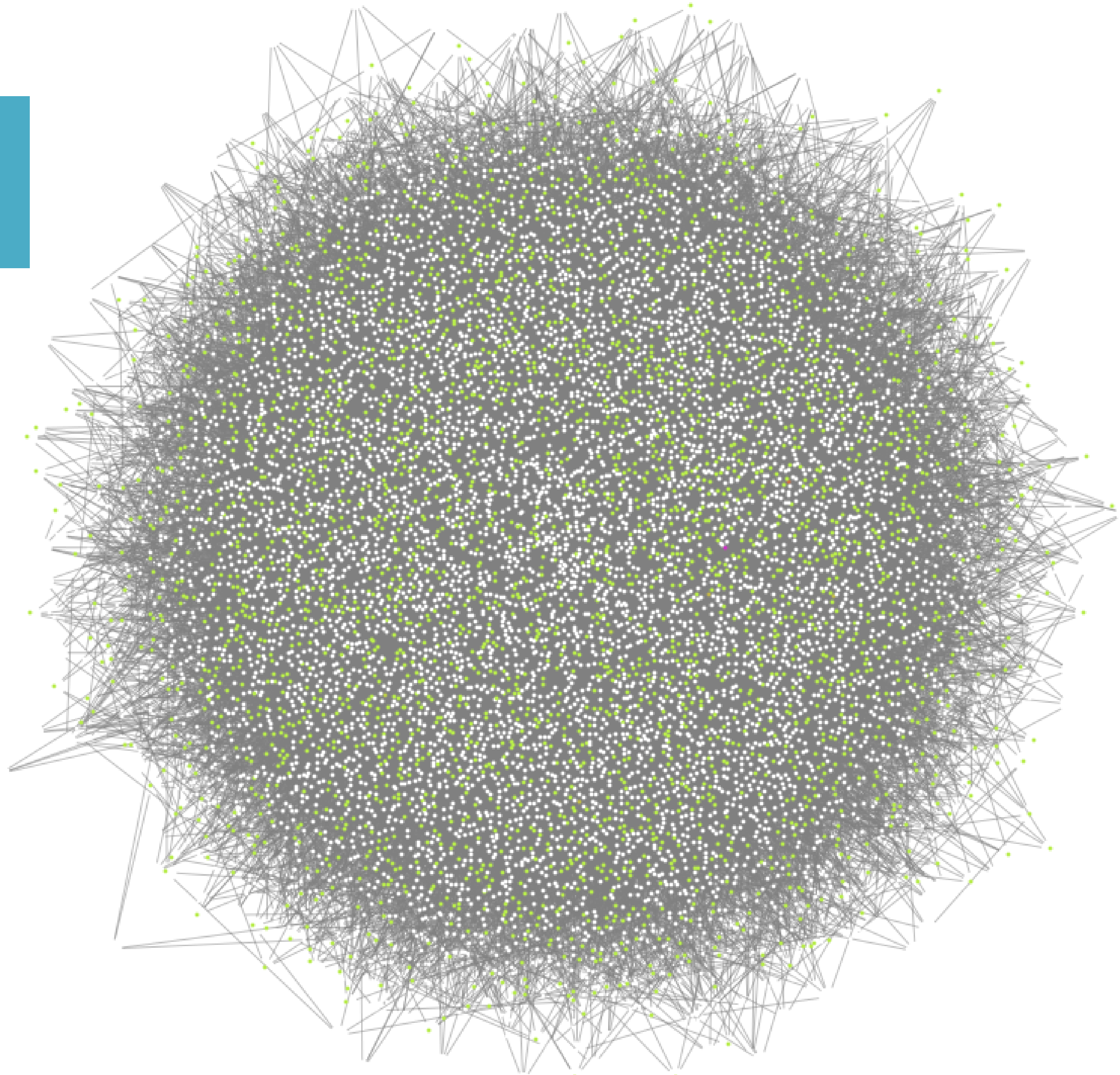
$p = 1.00, E_e \approx 4.0$



Results

Barabasi-Albert Model

BA model with $N = 10^4$,
$\langle k \rangle = 4 \times 10^4$.
$p \approx 0.3$, $E_g \approx$-8.7.

