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Multifractal analysis of stock exchange crashes

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ABSTRACT

We analyze the complexity of rare events of the DJIA Index. We reveal that the returns of the time series exhibit strong multifractal properties meaning that temporal correlations play a substantial role. The effect of major stock market crashes can be best illustrated by the comparison of the multifractal spectra of the time series before and after the crash. Aftershock periods compared to foreshock periods exhibit richer and more complex dynamics. Compared to an average crash, calculated by taking into account the larger 5 crashes of the DJIA Index, the 1929 event exhibits significantly more increase in multifractality than the 1987 crisis.

1. Introduction

It is widely accepted that financial markets illustrate strong signs of complex dynamical systems and the distribution of returns of high frequency data follows a power law. In this context, financial time series exhibit non-linear properties and the stylized facts call for long-memory, fat tails and multifractality [1,2]. Particularly for stock exchange time series, fat tails, power-law correlations and multifractality have been documented in a number of cases [3–5]. These results are in disagreement with the traditional economic notion which states that markets act in accordance with the Efficient Market Hypothesis (EMH).

The majority of the empirical financial studies aimed in identifying long term correlations either in single or multiple time series data [6–10]. Unlike large and intraday time series data examination, extreme events of stock exchanges have received little attention. In our situation we are interested in investigating the statistical properties of stock exchange indexes during periods of high stress and namely periods of stock market crashes with emphasis given to 1929 and 1987 crashes. Ref. [11] analyzed similar extreme event impact but on the exchange rate markets, while [9] for 88 companies that contribute to the S&P 500 index, during the 26-year period 1983–2009, apply time-lag Random Matrix Theory (TLRMT) for each year and show pronounced peaks in TLRMT singular values during the largest market shocks and economic crises: Black Monday, the Dot-com bubble and the 2008 crash.

The purpose of investigating market crashes is based on scientific evidence that such complex systems reveal their structure better when they are under stress than in normal conditions. According to Sornette, [12] "such extreme events express more than anything else the underlying 'forces' usually hidden by almost perfect balance and thus provide the potential for a better scientific understanding of complex systems". Consequently the examination of these specific great crashes will provide an understanding about the dynamics and complexity of the stock exchange markets will assist especially institutional investors to correctly assessed market risk and, also, it will provide to the policy makers the information needed to put in place the appropriate mechanism in encountering future problems.

In this vein, the work of Refs. [13,14], using intraday data, report a correlation between the width of the estimated multifractal spectrum and future price fluctuation, putting the basic for price fluctuations prediction and thus to future

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crashes. But Ref. [15] after testing the above statement, with rigorous research, has come to the conclusion that the multifractal nature in the indexes is not a fact but fiction. This result was further supported by analyzing two additional indexes (S&P 500 and NASDAQ) in developed stock markets. Furthermore Ref. [16], suggest that it is valuable to apply the partition function approach to the multifractal analysis of stocks and indexes and to the possible application of multifractal properties in market forecasting and managing risk, but by using returns series rather than stock prices or indexes.

In this paper, we first calculate the market complexity of the two crises and later we divide the total sample into periods before (foreshocks) and after (aftershocks) the crash in an attempt to identify the changes (if any) in the market dynamics and complexity. In addition we analyze three other main crashes and then we calculate the average crash based on all these rare Dow Jones Industrial Average (DJIA) market events. Last we present the generalized Hurst exponent and we provide some concluding remarks.

2. Methodology

We examine the nonlinear features of extreme events of DJIA, starting with the 1929 and 1987 crashes. The data consist of daily returns of the stock market index before and after the market crash and spans from August 1928 to January 1931 and August 1986 to December 1988 respectively. We are interested in investigating the whole process of the event, i.e. energy accumulation, or bubble rising and the expansion process after the eruption. Therefore the data consist of around three years of trading days, sufficient enough in revealing the statistical properties and relevant information. The dates of the two crashes were Oct. 28, 1929 with the DJIA losing 14.5% of its value and Oct. 19, 1987 with the loss amounting to 25.6%. The daily rate of returns of the stock market is calculated as follows:

$$r_t = \ln p_t - \ln p_{t-1} \tag{1}$$

where $p(t)$ is the price of the index on day t and r is the rate of return.

