Goals:

- Explain the abundance of Pareto law in economics and other branches of science and humanities.
- A man should look for what **is,** and not for what he thinks **should be** (A. Einstein)
- Investigate how few simple assumptions give rise to various statistical patterns in socio-economics systems.

The grand aim of all science is to cover the greatest number of empirical facts by logical deduction from the smallest number of hypotheses or axioms (Einstein)

- Learn how to simulate models of socio-economic phenomena on the computer
- Build a comprehensive theory of firm growth.

PART II: "THE PROBLEM OF COMPANY GROWTH"

Question: Are there "laws" quantifying how companies grow/shrink?
Answer: Economists know much, dating back to Gibrat (1930's)

Take home message

- P(growth rate) Laplace in Center: universal
- Width decreases as -1/6 power of size bin
- P(growth rate) crosses over to power law in wings
- No theory for -1/6 power law for width
- Theory (Buldyrev et al) for growth rate power law

Collaborators: Salinger, Buldyrev, Canning, Havlin, Amaral, Fu, Pammolli Yamasaki, Matia, Ponta, Riccaboni (also: Jeffrey Sachs!!!!!!)

Classical Problem of the Firm GrowthGrGrowGrowth

Firm at
time = 1

S = 5

Firm at
time = 2

S = 12

Firm at
time = 10

S = 33

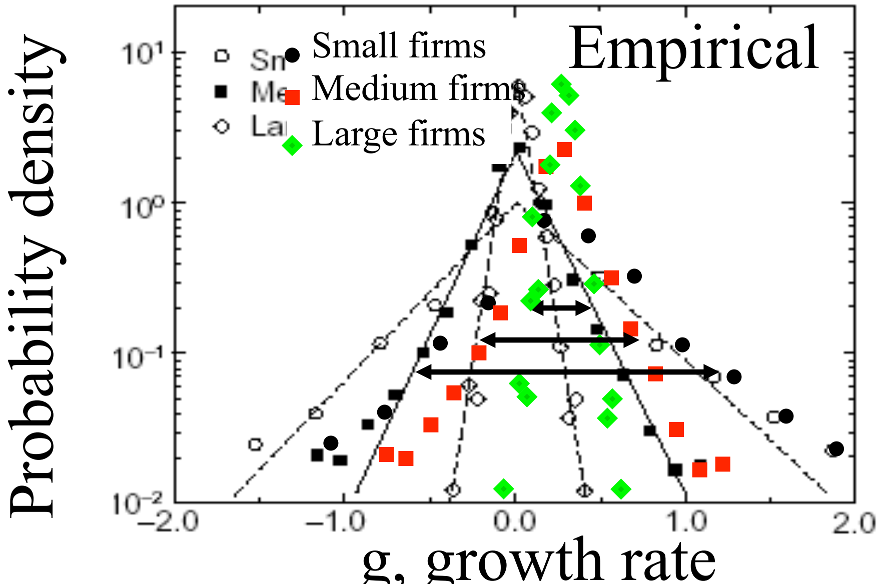
Firm growth rate **g**:

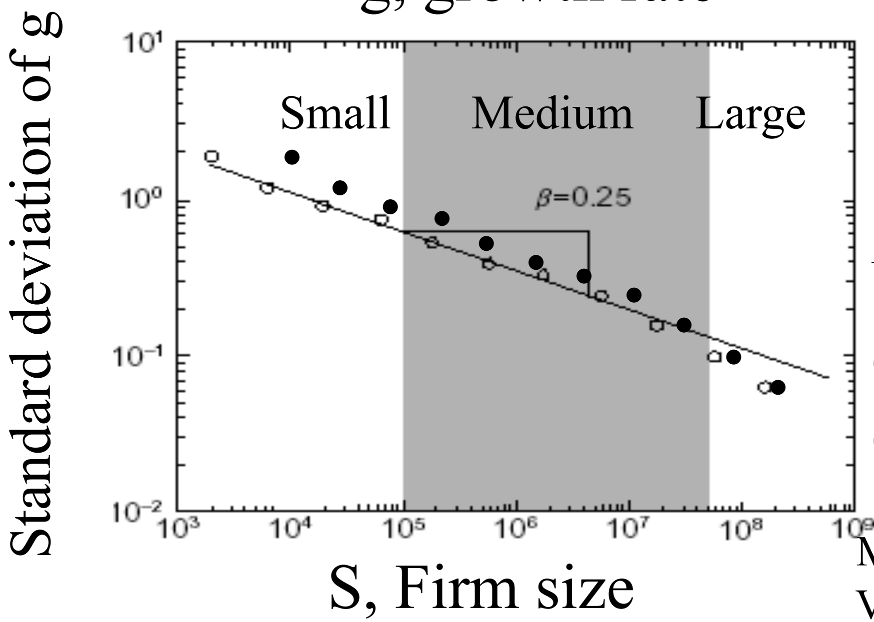
$$g \equiv \log \frac{S(t+1)}{S(t)} = \log\left(\frac{12}{5}\right)$$

