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Dissertation

DYNAMIC MODELING OF SYSTEMIC RISK IN FINANCIAL NETWORKS

by

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For Jess.
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ABSTRACT

Modern financial networks are complicated structures that can contain multiple types of nodes and connections between those nodes. Banks, governments and even individual people weave into an intricate network of debt, risk correlations and many other forms of interconnectedness. We explore multiple types of financial network models with a focus on understanding the dynamics and causes of cascading failures in such systems. In particular, we apply real-world data from multiple sources to these models to better understand real-world financial networks. We use the results of the Federal Reserve “Banking Organization Systemic Risk Report” (FR Y-15), which surveys the largest US banks on their level of interconnectedness, to find relationships between various measures of network connectivity and systemic risk in the US financial sector. This network model is then stress-tested under a number of scenarios to determine systemic risks inherent in the various network structures. We also use detailed historical balance sheet data from the Venezuelan banking system to build a bipartite network model and find relationships between the changing network structure over time and the response of the system to various shocks. We find that the relationship between interconnectedness and systemic risk is highly dependent on the system and model but that it is always a significant one. These models are useful tools that add value to regulators in creating new measurements of systemic risk in financial networks. These models could be used as macroprudential tools for monitoring the health of the entire banking system as a whole rather than only of individual banks.
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<td>Basel Committee on Banking Supervision</td>
</tr>
<tr>
<td>BIS</td>
<td>Bank of International Settlements</td>
</tr>
<tr>
<td>DBNM-BA</td>
<td>Dynamical Bank-Asset Bipartite Network Model</td>
</tr>
<tr>
<td>FDIC</td>
<td>Federal Deposit Insurance Corporation</td>
</tr>
<tr>
<td>HHI</td>
<td>Herfindahl-Hirschman Index</td>
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<td>PCV</td>
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<td>SUDEBAN</td>
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Chapter 1

Introduction

1.1 Motivation of Thesis

Since the financial crisis in 2008, the uncovering of the multitude of causes behind the crisis have made it evident that the standard microprudential methods and metrics for measuring and monitoring risk in the financial sector, while still necessary, are no longer sufficient for properly managing that risk, both internally within the financial institutions themselves and globally by the regulators whose job it is to prevent another crisis. The complexity of the modern financial industry requires newer macroprudential methods to be used in conjunction with the traditional methods of risk management. Such methods which look not just at microprudential (firm-level) oversight, but also incorporate macroprudential measures aimed at regulating risk at the system-wide level are the future of financial risk management, but such methods have yet to be incorporated in the framework of financial regulation in a relevant way. Macroprudential concepts like interconnectedness and centrality in financial networks will become ever more relevant in modern financial regulation. These newer methods will need much testing and calibration before they can be rigorously incorporated into standard financial regulation, but early models and tests can still tell us much about the fragility of the financial system, even in the post-2008 crisis world.
1.2 Background

1.2.1 The Banking Industry

In the global economy, bank holding companies and investment banks make up the majority of the world’s largest companies as measured by asset value. Many of the largest companies that aren’t banks still participate heavily in the financial sector. General Electric and Coca-Cola, for example, are some of the largest issuers of commercial paper worldwide [2]. Furthermore, these banks are bigger than ever and the wealth of the economy is concentrated in ever fewer corporations at the top of the ladder. The Glass-Steagall Act once served to keep commercial banks and securities firms separate but with its drawn-out reinterpretation culminating in its eventual repeal with the Gramm-Leach-Bliley Act in 1999, banks like Bank of America, Citigroup and JPMorgan Chase have become “super banks” able to participate in every aspect of the financial sector. While the motivation for these changes was that the consolidation of the banking industry would result reducing systemic risk, many believe the converse to be true [3]. While individual bank portfolios are indeed more diverse than ever before, the portfolios of these ever-growing banks have become more and more alike one another thus increasing their correlation to market movements and increasing systemic risk. To put it plainly, the major banks are all “in the same game”, so that if there were ever to be a large tail-risk event (such as the burst of the housing bubble that precipitated the financial crisis of 2008) that could affect one major bank, it would almost certainly affect nearly every other major bank as well.