We are investigating the multifractal properties of the above episodes. Numerous procedures have been produced in calculating multifractality. In our case we use the Multifractal-Detrended Fluctuation Analysis (MF-DFA) developed by Ref. [17] as this method reduces noise effects, removes local trends and avoids spurious detection of correlations that are artifacts of nonstationarities in the time series. Motivated by DFA, a detrended cross-correlation analysis (DCCA) was introduced [18,19] together with its multifractal extension [20,21]. It was also shown that for short time series, like in our case, and negative moments, the significance of results for MF-DFA is better than most of the other techniques. The multifractal generalization of the MF-DFA procedure can be briefly sketched as follows. The MF-DFA operates on the time series $x(k)$, where $k = 1, 2, \ldots, N$ and N is the length of the series. We assume that $x(k)$ are increments of a random walk process around the average $\langle x\rangle$ and the profile is given by the integration of the signal

$$Y(i) = \sum_{k=1}^{N}[x(k) - \langle x\rangle], \quad i = 1, \ldots, N. \tag{2}$$

Next, the time series $Y(i)$ is divided into $N_S \equiv \mathrm{int}(N/s)$ non-overlapping segments of equal length s, starting from both beginning and the end of the time series. Each segment υ has its own local trend that can be approximated by the least-squares fitting of the series. Then we determine the variance

$$F^2(\upsilon, s) \equiv \frac{1}{s}\sum_{i=1}^{s}\{Y[(\upsilon - 1)s + 1] - y_\upsilon(i)\}^2 \tag{3}$$

for each segment ν, $\nu = 1, \ldots, N_S$ and

$$F^2(\upsilon, s) \equiv \frac{1}{s}\sum_{i=1}^{s}\{Y[N - (\upsilon - N_s)s + 1] - y_\upsilon(i)\}^2 \tag{4}$$

for $\nu = N_S+1, \ldots, 2N_S$. Here, $y_\nu(i)$ is the fitting line in segment ν. Then, we detrend the series and average over all segments to obtain the qth order fluctuation function

$$F_q(s) \equiv \left\{\frac{1}{2N_s}\sum_{\upsilon=1}^{2N_s}[F^2(\upsilon, s)]^{q/2}\right\}^{1/q}. \tag{5}$$

The property of $F_q(s)$ is that for a signal with fractal properties, it reveals power-law scaling within a significant range of s

$$F_q(s) \propto s^{h(q)} \tag{6}$$

and the variable q can take any real value other than zero.

In general the exponent $h(q)$ will depend on q. For stationary time series, $h(2)$ is the well-defined Hurst exponent H and thus, $h(2)$ is the generalized Hurst exponent. Multifractal (MF) scaling exponent $\tau(q)$ is related to $h(q)$ through