t/year

1 2 10

(surprising) Empirical Observations (before 1999)





Reality: it is "tent-shaped"!

$$\mathrm{pdf}(g|S) \sim e^{-\frac{|g|}{\sigma(S)}}$$

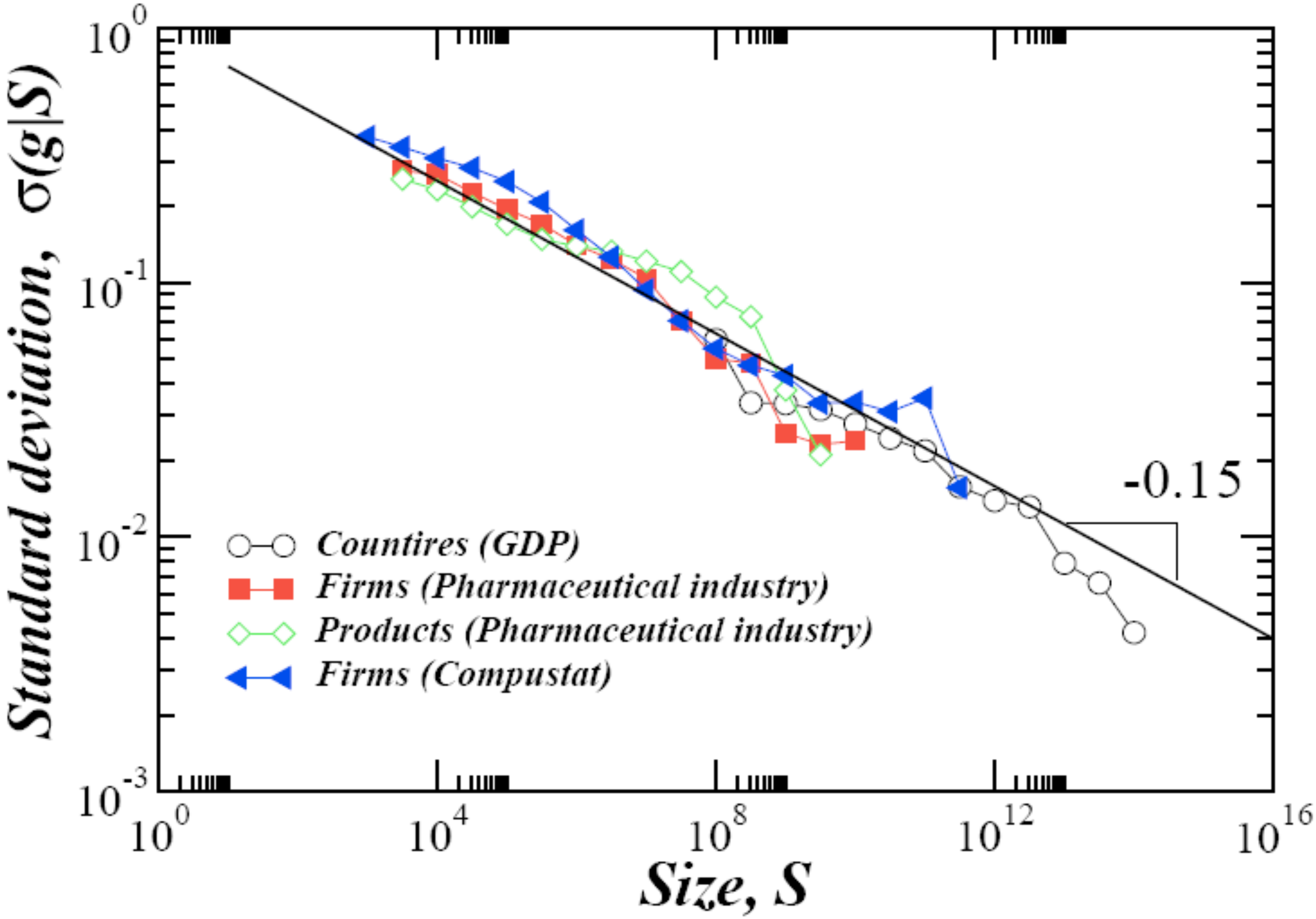
[[NOT log-normal (Gibrat theory)]]