These large companies, particularly the banks, also tend to be very highly leveraged. That is, they have a high ratio of assets \((A)\) to equity\((E)\). Equity is the book value of a company, equal to assets minus liabilities \((L)\), so a high leverage ratio \((\ell)\) means that the assets and liabilities are close enough such that the equity is relatively small.

\[
A = L + E \quad (1.1)
\]

\[
\ell = \frac{A}{E} = \frac{A}{A - L} \quad (1.2)
\]
In the case of financial institutions, this indicates a lot of borrowing in order to make investments. A result of high leverage is that equity, and thus the risk of bankruptcy, becomes increasingly sensitive to movements in asset value. In the time leading up to the financial crisis of 2008, many banks saw leverage ratios at record highs [3]. Leverage ratios may be an indicator of the health of a given financial institution or even of the economy as a whole. Under the capital requirements of Basel III, the third iteration of the global, self-regulatory capital requirements for banks, banks will have leverage maximums though they won’t be mandatory until 2018.

To further the matter, these banks all have large, and mostly unknown, credit exposures to one another [3]. To fully model their own risk and capital requirements, banks would need information they don’t have about who is doing business with their business partners. This situation has only been magnified by the existence of credit derivatives such as credit default swaps (CDSs) and other more exotic financial instruments. Credit derivatives play an important role in the modern financial system. When a bank wants to make a loan to, say, the Ford Motor Company, but isn’t willing to take on the risk exposure of the full principal amount, credit derivatives allow the bank to make that loan, critical to the operations of the Ford Motor Company, and to hedge the risk by sharing the risk and return with other parties through CDSs. However, the lack of proper regulation of these credit derivatives can lead to their abuse. In fact, a bank could make a loan that it suspects will default and take out insurance on that loan in the form of CDSs with a notional amount greater than the principal. Thus, if the the loan did default as suspected, the bank would make back more money than they loaned. While betting against customers is clearly unethical, its legality is murky at best. In 2007, Goldman Sachs, one of the world’s biggest investment banks and known to be one of the most ethically sound [3] sold collateralized debt obligations (CDOs), a type of credit derivative, and then bet against them when the market went bad, saving them money at the cost of their clients. Furthermore, in the case of CDSs, third parties not involved in the primary loan are free to speculate on likelihood of a loan defaulting, betting on or against it, by selling or buying a CDS on a loan on someone else’s balance sheet, which
is known as a naked CDS. Selling naked CDSs allow less risk averse financial institutions, like some hedge funds, to synthetically increase their leverage, generally without having to report the full notional amount. Naked CDSs account for about 80% of the CDS market, indicating that they are primarily being used for speculation, not as insurance against a primary loan [4].

These three factors: 1) the largest banks having similar portfolios with highly correlated returns, 2) high leverage ratios which increase sensitivity to market shifts and 3) a complicated and largely unknown network of credit exposures. Each of these three conditions on their own could be worrisome, but combined the can lead to “brushfire” conditions in which a large shock can quickly spread throughout the rest of the financial sector and then through the economy as a whole triggering defaults the like of which we haven’t seen since the Great Depression. These shock could be endogenous or exogenous. An exogenous shock, such as a wave of terrorist attacks in financial capitals, could be enough of a trigger to expose weaknesses in the complex network structure of the financial system. Or an endogenous shock, such as with the housing crisis of 2008 or from questionable practices of a single highly connected financial institution as in the case of Long-Term Capital Management in 1998, could be bubbling beneath the surface at any moment.