$$\tau(q) = qh(q) - D_f \tag{7}$$

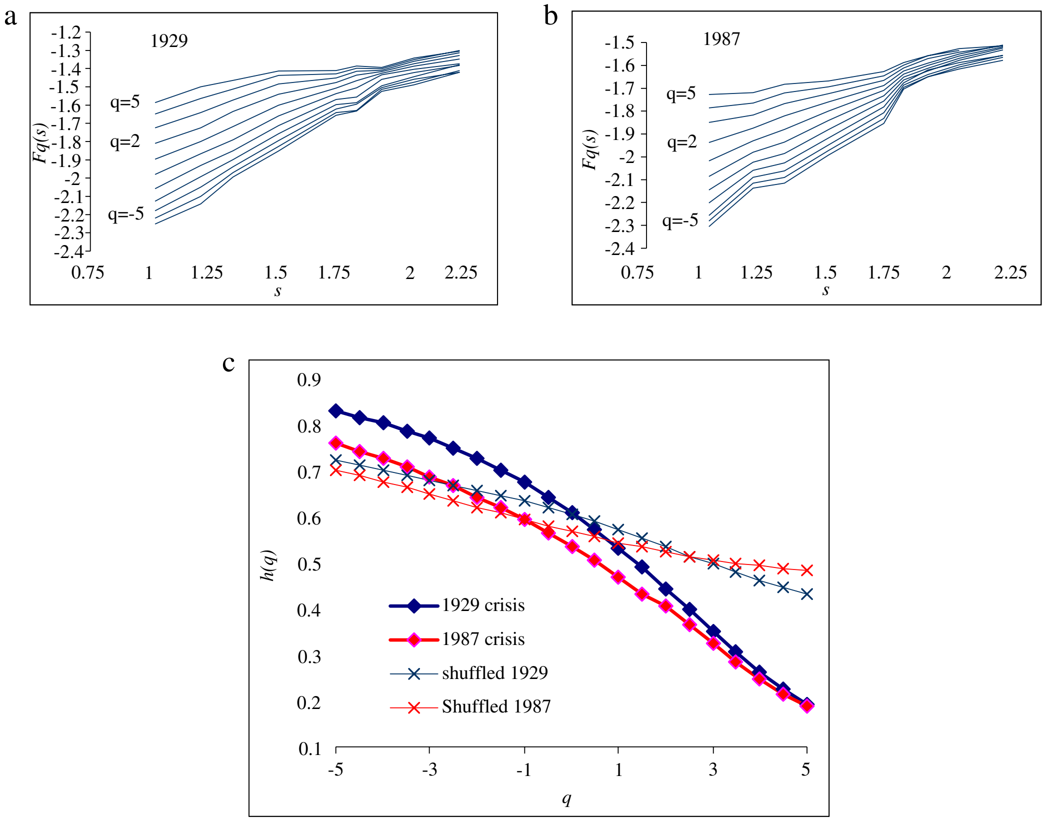

Fig. 1. The MF-DFA functions $Fq(s)$ versus the time scale s in log–log plot (a), (b) and the Hurst exponent $h(q)$ of the 1929 and 1987 events (c).

where D_f is the fractal dimension of a geometric support of the multifractal measure and $D_f = 1$. The exponent $\tau(q)$ represents the temporal structure of the time series as a function of the various moments q, or τ reflects the scale-dependence of smaller fluctuations for negative values of q, and larger fluctuations for positive values of q. If $\tau(q)$ increases nonlinear with q, then the series is multifractal.

3. Multifractal results

The analysis of the complexity of the 1929 and 1987 market crashes of DJIA will shed light how these extreme events may have affected their multifractality. We begin the analysis by calculating the fluctuation functions $f_q(s)$ obtained from MF-DFA. Fig. 1 panels (a) and (b) show the MF-DFA fluctuations for various qs for the returns of the times series logarithmic variations of the 1929 and 1987 events. One can clearly see that above the crossover region, the $f_q(s)$ functions are straight lines and the slopes change slightly when going from high positive moments to higher negative moments. The fluctuation function is calculated with the scaling parameter ranging from $s = 10$ to $s = N/4$, where N is the total length of the time series. Also, in the same figure panel (c) we depict the q-dependence of the Hurst exponent h_q. When q varies from -5 to 5, $h(q)$ decreases monotonously from 0.829 to 0.194 for the 1929 event and from 0.759 to 0187 for the 1987 event. In both cases $h(q)$ is not a constant, (for different q there are different exponents $h(q)$) indicating multifractality in time series. Also the richness in multifractality is associated with high variability of $h(q)$ and the degree can be quantified as

$$\Delta h = h(q_{\min}) - h(q_{\max}). \tag{8}$$

Therefore for the 1929 crash, $\Delta h = 0.635$ ($q \in [-55]$) and for 1987 $\Delta h = 0.572$ ($q \in [-55]$) indicating that the 1929 event exhibits richer multifractality than the 1987 crisis.