$$\sigma_g(S) \sim S^{-\beta}, \quad \beta \approx 0.2$$

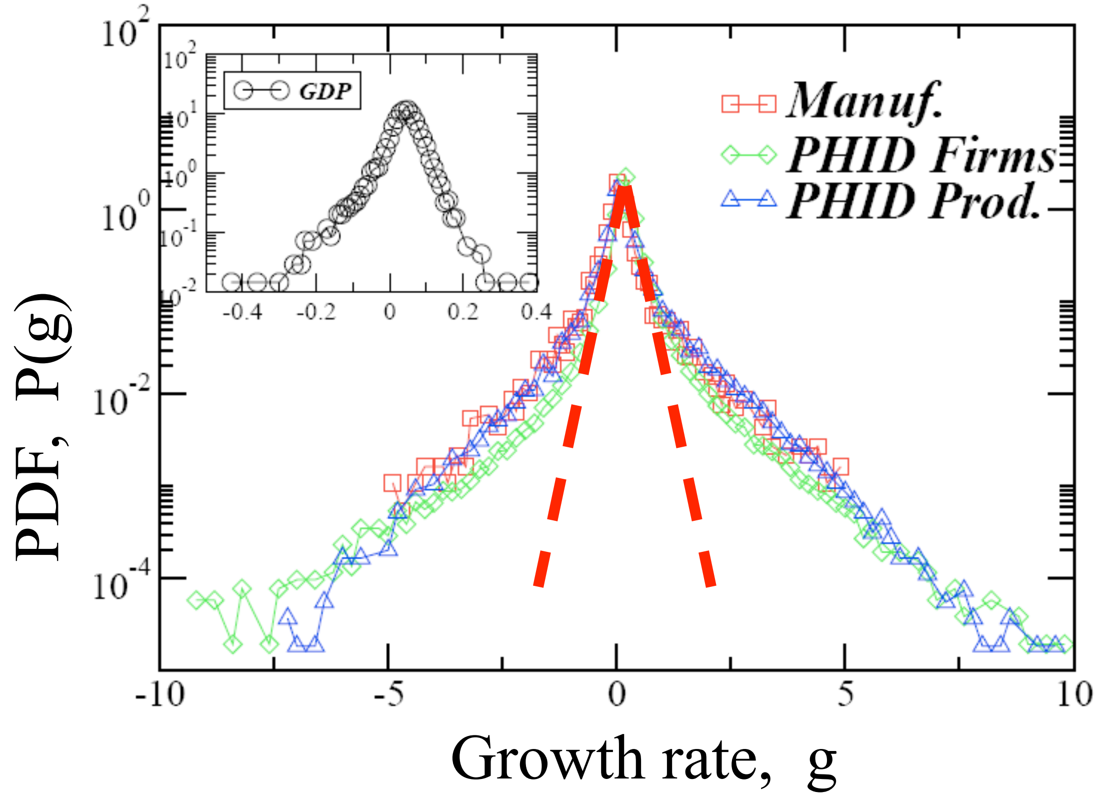
Universal for different economies (Takayasu) and organizations (university budgets, bird populations)

Michael H. R.Stanley, et.al. Nature 379, 804-806 (1996).
V. Plerou, et.al. Nature 400, 433-437 (1999).