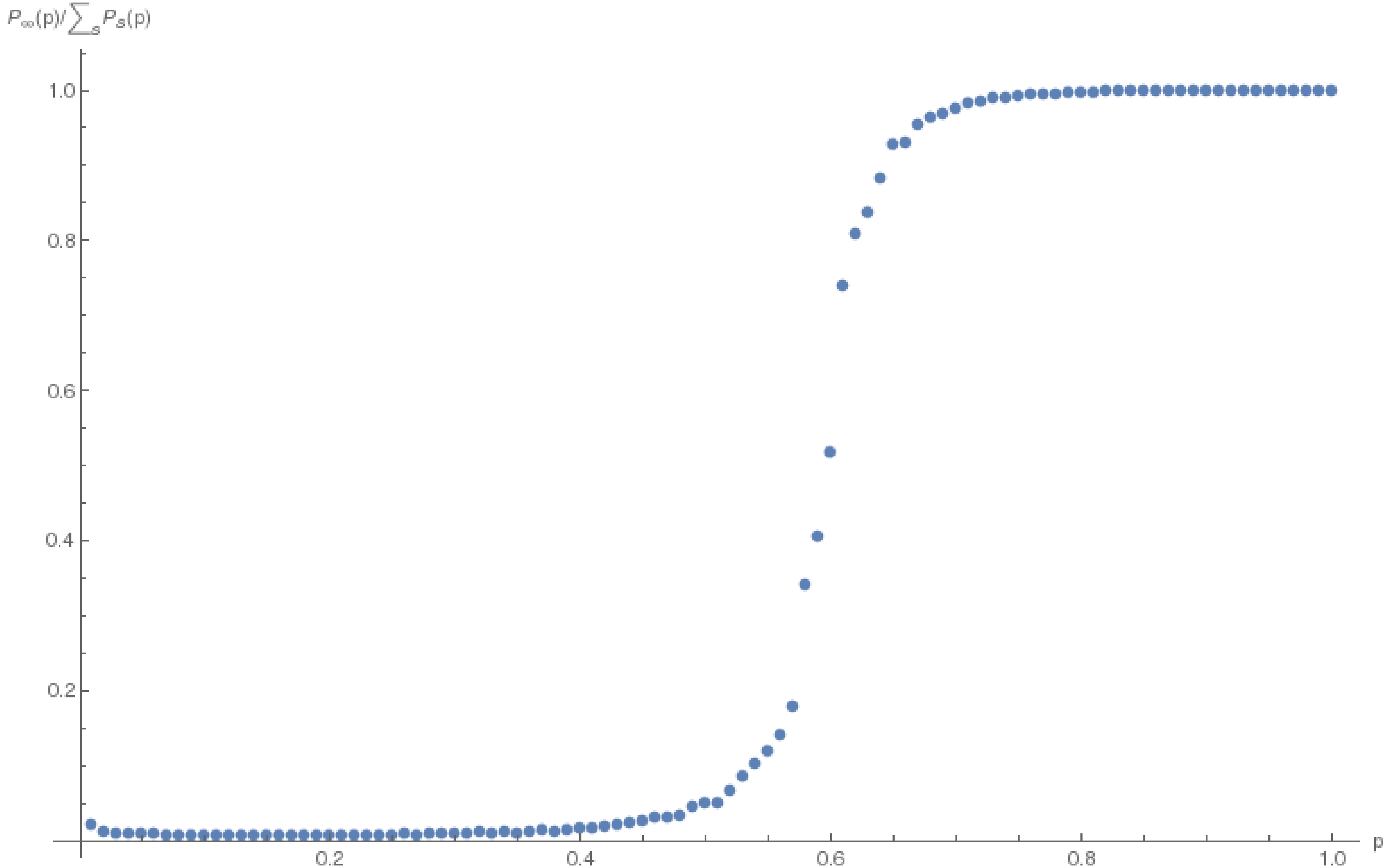


Results

2D Grid

Connectivity

2D Grid with $N = 10^4$. $p_c \approx 0.6$

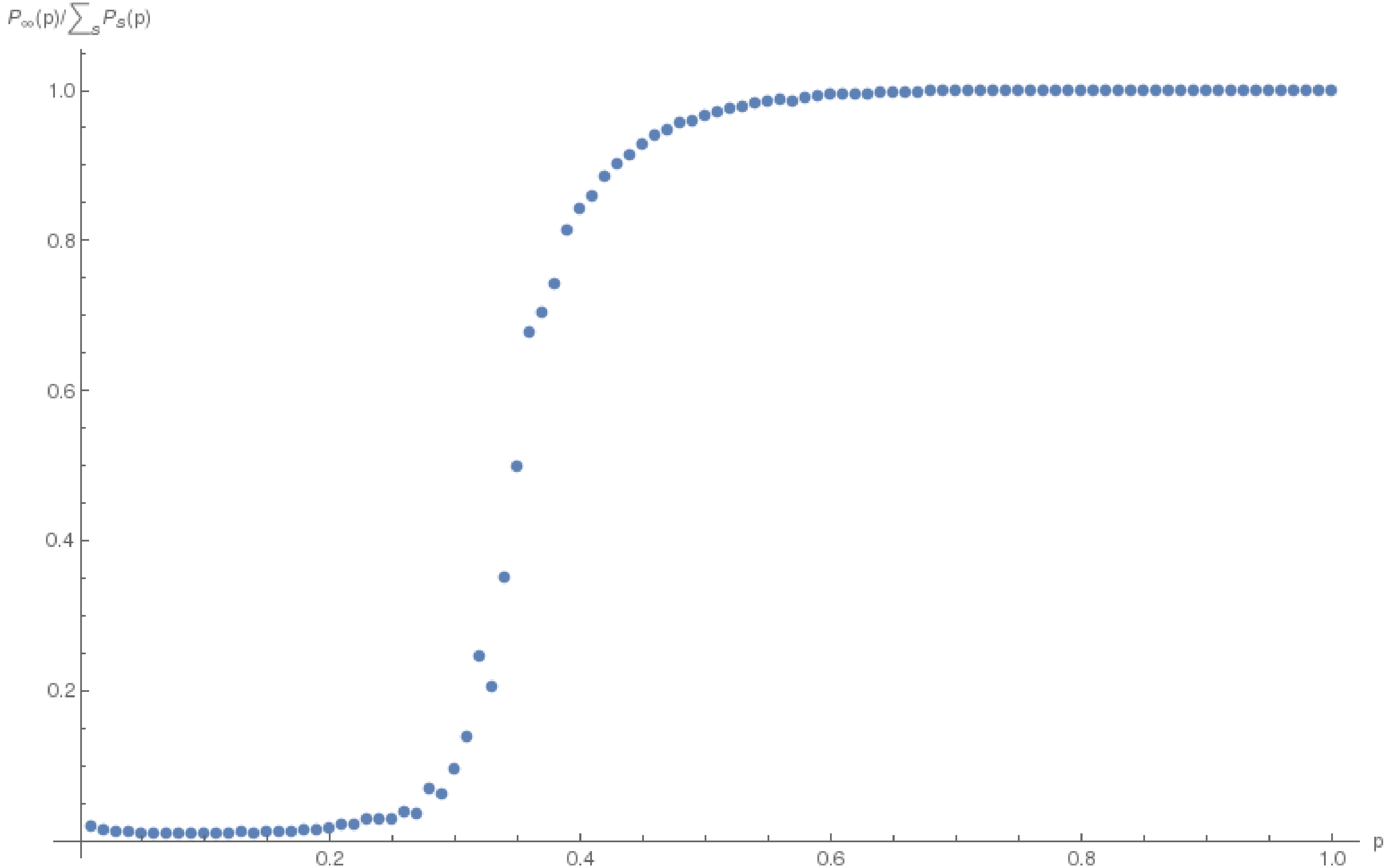


Results

3D Grid

Connectivity

3D Grid with $N = 10{,}648$. $p_c \approx 0.35$