Furthermore, another way to characterize the multifractality behavior is presented in Fig. 2 panel (a) with the multifractal scaling function $\tau(q)$ of the original and shuffled data calculated from the power-law relation between $f_q(s)$ and s, using scales in the range $10 \leq s \leq 150$. Fig. 2 presents the original and the shuffled time series created by randomly shuffling the data, in order to remove the temporal correlations but hold the same amplitude distribution as the original time series and test how significant is the multifractality.[1] Both crashes behave almost linear with q for negative moments, but show

[1] There is another method where a number of surrogate data sets can be created, i.e. 20 or 30 and then run on each of these surrogate data the MF-DFA procedure to obtain the appropriate values. The mean and standard deviation of α then can be calculated an compared with the value of the original data.

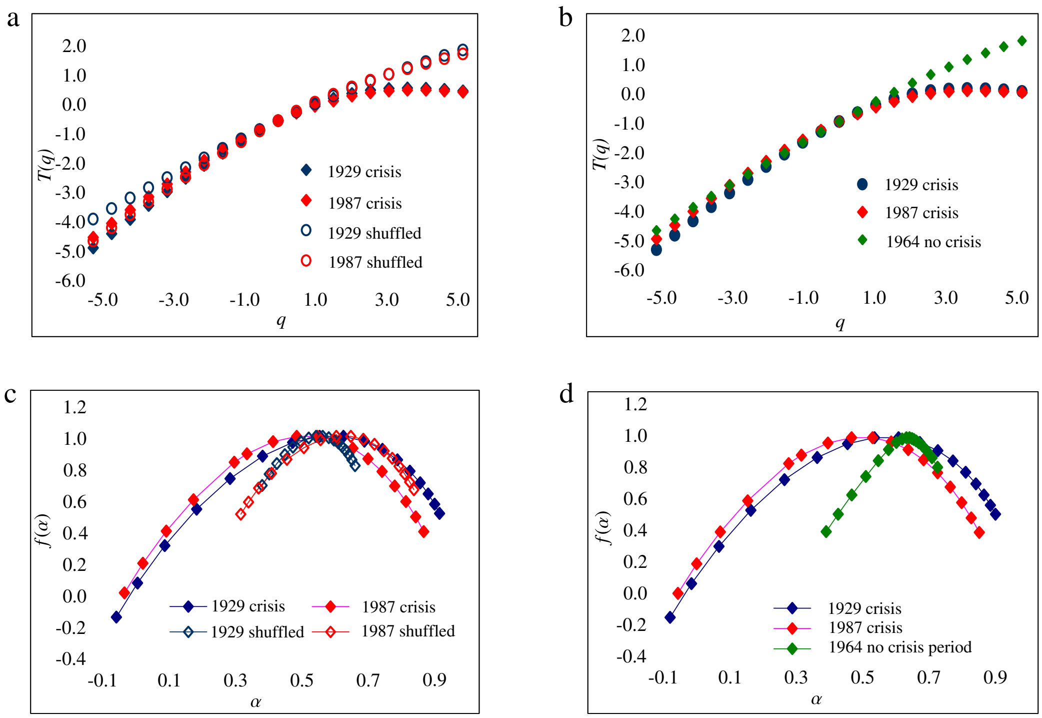

Fig. 2. Multifractal scaling exponents (panels a, b) and their fractal dimensions (panels c, d).

significant non-linearities for positive moments. This means that the temporal structure of the larger fluctuations play an important role in the multifractality and have changed dramatically in the shuffled series. In other words, the scaling exponents $\tau(q)|_{q<0}$ for all crashes and for the shuffled data are similar, whereas $\tau(q)|_{q>0}$ are significantly differ, suggesting that crashes and shuffled data are similar for small fluctuations and differ for large fluctuations. For robust purposes we perform the same analysis to a "non-crisis period" the period of 1964 where there were non significant shocks recorded. Fig. 2, panel (b) depicts the behavior of the nonlinearity of the two crashes and the almost linearity of the non-crisis period.