Size-Variance Relation: Universality



Empirical Findings: P(g)



Laplace fitting function:

$$P(g) \sim \exp(-|g-\bar{g}|/\sigma)$$

[Nature 379, (1996)]

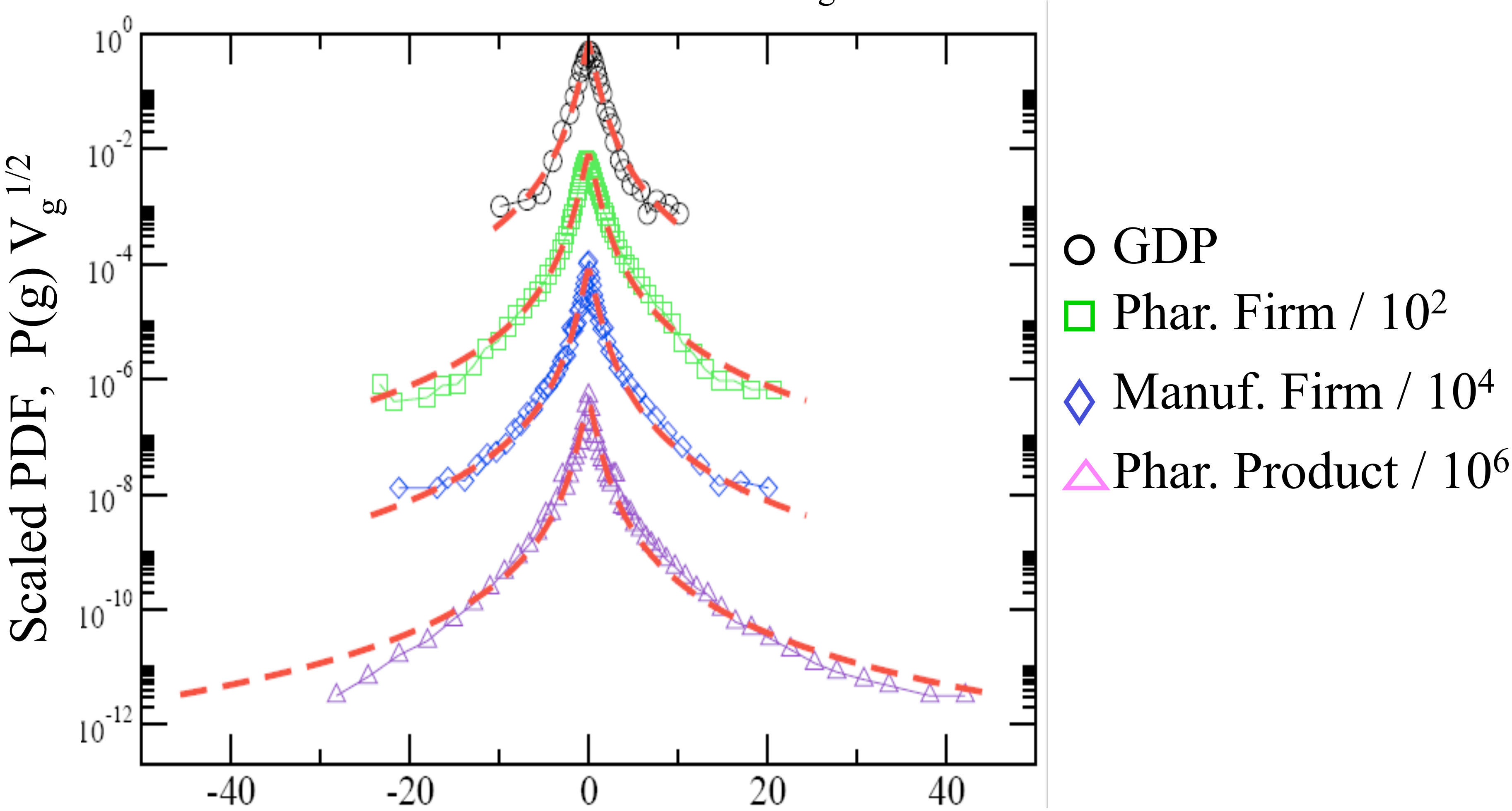
Our question: What is the function P(g), the PDF of growth rate?