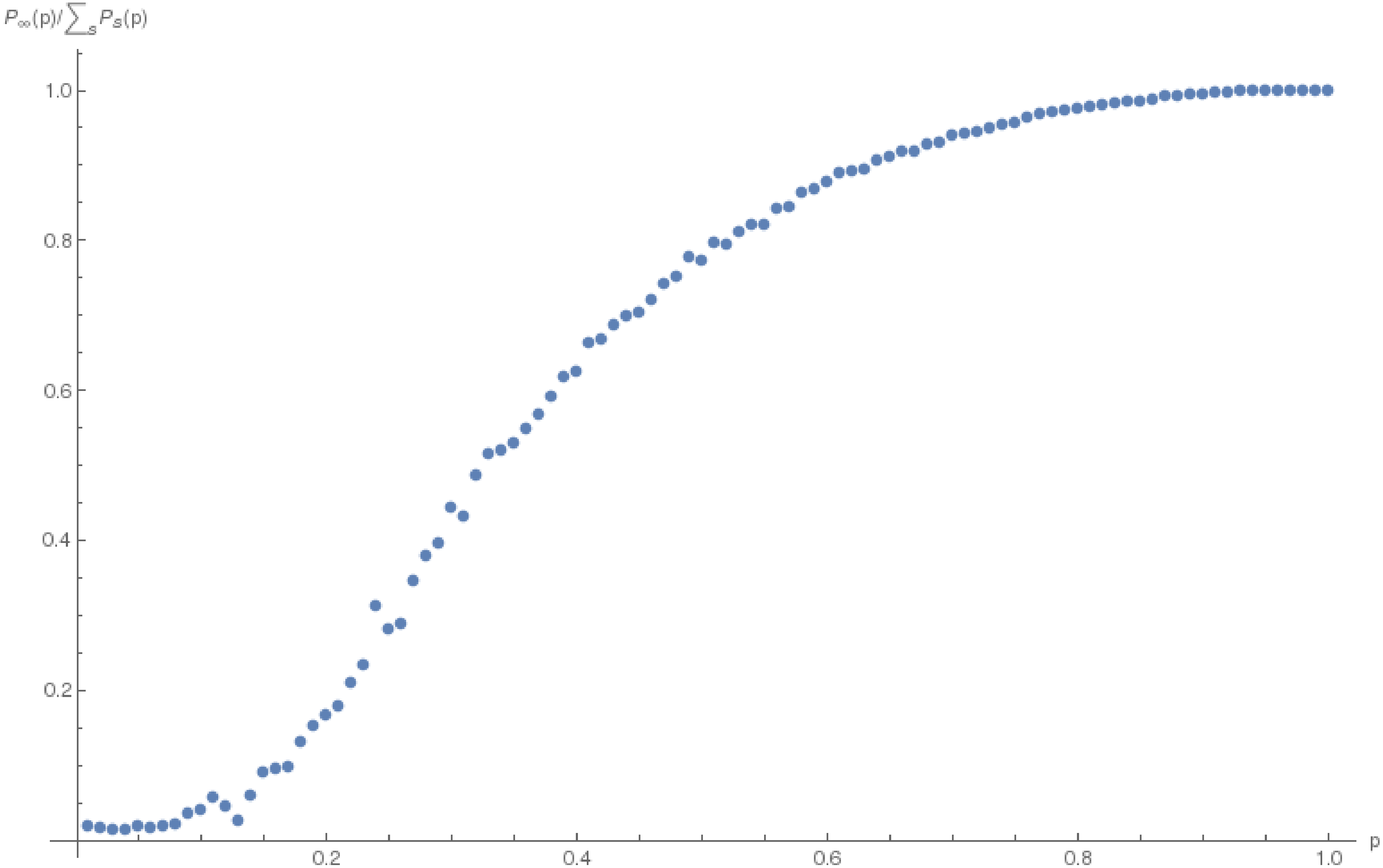


Results

Barabasi-Albert Model

Connectivity

BA model with $N = 10^4, \langle k \rangle = 4.\ p_c \approx 0.15$

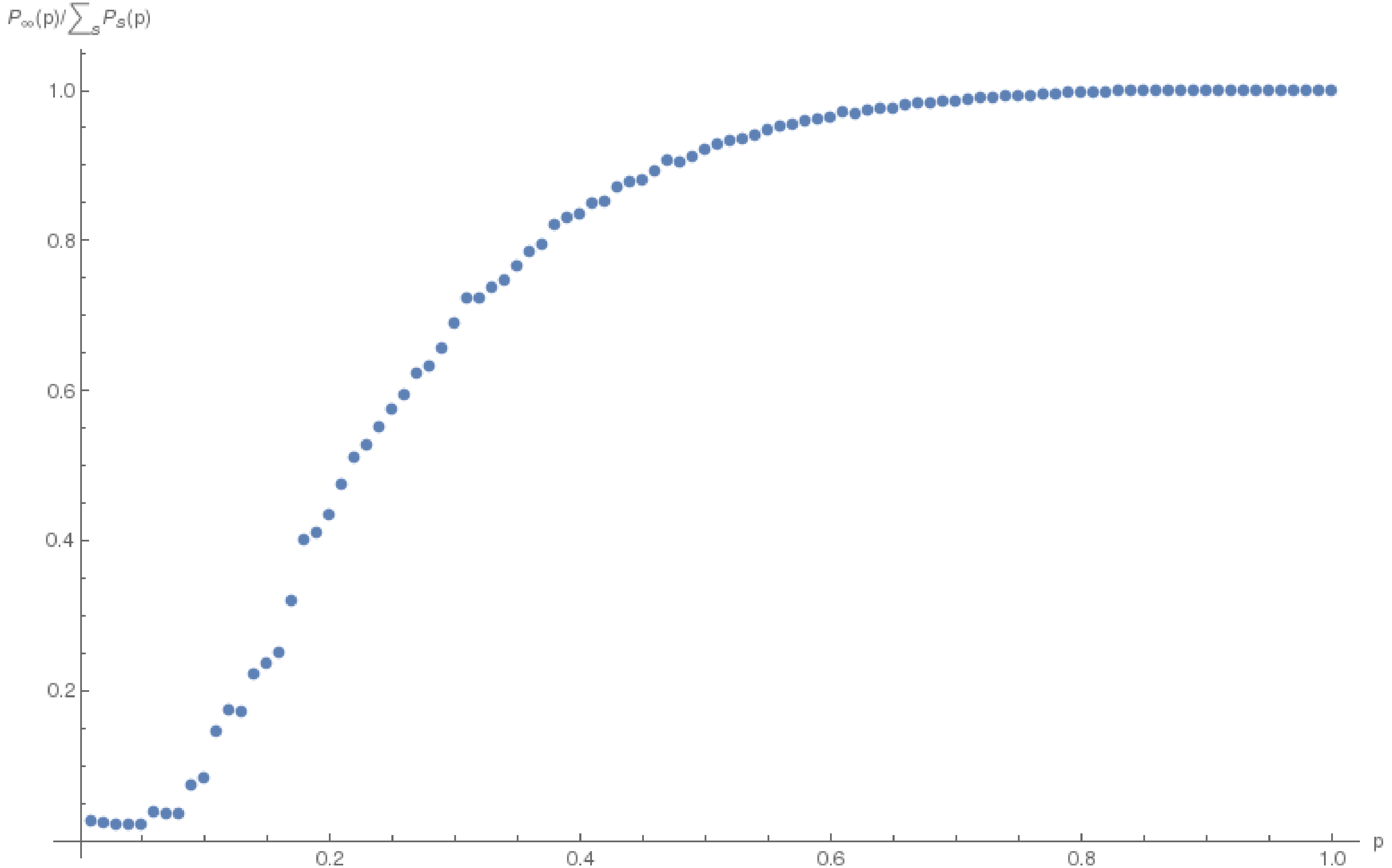


Results

Barabasi-Albert Model

Connectivity