Next, to explicitly observe the multifractality we can convert q and $\tau(q)$ to α and $f(\alpha)$ by a Legendre transform as

$$f(\alpha) \equiv aq - \tau(q), \quad a \equiv \frac{d\tau(q)}{dq} \tag{9}$$

where $f(\alpha)$ is the fractal dimension of the time series. Panels (c) and (d) of Fig. 2, present the multifractal spectra $f(\alpha)$ of the original and shuffled data. The spectrum, as an upside-down parabola, peaks at $f_{\max}$ (at $\alpha(f_{\max})$) and stretches from min to max. That is, the range $\alpha_{\min}$–$\alpha_{\max}$ quantifies the non-uniformity of the fractal, or that $\Delta\alpha = \alpha_{\max}$–$\alpha_{\min}$ conventionally quantifies the degree of multifractality which is the width of singularity spectrum, while $f(\alpha)$ tells how frequently events with α scaling exponent occur. Or the difference of the fractal dimensions of the maximum probability subset with the minimum $\alpha_{\min}$ and the minimum one with the maximum $\alpha_{\max}$ is Δf ($\Delta f = f(\alpha_{\min}) - f(\alpha_{\max})$).

We find that the singularity strengths of the markets lie within the following ranges: for the 1929 crash $-0.10 \leq a \leq 0.94$, for the 1987 crash $-0.07 \leq \alpha \leq 0.90$ while for the non-crisis sample is $0.19 \leq \alpha \leq 0.38$ and for shuffled data $0.18 \leq \alpha_{1929} \leq 0.34$ and $0.16 \leq \alpha_{1987} \leq 0.43$ respectively.[2] It is interesting that $f(a) < 0$ for some alpha's values. Refs. [22–24] argue that negative dimensions arise from the spatial distribution which fluctuates from sample to sample. In other words, the spatial concentration distribution of the disperse phase fluctuates randomly momentarily and shows intrinsic randomness. The results indicate that the 1929 and 1987 stock market crashes correlations show strong degree of multifractality. Also Fig. 2 panels (c) and (d) show that the multifractal spectra $f(\alpha)$ are narrower for the non crisis and shuffled time series, from which all temporal correlations have been removed. But surprisingly, although the multifractal spectra of the shuffled data are reduced there is still evidence of multifractality.

Since removing time correlations the return series still show some multifractality, it is interesting to extent the analysis and test if heavy-tailed distribution of the returns series has a crucial impact on the singularity. Therefore, based on the

[2] The tails with negative $f(\alpha)$ usually are produced by rare events (Anderson transition) that do not occur in a typical (wave) function. In the region of negative $f(\alpha)$, the average multifractal spectrum is dominated by rare samples and their anomalously localized or delocalized states.

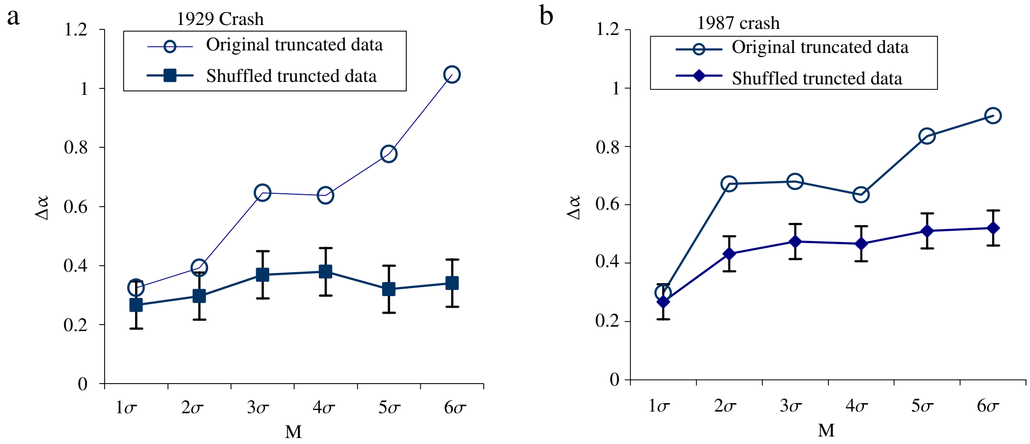

Fig. 3. Multifractality of time series where returns over the threshold level have been replaced for both truncated and shuffled truncated series.