Answer: **Not** Gaussian, [Gibrat (1930)].

Not Laplace, [M.H.R. Stanley, et al (1996)].

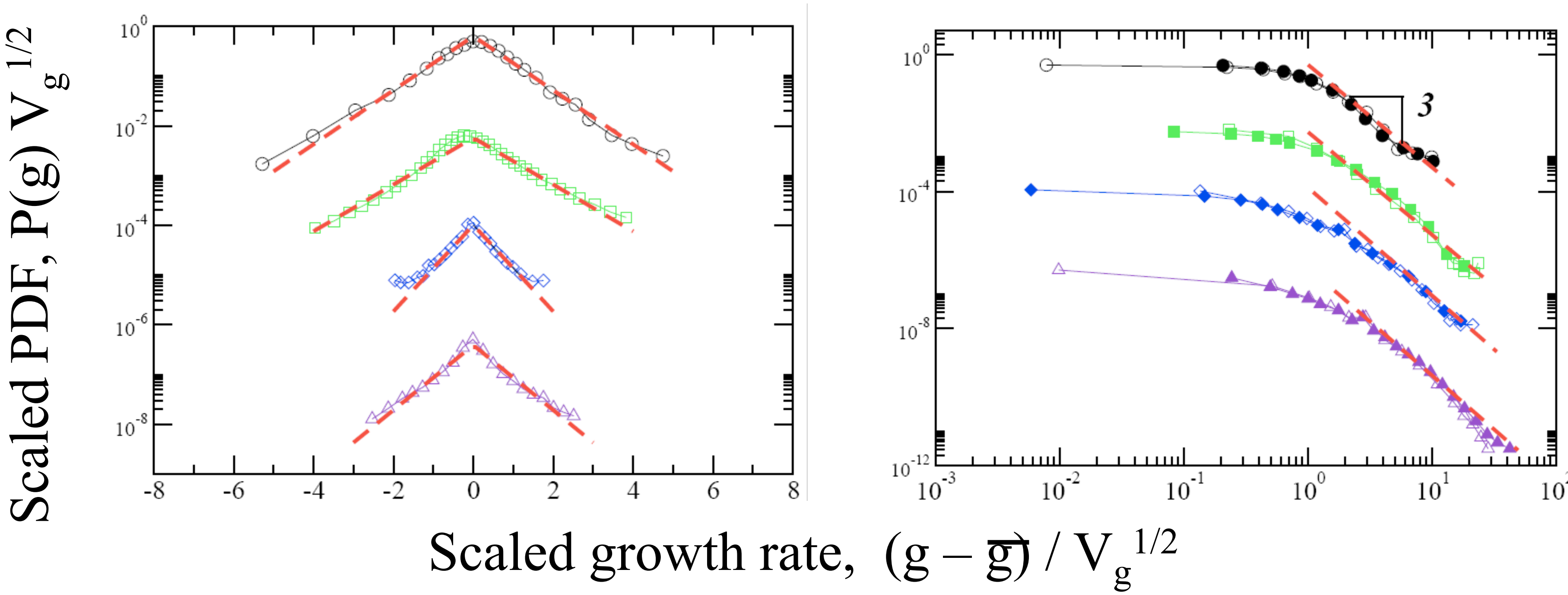
Test variance scaling with Empirical Data