BA model with $N = 10^4, \langle k \rangle = 6.\ p_c \approx 0.08$

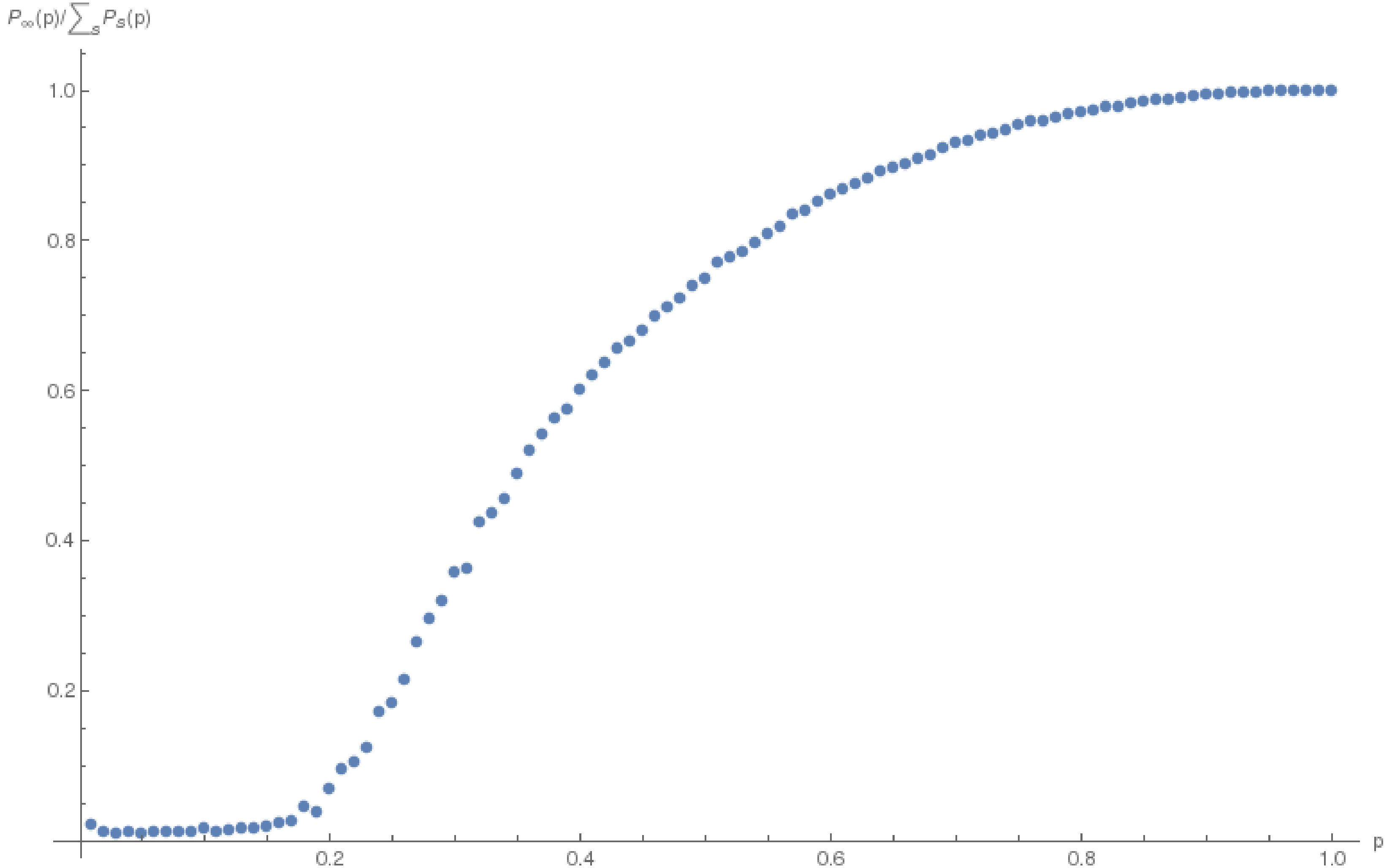


Results

Scale-Free Model

Connectivity

SF model with $N = 10^4, \langle k \rangle \approx 4, \gamma = 6.\ p_c \approx 0.2$

