Fractal Market Hypothesis

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Efficient Market Hypothesis (EMH) – Fama, Fisher, Jensen, and Roll, 1969

- Weak form: prices incorporate all past information.
- Semi-strong form: prices incorporate all past and publicly available information.
- Strong form: prices incorporates also private information.

Implication → prices are martingales (fair game), i.e. the best estimate of future prices is the current price. It is not possible to gain excess returns.

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What does martingale means?

- EMH implies that price changes are a random walk and the best estimate of the future price is the current one. This process is called a 'martingale' or a fair game.
- Bachelier (1990) firstly proposed that markets follow a random walk and that it can be modeled by standard probability calculus.
- If prices are a Random Walk then returns have to be IID → Gaussian distribution.
- If market returns are normally distributed (white noise), then they are the same at all investment horizons → no difference between speculation and investments.

Failure of Gaussian Hypothesis (1)

- The most stringent requirement is that the observations have to be independent or at least with very short memory. Empirical evidence shows that this is not the case, i.e. we observe long memory processes in price changes.
- Empirical evidence shows that returns are not normally distributed.
- If all information had the same impact on all investors , there would be no liquidity. When they received information, all investors would be executing the same trade, trying to get the same price. However investors are not homogenous. → different risk profile, time horizons and behavioral biases.
- The very source of liquidity is investors hetergeneity, i.e. different time horizons, different information sets, and obviosuly different concepts of a 'fair price'.

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Failure of Gaussian Hypothesis (1)

The fat tails are often evidence of a long memory system generated by a nonlinear stochastic process

What does this mean? The risk of of extreme events is much higher the the normal distribution implies.

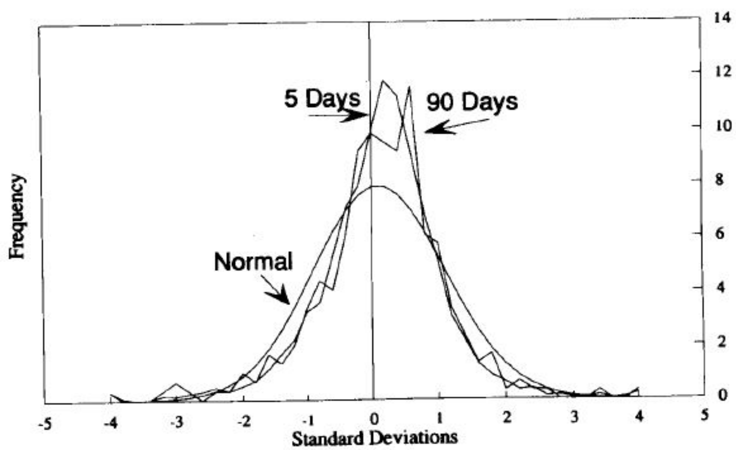

FIGURE 2.1 Dow Jones Industrials, frequency distribution of returns: 1888–1991.

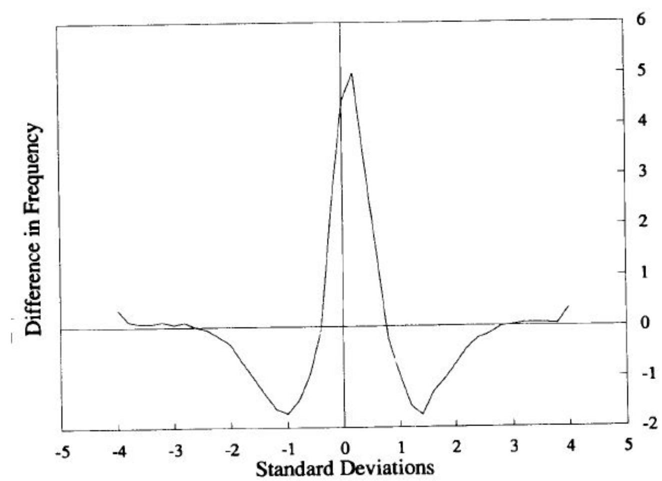

FIGURE 2.4a Dow Jones Industrials, 1-day returns – normal frequency.

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Failure of Gaussian Hypothesis (2)

- Another empirical evidence against the Gaussian hypothesis is related to the term structure of volatility.
- According to Einstein's 1905 observation, the distance that a particle in Brownian motion covers increases with the square root of time used to measure it.
- So, in order to annualize the standard deviation of daily returns we have to multiply it by the square root of 365 (or 360 or 250 depending on the model).
- Empirical evidence shows that volatility scales at a faster rate than the square root of time.

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Failure of Gaussian Hypothesis (2)

- Historical data on DOW show that volatility scale:
- Faster than the square root of time for horizons < 1000 trading days.
- Slower than the square root of time for horizons > 1000 days.
- Short term investors face higher risks with respect to long term investors. ◊ returns do not follow a white noise.

