HOMEWORK 2

Problem 1: Erdős-Rényi network

Given an Erdős-Rényi random graph which has $N = 3000$ nodes and linking probability $p = 10^{-3}$, answer the following questions:

1. Find $\langle L \rangle$, the expectation of how many links the graph should have.
2. Find the average degree $\langle k \rangle$.
3. Which regime should the graph be in? Subcritical regime? Supercritical regime?
4. Find the critical linking probability p_c.
5. The average path length between two arbitrary nodes of an Erdős-Rényi graph can be proved to be $d = \ln N / \ln \langle k \rangle$. Find it for this graph. Does it make sense to you?
6. How many nodes would this graph at least have if the graph had the same average degree but an average path length of $d = 100$? You can see that the "length" dimension of networks is usually very small compared to their size.
7. Find the degree distribution $P(k)$ (approximated by a Poisson distribution).

Problem 2: Cayley tree

A Cayley tree is a symmetric regular tree. It is constructed by starting from a central node of degree k; the k neighbors of the central node are the first layer and also have degree k each; the tree expands until the outermost layer (which is away from the central node by a distance of r) and the outermost nodes only have degree one and are called "leaves".

1. How many nodes are reachable in n steps from the central node (in terms of k and r)?
2. What is the average path length between two arbitrary leaves?
3. Show that the "length" dimension of a Cayley tree is also much smaller than its size.

-Sean.