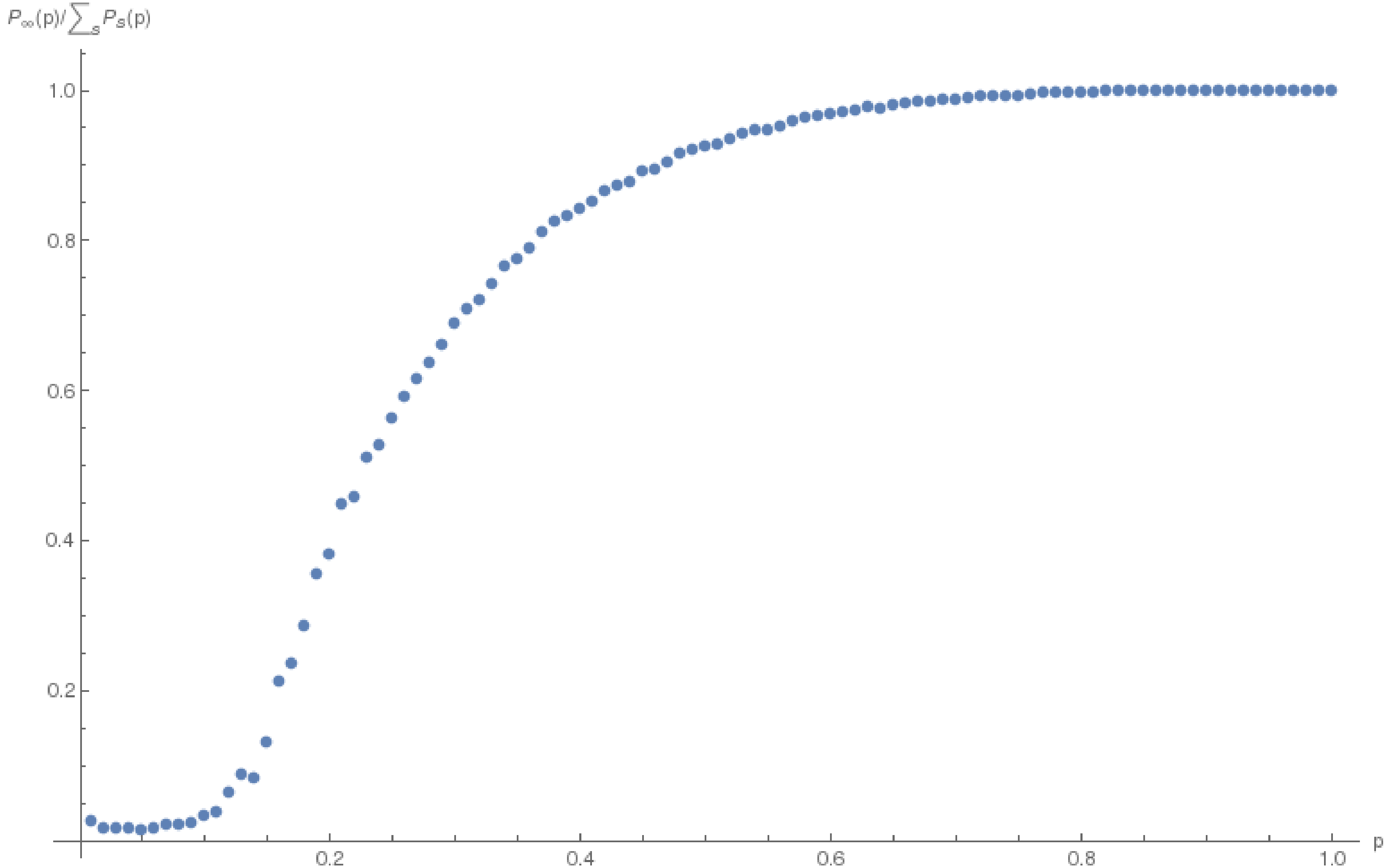


Results

Scale-Free Model

Connectivity

SF model with $N = 10^4, \langle k \rangle \approx 6, \gamma = 6.\ p_c \approx 0.15$