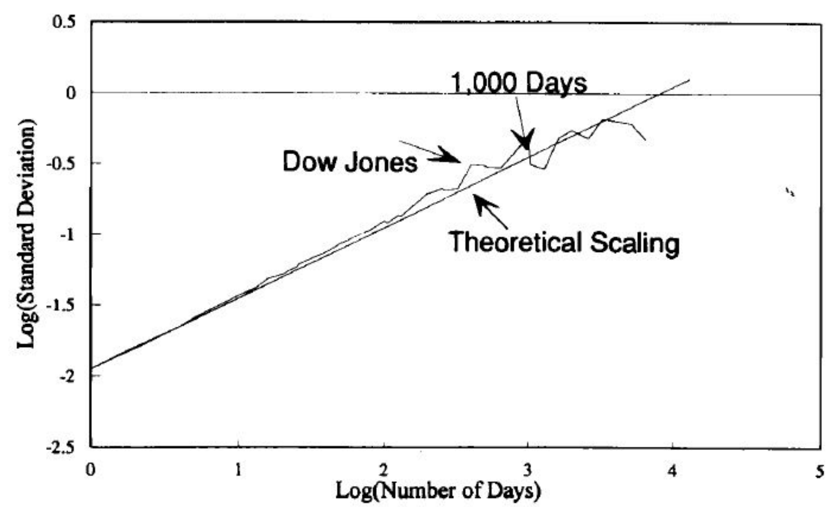

FIGURE 2.7 Dow Jones Industrials, volatility term structure: 1888–1990.

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The actual transition in Economics

	Gaussian	Non-Gaussian (Lévy?)
Rationality	EMH MPT (as CAPM) Neoclassical Economics	
Behavioral Biases		FMH Behavioral Economics

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Fractal Market Hypothesis

What is a fractal?

- Property one: self similarity
- Property two: power law scaling feature (the fractal dimension)

$e \uparrow x \quad VS \quad a \uparrow x$

For physical (or geometric) fractals, this scaling law takes place in space. A fractal time series scales statistically, in time.

The fractal dimension of a time series measures how jagged the time series is. Eg. a straight line has a fractal dimension of 1; a random time series has a fractal dimension of 1.5 ◊ Gaussian time series. Values far from 1.5 signal departures from the Gaussian distribution.

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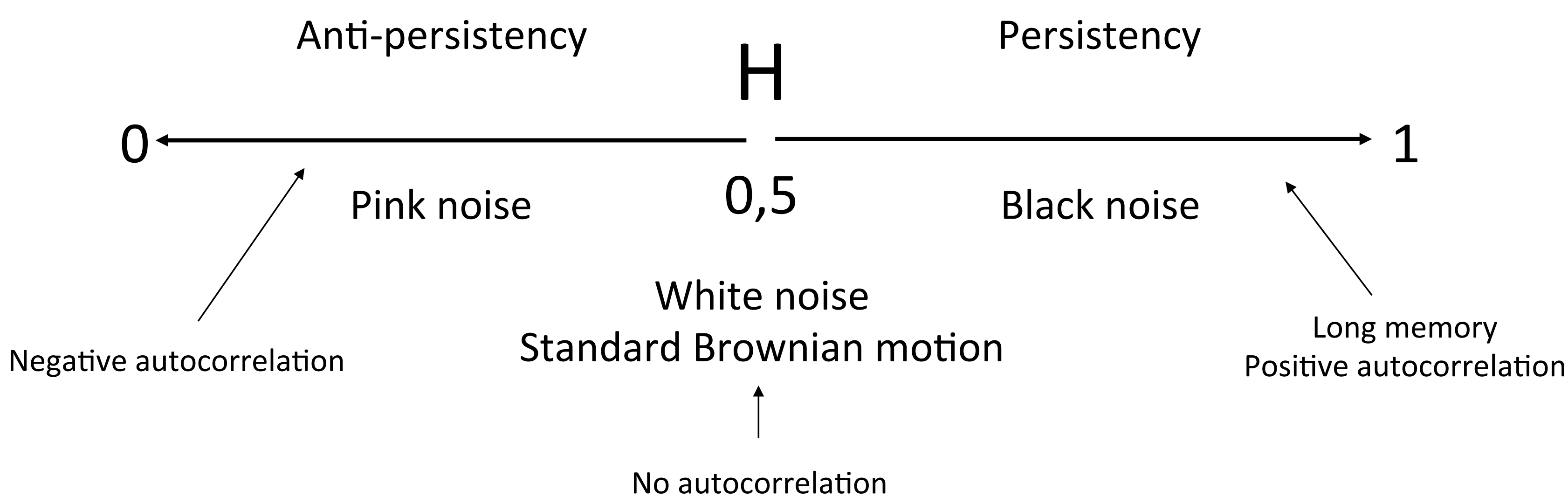
Fractional Brownian Motion

FBM is a generalization of the Gaussian Brownian Motion for different values of H (Hurst exponent)



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Lévy Distributions

Stable distributions have 4 parameters:

- Gamma: location parameter. Some distributions can have mean different than zero. In most cases the distribution under study is normalized and gamma = 0, that is, the mean of the distribution is set to zero.
- C: Scale parameter. It is the measure of dispersion, i.e. it sets the units by which the distribution is expanded or compressed around Gamma.
- Beta: Skewness parameter. When beta = 0 the distribution is symmetrical around Gamma. For beta<(>)0 the distribution is negatively (positively) skewed.
- Alpha: Characteristic exponent. It determines the peackedness at Gamma and the fatness of the tails. It can take values between 0 and 2. for alpha=2 the distribution is normal with cariance = 2*C^2. For 1<alpha<2 the distribution has infinite or undefined second moment (variance) and finite first moment (mean). For alpha<1 also the mean is infinite.