PCA as a Tool for Analyzing the Market

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Modeling the Market

• We have to find a way to choose parameters for our models.

• Our parameters should be invariant in time.

• We should be able to reconstruct prices from invariants.
Invariants

• Stocks:
  • Price is not an invariant.
  • Logarithmic returns is.

• Bonds:
  • Price is not an invariant.
  • Yield to maturity is.
Invariants

- The difference between the invariants and prices:
Invariants

• Logarithmic (compounded) returns for a stock:

\[ C_{t,\tau} \equiv \ln \left( \frac{P_t}{P_{t-\tau}} \right) . \]

• Yield to maturity for bonds:

\[ Y_t^{(u)} \equiv -\frac{1}{u} \ln \left( Z_t^{(t+u)} \right) . \]
Dimension Reduction

• In general our invariants can be expressed as a single multivariate parameter:

\[ X_{T \times K}(Y, C) \]

• Price will be a function of this parameter:

\[ P_{T \times K} = f(X_{T \times K}) \]
Dimension Reduction

• Usually the dimension of this parameter is larger than the actual degree of randomness in our market.

\[ K > \text{Number of independent dimensions} \]

• Reasons:
  
  • Derivatives.
  
  • Hidden dependencies.
Dimension Reduction

• One method of reducing dimensions is Principal Component Analysis (PCA).

• We assume our parameter is a combination of some common factors and small perturbations:

\[ X \equiv q + BF (X) + U. \]
Principal Component Analysis

• Construct the covariance matrix:

\[ C_{K \times K} = \frac{X'X}{T} \]

• Find the eigenvalues and eigenvectors of the covariance matrix.
Principal Component Analysis

• The eigenvectors are the principal axes of the location-dispersion ellipsoid.

• They represent the directions of most variance.

• The dimension of the ellipsoid can be reduced depending on the eigenvalues.
Principal Component Analysis
An Example: Stock Market

- 26 stocks from the Dow-Jones Index.
- Daily close prices from: 1/1/1990 to: 4/20/2015
- Use R-squared for measuring the explanatory power of the first N eigenvalues:

\[
R^2 = \frac{\sum_{i=1}^{N} \lambda_i}{\sum_{i=1}^{K} \lambda_i}
\]
An Example: Stock Market

A look at eigenvalues

![Graph showing eigenvalues and R-squared values.](image)
An Example: Stock Market

A look at eigenvectors

Bar plot of the first eigenvector

Bar plot of the third eigenvector
Another Example: Bond Market


• Maturities: 1 Mo, 3 Mo, 6 Mo, 1 yr, 2 yr, 3 yr, 5 yr, 7 yr, 10 yr, 20 yr, 30 yr.

• Using the same R-squared as stock market.
Another Example: Bond Market

- The covariance matrix has some nice properties:
  - Smooth in both of its arguments.
  - Nearly constant diagonal terms.
  - Only depends on one parameter.
  - Infinite dimensional case -> Toeplitz Operator.
Another Example: Bond Market

- **Eigenvectors**: The graphs show the distribution of eigenvectors across different frequencies and time to maturity.

- **Eigendensities**: The distribution of eigendensities is visualized across a range of frequencies.

- **R-Square**: The r-square values are illustrated across the frequency cut-off range.

- **Time to Maturity**: The graphs also depict the relationship between time to maturity and various parameters such as $\omega$. For instance, $\omega = 0.2$ and $\omega = 0.1$ are highlighted.
Another Example: Bond Market

Representation of covariance matrix of stocks

Representation of covariance matrix of bonds
Another Example: Bond Market

A look at eigenvalues
Another Example: Bond Market

A look at eigenvectors:

First 4 eigenvectors
Another Example: Bond Market

- How about different countries?
- Monthly Spanish Government Bond yields
  - from: 10/1/2004 to: 4/1/2015
Another Example: Bond Market

A look at eigenvalues:
Another Example: Bond Market

A look at eigenvectors:

First 4 eigenvectors

![Graph showing eigenvectors with bond maturity rank on the x-axis and eigenvector on the y-axis.](image-url)
Another Example: Bond Market

• They look similar.

• Maybe we can look for specific years:
  • Before 2007.
  • After 2007.
Another Example: Bond Market

Eigenvalues for two different years:
Another Example: Bond Market

First eigenvectors for two different years:

First eigenvector in 2005

First eigenvector in 2008
Conclusions

• Stocks:
  • Dimension of the stock market is not as easily reducible as the bond market.
  • Gives valuable information about common factors, and correlation between different stocks (sectors?)
  • Might give better results for larger number of stocks.

• Bonds:
  • Number of important dimensions is usually less than 3.
  • Different markets and different years tend to give similar results.
  • Relation to the continuous case can make it easier to analyze.
Thank you!