Table 1
Degree of multifractality, $\Delta\alpha$ for major events.

Event	$\Delta\alpha_{or} - \Delta\alpha_{shuf}$	$\Delta\alpha_{after} - \Delta\alpha_{fore}$	$\Delta\alpha_{after} - \Delta\alpha_{shuf}$	$\Delta\alpha_{fore} - \Delta\alpha_{shuf}$
1929	0.35	**0.79**	0.55	0.03
1987	0.42	**0.45**	0.04	0.13
1899	0.38	**0.27**	0.18	0.08
1914	0.24	**0.07**	0.21	0.15
1997	0.31	**−0.03**	0.15	0.16
Avg. crash	0.33	**0.31**	0.24	0.12

Notes: *or* stands for original data, *shuf* for shuffled data, *after* is aftershock period and *fore* for foreshock.

fact that the shuffled series exhibit some singularity width we ask if the distribution of returns has any impact on the $f(\alpha)$ curve. We follow the work of Refs. [11,24,25] and test this hypothesis by constructing surrogate data. Basically, there are two methods of generating surrogate data. The first method, which is called truncated method, eliminates large returns, above a certain threshold value, and replaces them with random numbers, with values less than the threshold level, drawn from a normal distribution. The second method is the phase-randomization process where surrogates are created for testing the Gaussianity and one can eliminate any sort of nonlinearity [26–28].

In our case we apply the truncation method, by constructing the time series with the returns with magnitude greater than a threshold, M, to be replaced with returns of re-sampled randomly from the return series lower than the threshold level. Threshold M is set from 1σ to 6σ, and for each threshold value we generate a number of truncated data sets. The singularity width $\Delta\alpha_{\mathrm{trunc}}$, calculated from each spectrum, and its dependence on the threshold value M, is depicted in Fig. 3. Also we calculate the singularity width $\Delta\alpha_{shuf}$ of the shuffled data for each threshold value M, after we generate 20 shuffled truncated data sets for each point. For both crises the singularity width $\Delta\alpha$ increases as the threshold value M, increases and mostly the singularity width $\Delta\alpha_{trunk}$ is greater than the singularity width $\Delta\alpha_{shuf}$. The above analysis illustrates that large values in the stock market returns during crisis have significant impact on the width of multifractal $\Delta\alpha_{shuf}$ singularity and the temporal structure becomes a stronger factor on its multifractality.

As a next step we ask how stock market crashes influence the market complexity. In an attempt to give an answer and to determine (if there were any) changes in market complexity we divide the series into two equal periods, with each period comprised of 300 trading days. The period before the crash is named the foreshock period and the period after the crash, the aftershock period. Fig. 4 presents the estimate the q-dependence of the Hurst exponent h_q as well as the multifractal spectra $f(\alpha)$ of the aftershock and foreshock periods. We find that the richness in multifractality is associated with high variability of $h(q)$ and the degree of multifractality for both aftershock periods is significantly greater than the degree of multifractality for the foreshock periods. The broadness of the singularity spectrum, as a measure of the degree of mutifractality or complexity, signifies a transition from homogeneous to heterogeneous pattern. Hence, in Table 1 we measure the degree of multifractality as $\Delta\alpha$, for each crash, original and shuffled data and we compare the foreshock and aftershock periods. It seems that the 1929 crash display a greater transition from homogeneous to heterogeneous pattern exhibited by the larger change in α, 0.79 compared with 0.45 of the 1987 crash event. Tentatively a comparison of the singularity spectra of the two aftershock periods reveals a $\Delta\alpha = 0.24$ indicating the 1929 aftershock period impacted more by its main shock than the 1987 aftershock sequence.