ONLY **One** Parameter: V_g



Scaled growth rate, $(g – g) / V_g^{1/2}$

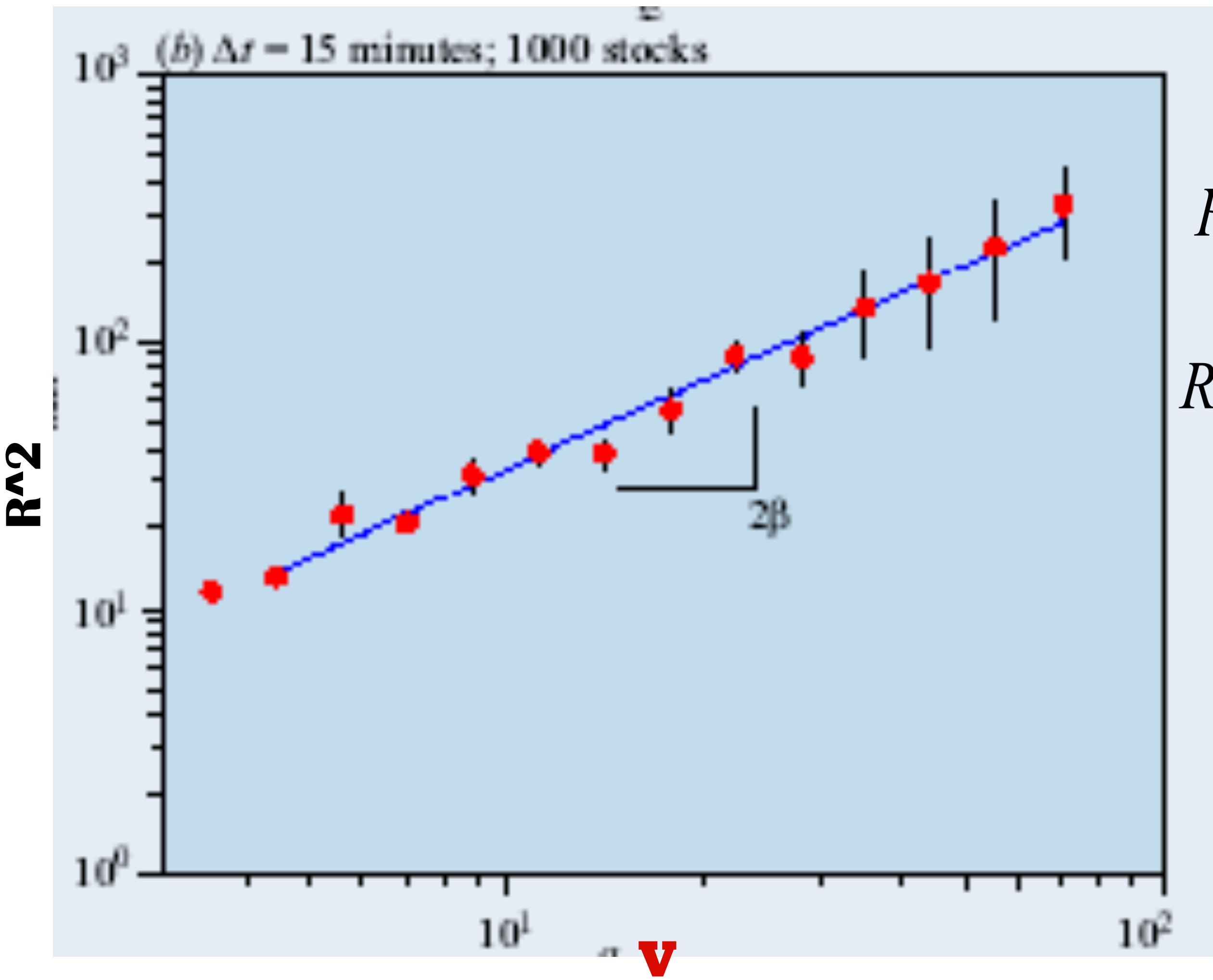
The Test of Central & Tail Parts of P(g)



Central part is Laplace.

Tail part is power-law with exponent -3.

Returns & Volume: Square-root form of Price Impact



$$R^2 \sim V^{2\beta} \qquad 2\beta \approx 1$$

$$R^2 \sim V \;\Rightarrow |R| \sim \sqrt{V}$$

$$\Rightarrow \zeta_R = 2\zeta_V$$

"Volume drives price"

Summary of results

- Distribution of returns consistent with a power-law functional form

$$P(R > x) \sim x^{-\zeta_R} \qquad \zeta_R \approx 3$$

- Two more power-law relationships for market activity (N) and the volume traded:

$$P(N > x) \sim x^{-\zeta_N} \qquad \zeta_N \approx 3$$

$$P(V > x) \sim x^{-\zeta_V} \qquad \zeta_V \approx \frac{3}{2}$$

- Square-root form of price impact

$$R^2 \sim V \Rightarrow |R| \sim \sqrt{V} \qquad \zeta_R = 2\zeta_V$$

- Power-law tails of returns arise from volume
- Long-memory of |R| arises from N