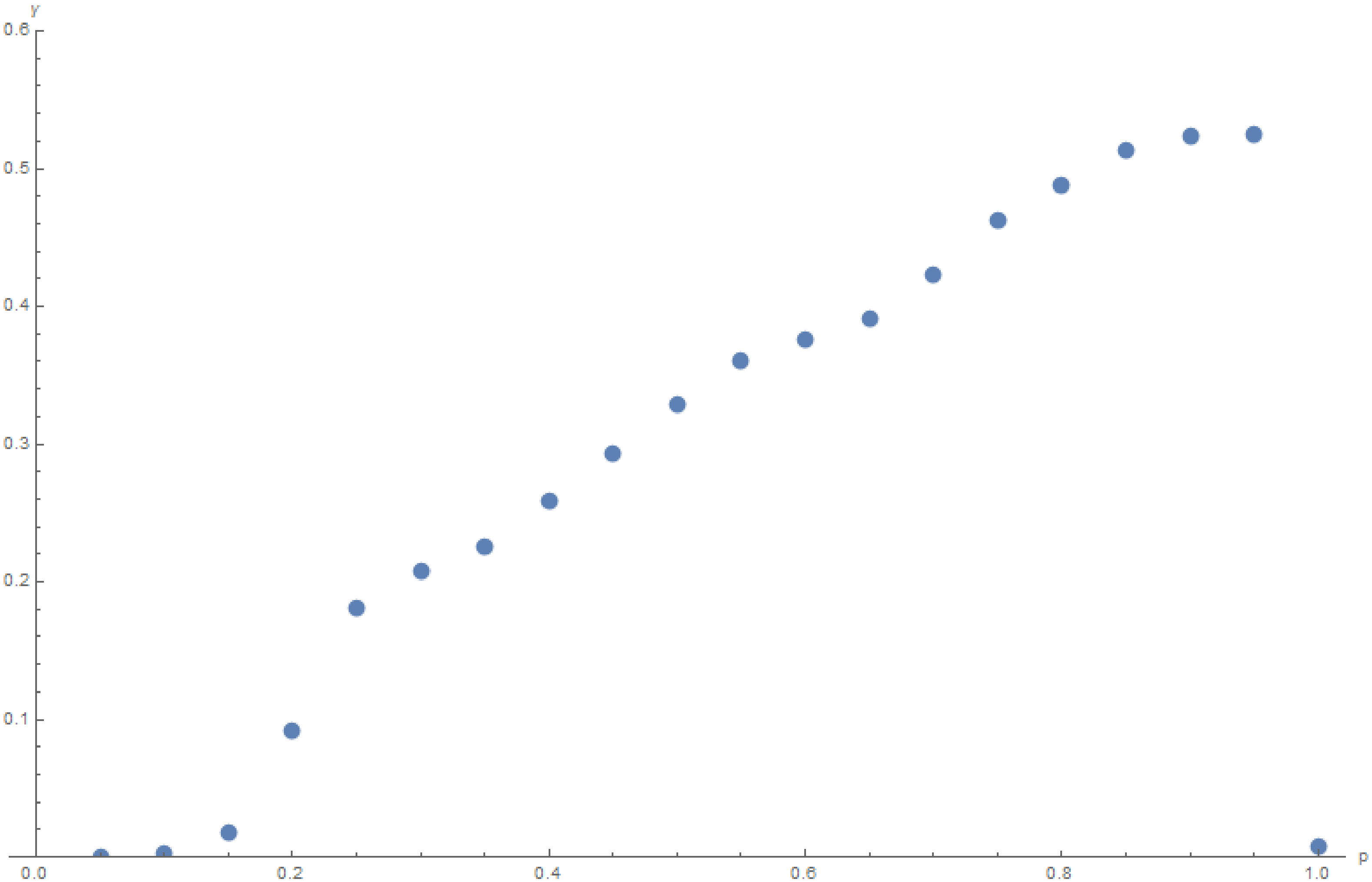


Results

2D Grid

Wave Localization

2D Grid with $N = 10^4$. $p_q \approx 1.0$

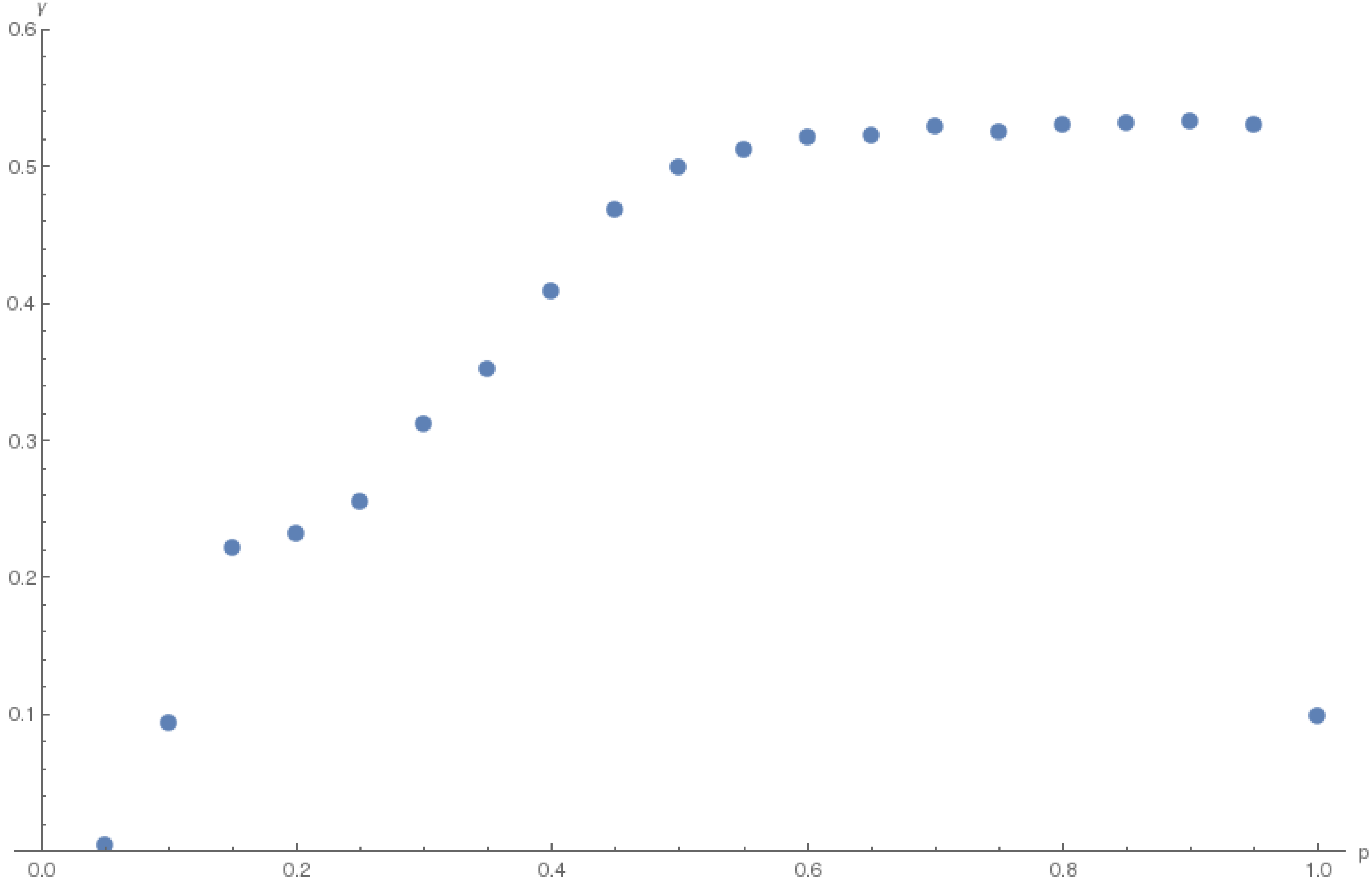


Results

3D Grid

Wave Localization

3D Grid with $N = 10{,}648$. $p_q \approx 0.44$

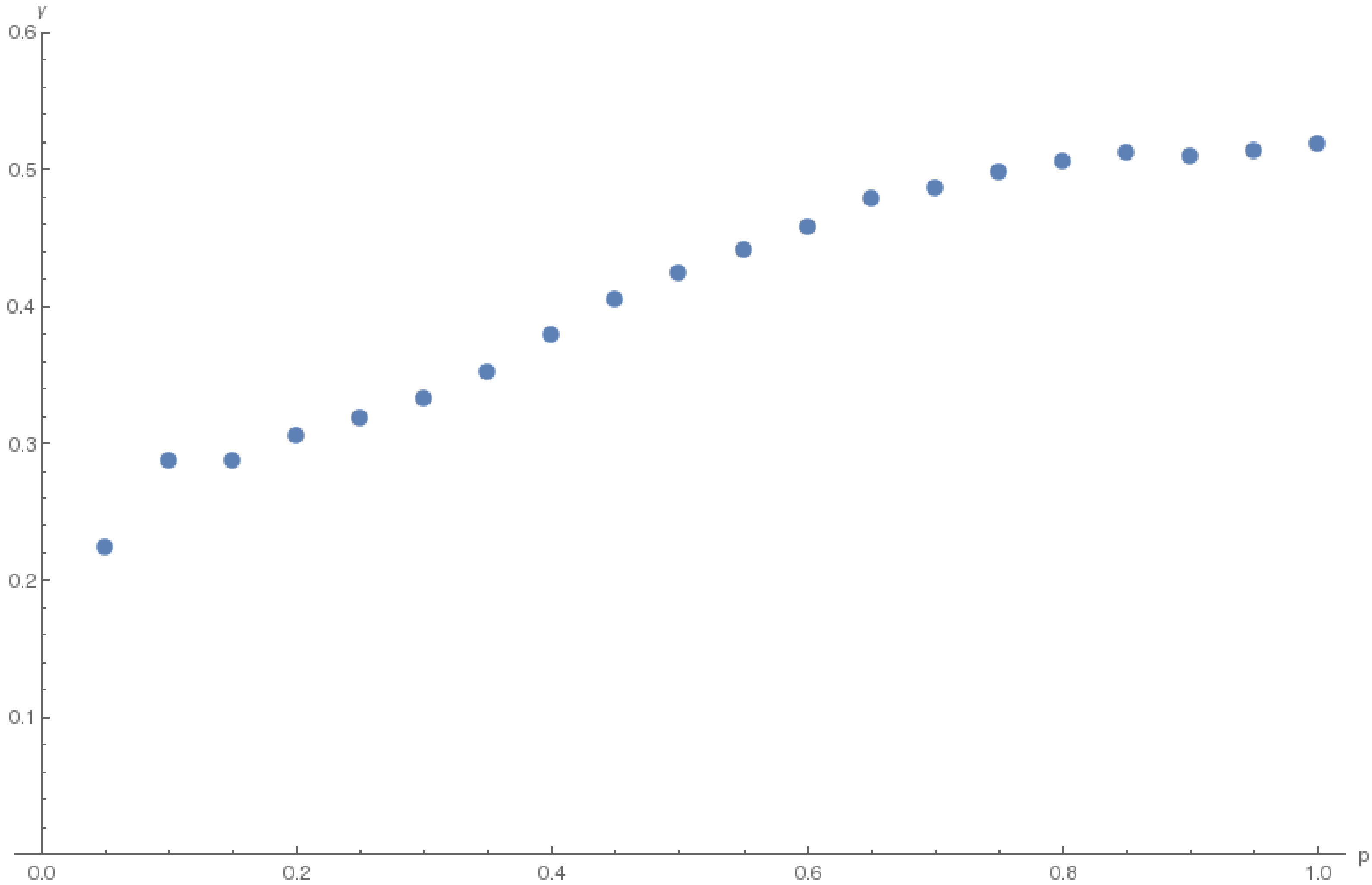


Results

Barabasi-Albert Model

Wave Localization

BA model with $N = 10^4, \langle k \rangle = 4.\ p_q \approx 1.0$

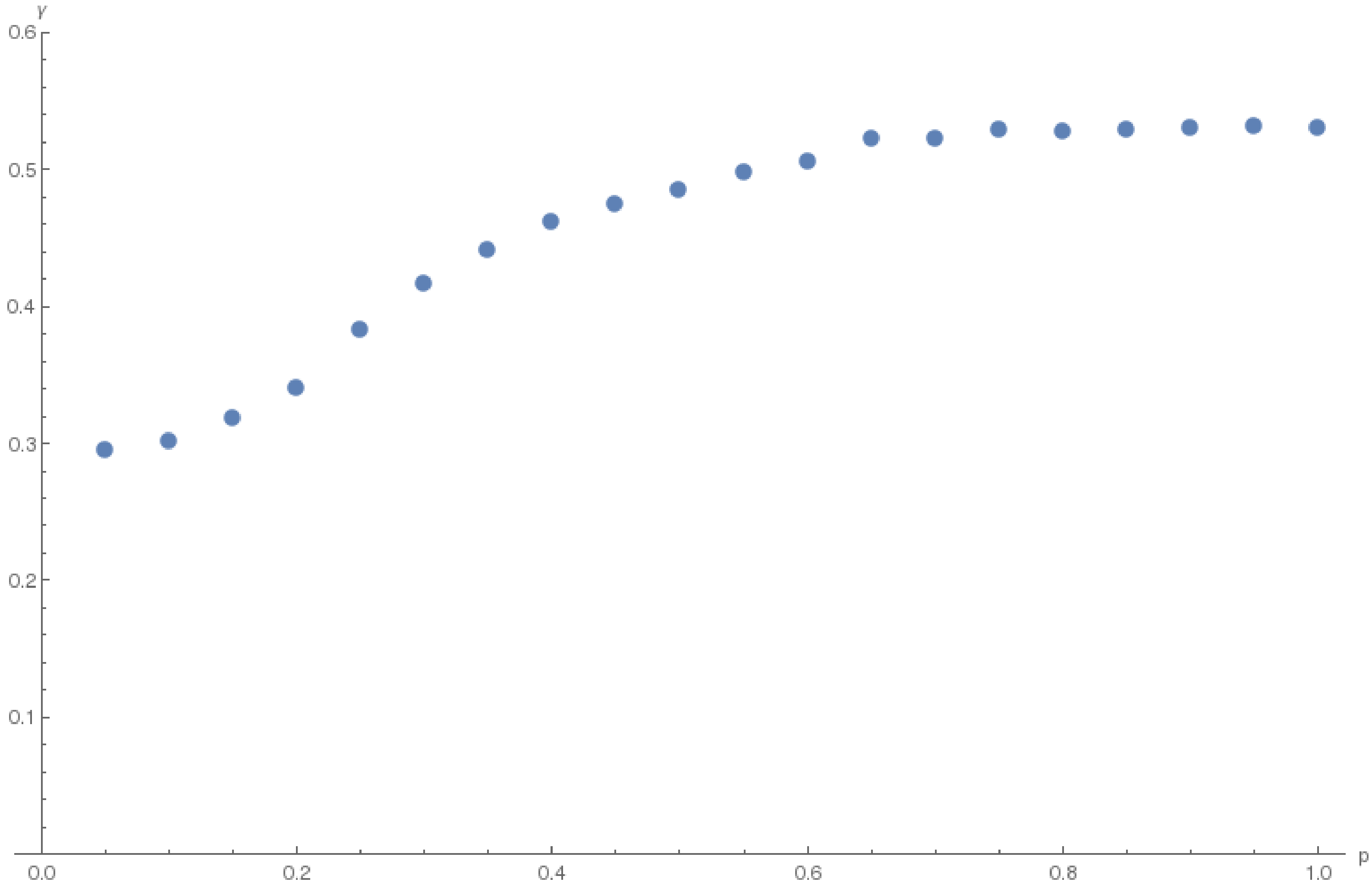