Furthermore in Table 1, we analyze three more major crashes and namely the crash of 12/18/1899 where Dow Jones Index lost 12% of its value, the 12/14/1914 and 10/27/1997 crashes with DJIA loosing 21% and 7% respectively. Based on

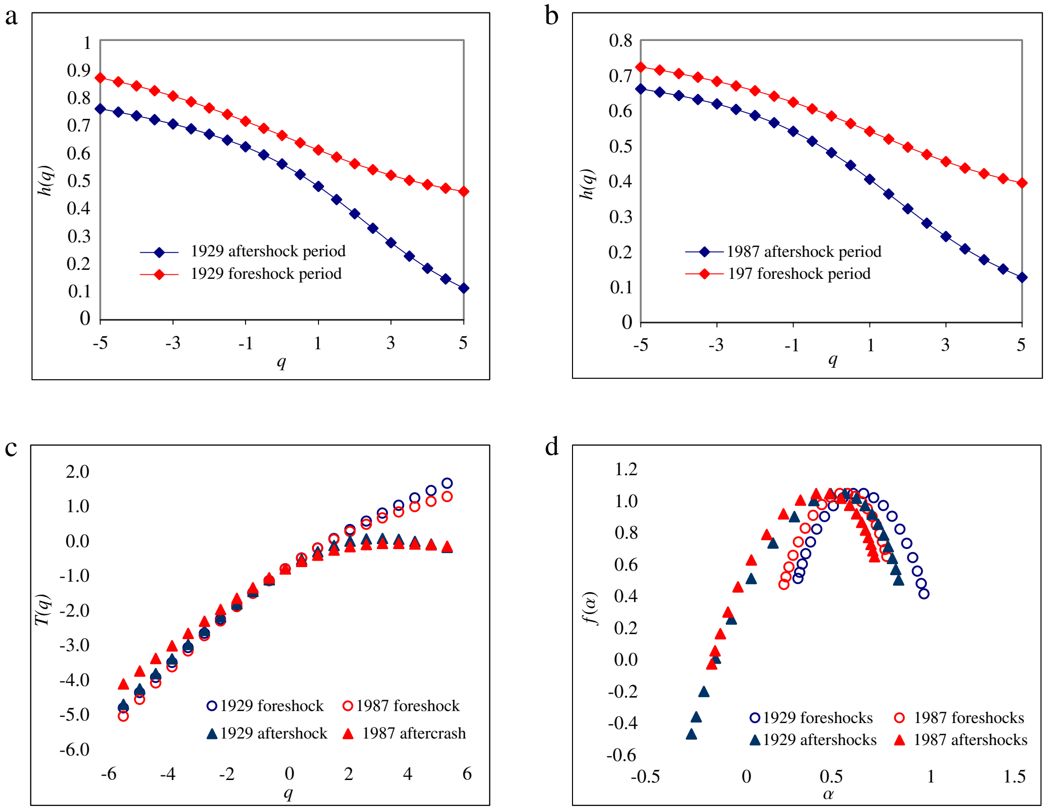

Fig. 4. Multifractal scaling exponents for foreshock and aftershock periods and their respected spectra.

these five major events we also calculate an average crash as the point of reference. The calculation is as follows.

$$\tau_{av}(q) = \frac{1}{5}\sum_{i=1}^{5}\tau_i(q). \tag{10}$$

Then the multifractal spectrum can be computed from the $\tau_{av}(q)$ vs. q for the average shock. As in Table 1, the average aftershock singularity spectrum is broader by 0.31 than the foreshock spectrum. Interesting enough is the 1997 event where the foreshock spectrum is marginally broader than the aftershock spectrum as $\Delta\alpha = -0.03$ indicating that the main shock was probably transitory and did not change the complexity of the market of that specific time period.