1.2.2 Complex Networks

Despite all the reforms and progress made, main monitoring standards still rest on the microprudential aspects and attend the strength of units of the system, leaving its systemic relationship as a simple consequence of the above. This is a weakness that remains a crucial issue that must be seriously addressed [5]. In this regard, a greater understanding of the externalities of economic and financial networks could help to design and adopt a framework of prudential financial supervision in such a way of considering both the actors of the system (financial institutions) and the vulnerabilities that emerge from their interdependence in network and thus try to improve investment and corporate governance decisions and mainly, help prevent crises or minimize their negative impacts.
Network science has greatly evolved in the 21st century, and is currently a leading scientific field in the description of complex systems, which affects every aspect of our daily life [6–10]. Famous examples include the findings about sexual partners [11], Internet and WWW [12, 13], epidemic spreading [14], immunization strategies [15], citation networks [16], structure of financial markets [17], social percolation and opinion dynamics [18, 19], dynamics of physiological networks [20], structure of mobile communication network [21], and many others. Among the phenomena that have been shown to fall in this conceptual framework are: cascading failures, blackouts, crashes, bubbles, crises, viral attacks and defense against them, introduction of new technologies, infrastructure, understanding measuring and predicting the emergence and evolution of networks and their stylized features, spreading phenomena and immunization strategies, as well as the stability and fragility of airline networks [9]. Current and past research has shown that in real life systems, there is a strong feedback between the micro states and macro states of the system. This description of nature can be well represented by network science – in which the micro is represented by the nodes of the network and the links between them, and the macro by the network itself, its topology, dynamics and function. Thus, network science, present and future, is the leading framework to investigate real life systems. For example, as opposed to physical systems where the dynamics is usually bottom-up, in social and economic systems there are interplays on all levels with singular top-down feedbacks. Thus, in many practical realizations, in addition to the bottom-up contagion propagation mechanisms one finds that there is a global-to-local feedback: individuals, their interdependence and behaviors build up the system that finally affects back on individuals’ choices. It has been proposed that the bottom/up – top/down feedback has the capability to change completely the character of a phase transition from continuous to discontinuous, thus explaining the severity of the economic crises in systems where the collective interacts as such with its own components [22].

Network theory provides the means to model the functional structure of different spheres of interest, and thus, understanding more accurately the functioning of the network of relationships between the actors of the system, its dynamics and the scope or degree of
influence. In addition, it measures systemic qualities, e.g., the robustness of the system to specific scenarios, or the impact of policy on system actions. The advantage offered by the network science approach is that instead of assuming the behavior of the agents of the system, it rises empirically from the relationships that they really hold; hence, the resulting structures are not biased by theoretical perspectives or normative approaches imposed ‘by the eye of the researcher’. On the contrary, the modeling by network theory could validate behavioral assumptions by economic theories and further, channeling the attention of policy instruments in quantity and quality highly focused. Network theory can be of interest to various edges of the financial world: the description of systemic structure, analysis and evaluation of the penetration or contagion effects [23–34]; studies that assess the impact of the insolvency of one or a particular group of actors in the system, depending on its relevance and connectivity within the structure [35, 36]; and those that allow to evaluate the impact of liquidity problems at specific times and initiated in different nodes of the system [37–42]. In a nutshell, it becomes not only an alternative perspective, but provides tools allowing to compare and to contrast the structure of the systems in a static way and project different dynamic scenarios.

In this sense, the payment system can be seen as an example of complex network, and thus, considered as a network, derive its stability, efficiency and resilience features (see for example [43]). Analytical frameworks for the study of these structures are varied, and range from the identification of the type and properties of the network, to the analysis of impact of simulated shocks, in order to quantify the risks inherent in its operations to some extent and design policy proposals to mitigate them. For example, once the payment system can be mapped as a network, such as the recently introduced funding map [43], then the structure of the network can be used as input for models that simulate the dynamics of the system [44].

Recent studies by [45], [46], [47], and [48], investigated the interbank payment system using network science. considering the system as a network, these authors were able to uncover the structure of the system and allowed the design of scenarios and the visualization
of specific effects. Meanwhile, [49] analyze the overnight money market. The authors developed networks with daily debt transactions and loans with the purpose of evaluating the topological transformation of the Italian system and its implications on systemic stability and efficiency of the interbank market.

Focusing on liquidity, [50], explore the properties of the network of global banking using information from bilateral loans from 184 countries and their quarterly direct investment flows. Coinciding with several papers on capital flows, they conclude that advanced economies are the major players in the global banking market with 10 times more flows between them than to developing or emerging countries, making up the core of the network with other countries in the periphery. After describing the topology of the network and evaluating its dynamics in the period 1978–2009, they found volatility in the network topological properties: the interconnection between nodes is unstable and connectivity tends to decrease during periods of crisis.

Considering the problem of contagion, [51] study how shocks can spread in the banking system when it is structured in the form of a network. [52] develop a measure that captures the importance of an institution, in term of its systemic relevance, in the propagation of a shock in the banking system. More recently, Acemoglu et al. [53–55] develop a model of a financial network through its liability structure (interbank loans) and conclude that complete networks guarantee efficiency and stability, but that when negative shocks are larger than a certain threshold, contagious prevails and so does the systemic instability. The critical issue remains identifying such a threshold, and calibrating such models with real data.