Results

Barabasi-Albert Model

Wave Localization

BA model with $N = 10^4, \langle k \rangle = 6.\ p_q \approx 0.70$

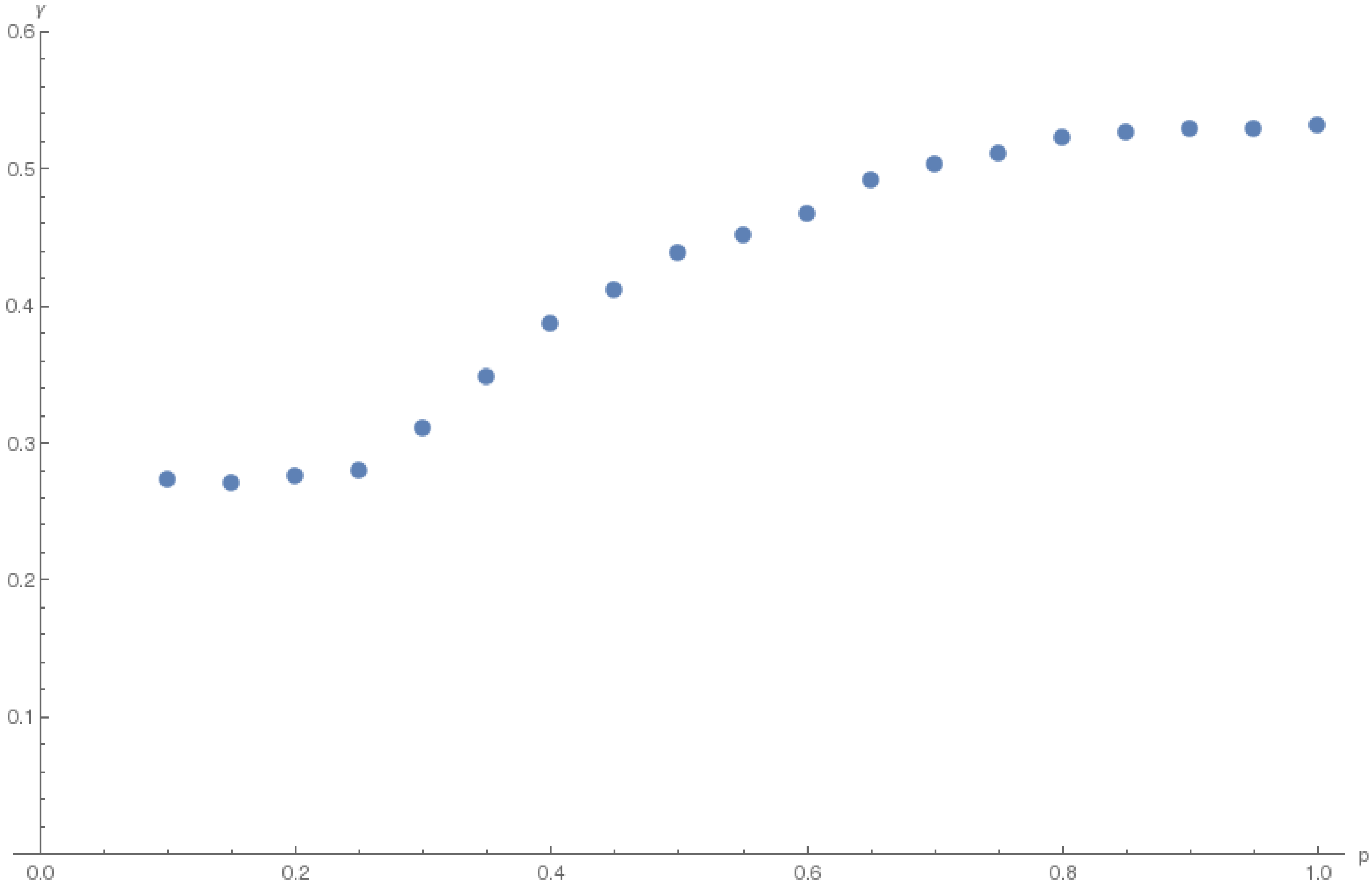


Results

Scale-Free Model

Wave Localization

SF model with $N = 10^4, \langle k \rangle \approx 4, \gamma = 6.\ p_q \approx 1.0$

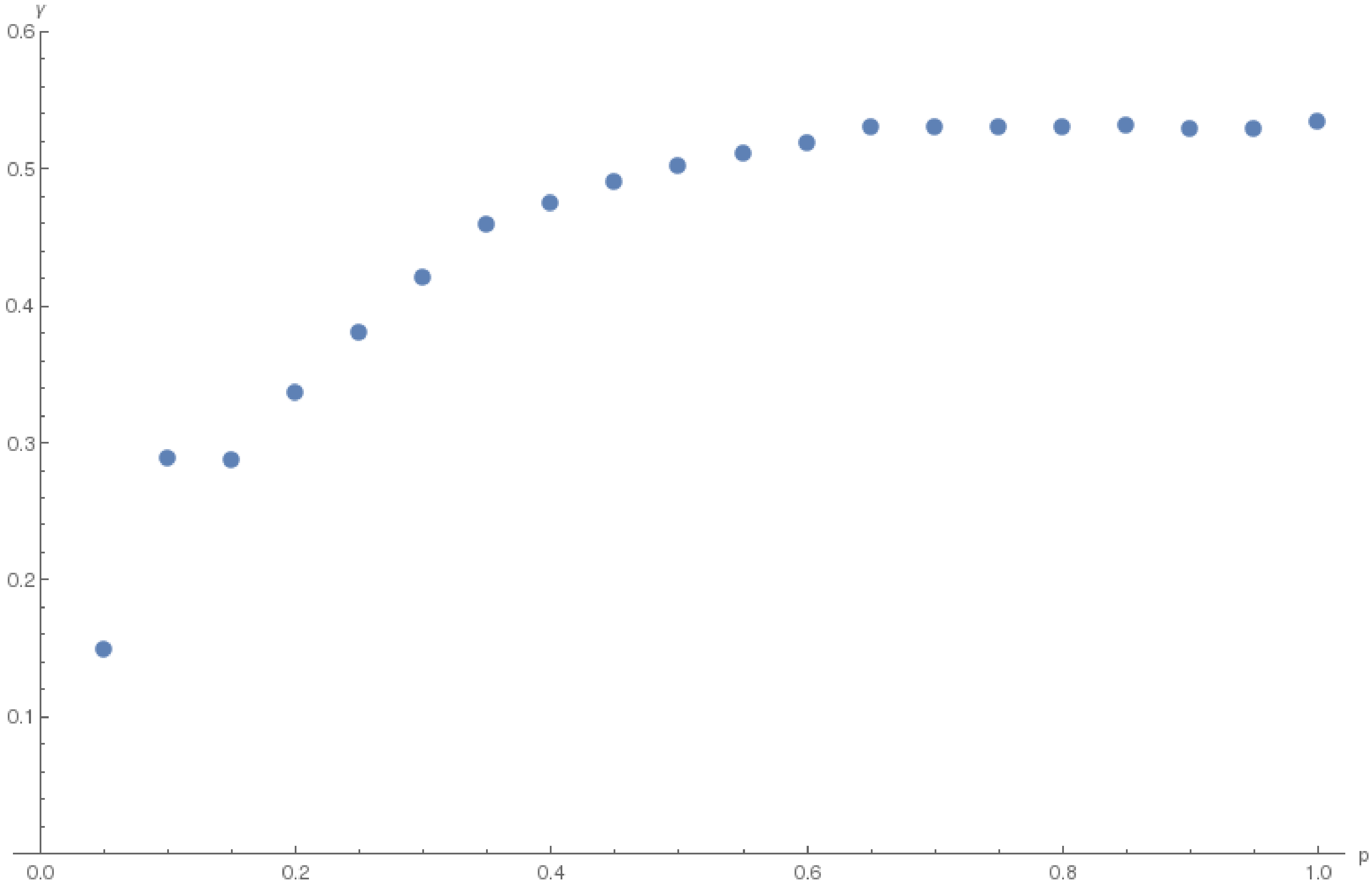


Results

Scale-Free Model

Wave Localization

SF model with $N = 10^4, \langle k \rangle \approx 6, \gamma = 6.\ p_q \approx 0.65$



Summary

Conclusion & Future Work

Conclusion

- **Wave transmission can be blocked even if the network is fully connected.**
- **Compared with grids, scale-free networks are more connectable with classical percolation, but less connectable with quantum percolation.**

Future Work

- **Critical exponents.**
- **Quantum entanglement network: dimension of Hilbert space $n \to 2^n$.**
- **Open system dynamics: stability of hubs of networks.**

Thank you