A Study on the Foreign Exchange Market

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Outline

• Intro to foreign exchange
• Time series analysis
  – Exchange rate and correlation of return
  – Arbitrage of return and bid-ask spread
Introduction:
Foreign Exchange (Forex)

• Decentralized (as oppose to stock exchange)
• Trade 24 hours a day, 5 days a week
• High liquidity
• Spot market
• Future/forward market
Mechanism

- **USD/EUR:** how much EUR does 1 USD buy
- **Ask:** price to buy the base currency
- **Bid:** Selling price of the base currency
- **Bid-ask spread:** difference between bid and ask

**Example:**

\[
\text{USD/CAD} = 1.2000/05
\]

Bid = 1.2000
Ask = 1.2005
Spot market

• Exchange rate depends on supply and demand

• Factors:
  – Inflation rate: high inflation=>lower money value
  – Interest rate: high interest rate=>higher money value
  – Terms of trade: import > export => lower money value
Time series analysis I: return of exchange rate and correlation
Return of exchange rate

JPY/CNY

JPY/HKD
Time series of log return of JPY/EUR (black) and JPY/USD (red)

$R =$ Log return plot

Volatility $= R^2$
2-d scatter plot could give us more information of how two currency correlate

Four categories

C and G shows more fuzziness
EUR shows some uncertain correlation
H and J shows “Discreteness “ along with correlation

2-d scatter plot can tell us more about how two currency correlate via a third currency
Most stable one if USD
Time series analysis II: arbitrage of return and bid ask spread
The triangle arbitrage

At particular moment, if \((\text{USD/EUR})_b \times (\text{EUR/GBP})_b \times (\text{GBP/USD})_b \neq 1\), an arbitrage opportunity exist
The triangle arbitrage

• Our data: daily average exchange rate
• Not high frequency data (of order ms→s)
• Arbitrage does exist, but dissipates very quickly, of order of 10s (Danial Fenn, University of Oxford)
• Look at ‘arbitrage’ of return instead:

\[
\text{Arbitrage} = R(\text{USD/EUR})_t \times R(\text{EUR/GBP})_t - R(\text{USD/GBP})_t
\]

Where \(R(x)_t = \ln(x(t+1)) - \ln(x(t))\)
Time series of (return) arbitrage

- notation: USD=U, EUR=E, GBP=G

$$UEG = R(USD/EUR)_t \times R(EUR/GBP)_t - R(USD/GBP)_t$$
Time series of (return) arbitrage

- Periodic feature (period = 7 days/1 week)
- Mean of arbitrage $\sim 10^{-8} \rightarrow 10^{-7} \sim 0$
- Unexpected spikes
Time series of bid ask spread (BAS)

- Bid price – ask price ≤ 0 (always)
- Also exhibit period behavior (period = 7 days)
- Spread is larger in weekends
Arbitrage and BAS

• Comparing the time series of BAS (of UE, EG, UG) and arbitrage of UEG

• Similar pattern: periodic, some spikes in the BAS time series coincide with those in the arbitrage time series
Autocorrelation of arbitrage

Negative autocorrelation at lag-1 (day)
Autoregressive (AR) model

• Negative lag-1 autocorrelation of arbitrage:
  \[ R(t+1) - R(t) = -\alpha R(t) + \beta \varepsilon(t) \leftarrow \text{noise term} \]
  \[ R(t+1) = (1-\alpha)R(t) + \beta \varepsilon(t) \leftarrow \text{AR(1)} \]

• Can explain the dynamics of the arbitrage time series only in a short observation window (several days)
AR(1) model

• Can only explain data up to lag 1
• Still cannot explain the periodic behavior of the arbitrage time series and the existence of sudden spikes
• Due to the similarities between the BAS time series and the arbitrage time series, we attempt to model these other features using the BAS time series
BAS vs arbitrage

• Both are periodic with period of 1 week
• Both deviate from zero significantly around weekends
BAS vs arbitrage

- some huge spikes (>0.005) in the arbitrage coincide with those in BAS. The reverse are seen not to be necessarily true
‘return’/change of BAS vs arbitrage (UEG)

• We now look at the change (return) of BAS
• Both has a periodic behavior
• Micro-profile (weekend oscillations) are also quite similar
‘return’/change of BAS vs arbitrage(UEG)

- The macro-profile (spikes) are also similar, but magnitude does not have a right scale
Correlation of $\Delta$BAS and arbitrage (UEG)

- Correlation are strong for UE and EG, not so for UG (2007-2015)
ΔBAS vs arbitrage (UCG)

- \( R(USD/CNY) \times R(CNY/GBP) - R(USD/GBP) \)
- \( ΔBAS(UC) \)
- \( ΔBAS(CG) \)
- \( ΔBAS(UG) \)
Discussion

• Possible that both ΔBAS and arbitrage are driven by the same underlying factor(s) (e.g. trading volume) which might affect both time series in similar fashion

• Underlying factors that affect the arbitrage might affect the BAS with a different magnitude

• The resolution of our data (daily average) covers up features which might otherwise be visible in high frequency data
Conclusion

• Both arbitrage and BAS deviate from zero largely during weekends
• Macro-profile (huge spikes) are seen to be related in both arbitrage and BAS time series
• Micro-profile (weekly oscillation) are also seen to be related in the two time series
• Other factors are also needed to be consider to understand the arbitrage time series
Reference

• Investopedia

• *Network communities and the Foreign exchange market* - Daniel Fenn, University of Oxford