In summary, there are two main channels of risk contagion in the banking system, both of which we will explore. The first is direct interbank liability linkages between financial institutions. It has been studied, mostly theoretically, by [56], [57], [58], [30], and [59]. These studies focus on the dynamics of loss propagation via the complex network of direct counterparty exposures following an initial default. However, data on the exact nature of these obligations are generally not publicly available. The most common practice is
to take known data about given banks’ total obligations to other banks and any other available data and use that information as a constraint on the possible structure of the complete network of obligations and then make an estimation assuming maximum entropy. This procedure results in an obligation network where all unknown obligations contribute equally to the known total obligations for each bank [30]. However, the magnitude of the systematic error is not entirely clear because of this lack of data, consensus seems to be that the maximum entropy estimation underestimates contagion [60]. The second is contagion via changes in bank asset values, which has been studied less extensively and when studied, is more commonly through indirection correlation-based networks, rather than the more direct bipartite bank-asset network model we employ.

1.3 Organization of Thesis

The following chapters of this thesis are organized as below.

In Chapter 2, we study direct interbank network models of financial systems. We primarily apply the results of the “Banking Organization Systemic Risk Report” (FR Y-15) to study how the direct credit obligations between banks affect the risk if system failure. We also look at the effect of global shocks to asset values to the US banking system and changes to the structure of the bank network result in varying levels of systemic risk. We also look at how changing assumptions about the network structure affect the resiliency of the system.

In Chapter 3, we study bipartite bank-asset network models of financial systems. In particular, we apply our model to the Venezuelan banking system using historical balance sheet data. We study how external shocks to asset prices and liquidity parameters can lead to critical phenomena in the survival rates of banks in the system and how changing network structures over time can lead to large changes in this critical behavior.

In Chapter 4, we summarize the results of the previous chapters and propose new avenues of research and next steps in continuing this research.

In the appendices, we provide relevant code developed and used for some of the algo-
rithms described in the text.
Chapter 2

Direct Interbank Network Models

2.1 Stochastic Proportional Asset Network Model

We begin with a relatively simple stochastic model in which we study the dynamics of an economy as a function of an assumed underlying credit exposure network. In this model, the asset values of the vary stochastically. When the assets of a firm drop below its liabilities, it becomes insolvent and fails. The value lost in such a failure has to be eaten by other firms in the system. Doing so can cause those banks to fail, and so on results in cascading bankruptcies. Because the assets value stochastically, eventually all banks’ assets should dip sufficiently to cause failure. We measure the speed and degree to which the system fails as we change our assumptions in the model.

2.1.1 Data

We begin with basic balance sheet data from the largest 200 global companies as ranked by Forbes in 2010. The distribution of assets for these 200 companies is approximately log-normal as can be seen in Figure 2.1. There is information on which economic sectors each of these firms operate in, but this analysis does not use that data. Future work could use this information to create subnetworks linked by the financial firms. By including the largest 2000 companies we are including the vast majority of the major players in the global economy. Each company has a certain initial leverage ratio, which is determined by the
Using the 2010 Forbes Data, we see a strong positive correlation between asset size and leverage. The larger firms with more assets tend to be more highly leveraged. This relationship can be seen in Figure 2.2.

2.1.2 Model

Our model begins by taking the top 200 Forbes-ranked companies and varying their asset values stochastically. Whereas liabilities tend to be relatively fixed in value over time, some types of assets can vary a great deal depending on a host of market factors. When the asset value of a company drops below the liabilities, we trigger a bankruptcy. In this model we do not attempt to distinguish between short-term and long-term debt or account for any payment schedules. We also do not consider cash or cash equivalents on the asset side. It is assumed that there is a reasonable match between short-term debt and liquid assets such that only when the overall assets drop below the overall liabilities must the company default. When a company defaults, other companies connected to the defaulted company via the credit exposure network will then have their assets devalued based on the recovery rate and severity of default. It is assumed that all assets are sold off to repay as much of the debt as possible. We do not consider the time required for resolution of the bankruptcy and allow recovery under the default to happen instantly. The devaluing of assets associated with this recovery may trigger a bankruptcy in the connected companies or it may only bring them a step closer to bankruptcy. Either way the model continues as we track the number a size of bankruptcies over time as well as the distribution of leverage ratios across the economy.