Last we investigate the level of correlation with the use of the generalized scaling Hurst Exponent, $H(q)$, where $q = 2$. If $H = 0.5$, the system displays "Markovian" behavior and there is no long-term correlation or memory. If $H < 0.5$, the system displays fractional Brownian motion and anti-correlation and finally, if $H > 0.5$, there is a positive long-term correlation or memory exist in the series.[3] From Table 2 and Fig. 5 both total sample events exhibit antipersistent behavior, $H(2) < 0.50$ with 1987 crisis to show more antipersistent behavior, $h(2) = 0.41(\pm 0.011)$ than the 1929 crisis, 0.44 ($\pm$0.025). The antipersistent behavior and deviations of one sign generally followed by deviations with the opposite sign is an indication of the high degree of market nervousness and uncertainty. In addition the value of $H(2)$ of the aftershocks events revealed a much greater tendency to be <0.5 and with greater level of anticorrelation compared to the foreshock periods. As for the foreshock periods the 1929 event exhibits persistence, with $H(2) = 0.56(\pm 0.025)$, indicating possibly a herding behavior where investors expecting sequential positive marks in the stock exchange a behavior that led to the great stock market bubble. In Table 2 and Fig. 5 all analyzed crashed events are presented.

For all crashes excluding the 1929 and 1987 (and including the average crash) the value of the generalized Hurst exponent fluctuates around ∼0.50 indicating Markovian behavior with no long-term correlation or memory. In other words, the analysis of these events give support to the Efficiency Market Hypothesis whereas the results of the 1929 and 1987 crashes exhibit antipersistence behavior therefore rejecting the efficient operation of the market. This is more evident in the aftershock periods where most of the aftershock generalized Hurst exponent is less than 0.50, meaning that the market did not absorb all available information efficiently or investors acted in a state of panic.

3 We test the detrended series for stationarity by employing the Phillips–Perron test for unit root. All series are stationary at 1%.

Table 2
The generalized Hurst exponent ($q = 2$).

Event	Total sample	Foreshock	Aftershock
1929	0.44 (0.025)	0.56 (0.025)	0.38 (0.04)
1987	0.41 (0.011)	0.50 (0.04)	0.32 (0.02)
1899	0.52 (0.038)	0.50 (0.036)	0.53 (0.038)
1914	0.52 (0.07)	0.52 (0.073)	0.51 (0.038)
1997	0.48 (0.01)	0.44 (0.04)	0.47 (0.02)
Avg. crash	0.48 (0.04)	0.51 (0.04)	0.44 (0.03)

Note. Standard errors are in parenthesis.

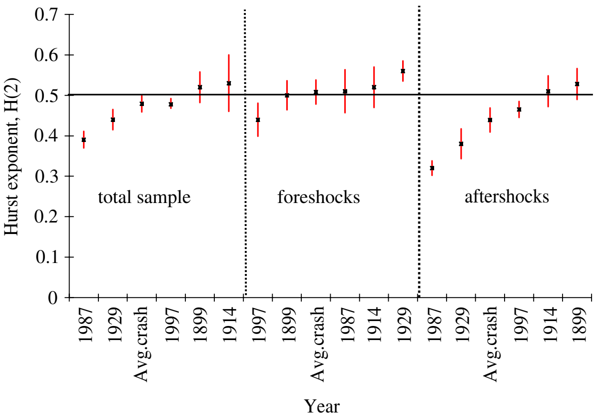

Fig. 5. The generalized Hurst exponent $H(2)$ for major crashes of the DJIA. Total sample, foreshock and aftershock periods. Notes: Red lines are $\pm$ standard errors. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

4. Conclusion

We investigated the complexity of major stock market crashes. We have observed that temporal correlations are not linear and with the MF-DFA method we have revealed the existence of multifractality in both stock market crashes. Even the shuffled series, removing the time correlations, still exhibit some degree of multifractality.

In addition, in order to observe how the market crashes influenced the stock market complexity, we studied their multifractal properties before and after the crisis. We found that the eruptive event (crash) changes the multifractality, which is increased significantly after the crash. The increase of the multifractal degree in the aftershock period, measured by the width of the multifractal spectrum indicates a gain of heterogeneity, signifying a transition from homogeneous to heterogeneous pattern. A comparison of the two aftershock periods and the average crash as appoint of inference shows a greater degree of multifractality for the 1929 aftershock period, meaning a richer and more complex structure. Lastly based on the generalized Hurst exponent we present evidence of failure of the Efficient Market Hypothesis for the two main crashes both for the total sample and especially for the aftershock periods where antipersistence behavior is presented.

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