Correlations in the stochastic movements

The health of any company is generally tied to the health of the economy overall. In particular, looking at the historical correlation of stock returns for various companies we find that the larger companies (often banks) tend to have larger correlations with market
Figure 2.1: The empirical distribution of assets among the 2000 largest public companies globally as ranked by Forbes in 2010. Of the largest 60 companies, General Electric is the only company not categorized as a financial company. However, GE is the largest issuer of commercial paper (short-term overnight corporate lending) in the world, including banks, accounting for approximately 3% of the market. Thus, we see that the largest companies should be the most connected in our credit exposure network.
Figure 2.2: The empirical distribution of leverage as a function of asset size among the 2000 largest public companies globally as ranked by Forbes in 2010 with power law fit.

\[
\exp(b) x^a
\]

Leverage

\[a = -0.619 \pm 0.017\]

\[b = -1.186 \pm 0.054\]
indices like the S&P 500. Thus as we stochastically vary the asset value of each company in the model, we consider both a component which moves with the overall market and an idiosyncratic component [and eventually a sector market movement - I’d like to evolve this model to include separate sectors that each have their own market movements which are themselves correlated to different degrees with the overall market, each of which will be closely tied to the financial sector which would be the hub of the network.]

\[ \log \left( \frac{A_{i,t+1}}{A_{i,t}} \right) = m_t c_i + \epsilon_{i,t}, \]  

(2.1)

where \( m_t \) is the market movement at time \( t \), drawn from a normal distribution, \( c_i \) is the correlation of company \( i \) with market movements and \( \epsilon_{i,t} \) is the idiosyncratic movement, also drawn from a normal distribution. For the purposes of this study, we limited our analysis to simulations with \( c_i = 1 \), in other words where asset value fluctuations are equal parts market movement and idiosyncratic. If the sector information were to be included, one could easily extend this analysis to include separate sector-based fluctuations as such:

\[ \log \left( \frac{A_{i,t+1}}{A_{i,t}} \right) = (m_t s_i + p_t) c_i + \epsilon_{i,t}, \]  

(2.2)

where \( p_t \) is the sector movement at time \( t \), drawn from a normal distribution, \( s_i \) is the correlation of the sector of company \( i \) with market movements and \( c_i \) is the correlation of company \( i \) with sector movements.

The credit exposure network

Whether two firms in the network are connected in the credit exposure network in determined by the total asset size of any two given companies, specifically the geometric mean of the asset values of any two given companies. If that value exceeds a fixed amount then we assume that they have credit exposure to one another. Given that the largest companies tend to be financial institutions, which are the most connected nodes in a credit obligation network, this assumption simulates that financial institutions will be the most connected in our network. We vary the threshold required for a connection a see how those assumptions
about the structure of the network change the risk of systemic failure. When the threshold is low, there will be more connections in the network, but at the same time the credit obligations will be more spread out. When the threshold is high, there will be fewer connections in the network, but at the same time the credit obligations will be more concentrated.

The credit exposure is a function of two elements: 1) The percentage of credit exposure that is endogenous to the entire system in the model and not companies outside the model and 2) The size (in terms of asset value) of the company at the other end of the link relative to other companies to which it is link. As asset values fluctuate, these links do not change. The initial structure of the network remains fixed for that given run of the model. The only changes to the network come when a company goes bankrupt and is thus removed from the network. We start with the assumption that our network of 2000 companies is the entire system and that all credit obligations and exposures are to other firms in the system.

When a bankruptcy occurs through the stochastic process, we have to sell off its assets to try to cover its liabilities to the other firms it is connected to in the network. Those assets will have to be sold at a discount and the unwinding process costs money as well. Resolving a bankruptcy requires significant legal fees that will eat into the payments made to those exposed to the failed bank. We simulate this through a recovery rate. When a bank fails, the assets will be worth a fraction of their value and we call that value the recovery rate. After applying this factor to the asset value, there will be a value lost which is the difference between the liabilities and the new asset value. This value lost is then spread out among the connected banks proportional to its size. So if a bank with $100M in liabilities fails and can only pay off $90M, then $10M is lost. Those losses are shared by connected banks in proportion to their size. So if all the connected banks have a total asset value of $1B, then a connected bank with an asset value of $100M (10%) will have its asset decrease by $1M (10% of the $10M lost.)

So while due to the stochastic nature of the model system failure is inevitable. The network structure affects the speed and severity with which this system failure will occur. If a stochastically-caused bankruptcy results in the cascading failure of neighboring firms in
the network, then the system will fail much more quickly than if we only rely on stochastic movements to cause failure.

2.1.3 Results

For a given simulation with random stochastic asset fluctuations, we measure the number of failed banks as a function of model runtime. The area under that curve, as seen in Figure 2.3, represents the severity of system failure for that simulation. For a given set of model parameters, we run the simulation 100 times and calculate the average severity of system failure (area under the bankruptcy-runtime curve).

![Graphs](a) Not severe failure  
(b) Severe failure

Figure 2.3: Fraction of banks bankrupt as a function of simulation runtime for two simulations. Greater area under the curve corresponds to a more severe system failure in which more banks failed more quickly.

For the purposes of this study, we vary the threshold for network connectivity and the recovery rate and plot the average severity of system failure as a function of number average number of links for the 2000 firms. In a fully connected network, each node would have 1999 connections. We see the results of this analysis in Figure 2.4. We can see that higher recovery rates result in much less risk of system failure as expected. However, there are a number of interesting features in the results. The first is that except in the case of high recovery rates (>80%), more interconnected results in more severe risk of system failure. The second interesting result is that for lower recovery rates (<60%) when the network is
relatively sparsely connected, the system is very sensitive to changes in connectivity. Small changes to the connectivity of the network results in large changes to the risk of system failure. We also find that there is a “tipping point” in recovery rate at around $\approx 70\%$. Small changes in the recovery rate about this value lead to drastic changes in the severity of bankruptcy cascades.

## 2.2 Direct Interbank Credit Exposure Network Model

We continue with a credit exposure network model in the Eisenberg-Noe framework [27] in which we study a smaller network limited to the largest US banks and extend the framework to measure the systemic risks as we impose global asset shocks and vary the network structure.

### 2.2.1 Data

Ideally, we would like to have knowledge of all interbank exposures between all banks in the financial system. However, this is unrealistic as this data is proprietary to each counterparty involved. In Poland and the Czech Republic, the regulatory bodies do have access to such information, but it is still not available to the public. In most other countries, not even the regulators have access to such detailed information. It’s important for the competitiveness and business operations of the banks that this credit exposure data remain not available to the public, but one could imagine a black box model designed by regulators where this information is inputted securely, and risk relevant risk metrics are output in which the private information cannot be restored from the output. Nevertheless, in an effort to begin collecting data on interbank exposures, the Federal Reserve has designed a survey for the 33 largest institutions in the US financial sector. This survey, the “Banking Organization Systemic Risk Report”, also known as the FR Y-15 report, requires banks to report total interbank credit exposures and obligations. No counterparty information is known, but we can use various assumptions to infer counterparty exposures from the data given. The full list of banks surveyed in the report appears in Table 2.2.1.
Figure 2.4: The severity of bankruptcy cascades for simulations with varying connectivity and recovery rate. The number of links in the network is determined by the combined asset threshold for linkage. The recovery rate remains fixed across all companies for a given simulation. More connections result in more severe bankruptcy cascades except in the case where recovery rates are exceptionally high (>80%). We also find that there is a “tipping point” around a ≈70% recovery rate about which small changes in the recovery rate lead to drastic changes in the severity of bankruptcy cascades.
<table>
<thead>
<tr>
<th>Bank name</th>
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<tbody>
<tr>
<td>BANCWEST CORPORATION</td>
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<tr>
<td>ZIONS BANCORPORATION</td>
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<tr>
<td>DEUTSCHE BANK TRUST CORPORATION</td>
</tr>
<tr>
<td>M&amp;T BANK CORPORATION</td>
</tr>
<tr>
<td>JPMORGAN CHASE &amp; CO.</td>
</tr>
<tr>
<td>KEYCORP</td>
</tr>
<tr>
<td>HUNTINGTON BANCSHARES INCORPORATED</td>
</tr>
<tr>
<td>PNC FINANCIAL SERVICES GROUP, INC., THE</td>
</tr>
<tr>
<td>FIFTH THIRD BANCORP</td>
</tr>
<tr>
<td>BANK OF AMERICA CORPORATION</td>
</tr>
<tr>
<td>BB&amp;T CORPORATION</td>
</tr>
<tr>
<td>BBVA COMPASS BANCSHARES, INC.</td>
</tr>
<tr>
<td>STATE STREET CORPORATION</td>
</tr>
<tr>
<td>U.S. BANCORP</td>
</tr>
<tr>
<td>WELLS FARGO &amp; COMPANY</td>
</tr>
<tr>
<td>SUNTRUST BANKS, INC.</td>
</tr>
<tr>
<td>RBS CITIZENS FINANCIAL GROUP, INC.</td>
</tr>
<tr>
<td>NORTHERN TRUST CORPORATION</td>
</tr>
<tr>
<td>COMERICA INCORPORATED</td>
</tr>
<tr>
<td>BMO FINANCIAL CORP.</td>
</tr>
<tr>
<td>TD BANK US HOLDING COMPANY</td>
</tr>
<tr>
<td>AMERICAN EXPRESS COMPANY</td>
</tr>
<tr>
<td>UNIONBANCAL CORPORATION</td>
</tr>
<tr>
<td>ALLY FINANCIAL INC.</td>
</tr>
<tr>
<td>CITIGROUP INC.</td>
</tr>
<tr>
<td>MORGAN STANLEY</td>
</tr>
<tr>
<td>CAPITAL ONE FINANCIAL CORPORATION</td>
</tr>
<tr>
<td>GOLDMAN SACHS GROUP, INC., THE</td>
</tr>
<tr>
<td>HSBC NORTH AMERICA HOLDINGS INC.</td>
</tr>
<tr>
<td>REGIONS FINANCIAL CORPORATION</td>
</tr>
<tr>
<td>BANK OF NEW YORK MELLON CORPORATION, THE</td>
</tr>
<tr>
<td>DISCOVER FINANCIAL SERVICES</td>
</tr>
<tr>
<td>SANTANDER HOLDINGS USA, INC.</td>
</tr>
</tbody>
</table>

Table 2.1: List of Banks included in FR Y-15 report.

The most common practice for taking data about given banks’ total obligations to estimate the complete network structure of counterparty obligations makes an assumption of maximum entropy. This procedure results in the most interconnected obligation network possible where banks’ total obligations and exposures are spread out as evenly as possible amongst the other banks within the given constraints [28–32]. The FR Y-15 Report gives
us the total interbank obligations ($L_i$) and the total interbank exposures ($A_i$) of each bank. If we imagine our credit obligation network as a matrix ($U$) where element $U_{i,j}$ represents the obligation of bank $i$ to bank $j$ and conversely, the exposure of bank $j$ to bank $i$, Then $L$ and $A$ tell us the sums of the rows and columns, respectively, of $U$. These constraints along with the maximum entropy (or cross-entropy minimum) assumption lead us to the most interconnected network possible. The details of this method can be found in Appendix 1. We will later impose further constraints to result in a different network structure.

2.2.2 Model

We use the Eisenberg-Noe framework for calculating payment clearing vectors from the interbank exposure network [27]. Given the full interbank obligation matrix and the equity values of each of the banks in our network (also derived from the FR Y-15 Report) this framework allows us to calculate a unique payment clearing vector in which each bank tries to fulfill it’s obligations to the best of it’s ability. If the incoming payments from other banks plus the bank’s equity are not enough to cover its obligations (i.e. it becomes insolvent and must file for bankruptcy), then the bank will pay the most that it can to all the other banks in proportion to its obligations to each bank. The details of this method can be found in Appendix 2.

If all the banks have enough equity plus incoming payments to cover all their obligations, then no banks fail and the network structure is secure. If any banks do not have enough equity plus incoming payments to cover all their obligations, then they fail and their deficient payments must be considered in recalculating the payment clearing vector and more banks may fail as a result. We measure the number of banks that fail and total dollar value lost (the difference between total obligations and total payment made) as we vary our assumptions about the network structure and impose shocks.
2.2.3 Results

Given the FR Y-15 Report data and the maximum entropy assumptions which results in a maximally connected network, we find that three of the 33 banks would fail initially (BNY Mellon, Amex and State Street) and that one bank would fail due to a cascading failure (Deutsche Bank.) In other words, BNY Mellon, Amex and State Street have high enough interbank obligations that their equity plus payments coming from their interbank exposures are not enough to cover them. Deutsche Bank does have enough equity plus expected payments coming from their interbank exposures to cover their interbank obligations. However, their actual payments from BNY Mellon, Amex and State Street are sufficiently lower than their expected payments to the degree that they cannot cover their obligations. All other banks can make full payment to their obligors. The value lost is $292.4M out of $2.38B, or about 12.3% of the total interbank obligations.

What would happen, however, if something happened to shock the value of the banks’ assets. This sort of stress-testing is important to understand how the system would respond to adverse scenarios. We employ a simple stress test in which total asset values of the banks are shocked globally by a given percentage. As a result there will be a decrease in each bank’s equity and thus there will be less of a cushion to absorb the difference between the incoming and outgoing payments arising from interbank obligations. In Figure 2.5 we see that total system failure doesn’t occur until at assets are shock a little over %10, which is a very large shock. At that point, in Figure 2.6 we see exponential losses starting to occur, at over $4B, more than the total interbank obligations of the banks. We are also interested if these defaults are initial defaults due to banks just being similarly leveraged or if they are due to cascading failures. In Figure 2.7 we see that indeed the spike in defaults occurring at a shock just over 10% are due to cascading failures.

What happens when instead of shocking asset values, we alter the structure of the network? We can impose new constraints in which we set elements of the interbank obligation matrix to zero and then re-balance the matrix so that the row and column constraints still apply and again assume minimum cross-entropy. So we are again assuming maximum inter-
Figure 2.5: Default rate due to global asset shocks.

Figure 2.6: Value lost due to global asset shocks.

c connectedness, but this time given there is a known broken link. So the new matrix is only maximally interconnected given this missing link. We are essentially breaking links in the matrix and rebalancing so that our constraints hold. We can do this to random links over and over again and observe how the new network structure affects the results.

In Figure 2.8, for each number of broken links up to 800 out of the 1056 maximum
Figure 2.7: Cascading default rate due to global asset shocks.

possible connections, we perform 120 sets of random link breakages. We find that there is very large variance in the value lost, but that there is an overall trend that as we break links (i.e. make a less interconnected network) there is a tendency for less money lost to bankruptcies. Figure 2.9 shows the average of money lost over the 120 simulations at each number of broken links. However, this tendency is clearly far from a rule. For some network structures, less interconnected means less risk and for some it means more risk.

What then distinguishes between two network structures with the same number of broken links and very different risk profiles? As is a weighted, directed network, there are many possible measures of network structure but there is not a lot of consensus in the limited literature on weighted, directed networks as to which measures are the most important. We have looked at transitivity, reciprocity, loop ratios and more. All have correlations to the losses we see, but all are also correlated to the overall connectivity of the network and none have shown a more meaningful relationship than just the number of links broken. Still, there is a wealth of new questions to ask and new veins of research to explore. We continue to search for new measures of network structure that may be the most impactful in determining the level of systemic risk inherent in the network structure.
Figure 2.8: Cascading default rate due global asset shocks.

Figure 2.9: Cascading default rate due global asset shocks.
Chapter 3

Bipartite network models of banks

3.1 Previous Research in Bipartite Networks

Bipartite network models, in which the nodes of the network are banks and asset classes, can be used to model asset price contagion. Models such as those in [61] and [62] have been able to show the importance of effects such as diversification and bank leverage on the sensitivity of the system to shocks.

Recently, [1] presented a model that focuses on real estate assets to examine banking network dependencies on real estate markets. The model captures the effect of the 2008 real estate market failure on the U.S. banking network. The model proposes a cascading failure algorithm to describe the risk propagation process during crises. This methodology was empirically tested with balance sheet data from U.S. commercial banks for the year 2007, and model predictions are compared with the actual failed banks in the United States after 2007, as reported by the Federal Deposit Insurance Corporation (FDIC). The model identifies a significant portion of the actual failed banks, and the results suggest that this methodology could be useful for systemic risk stress testing of financial systems.

Such models avoid the need for data on direct counterparty exposures by replacing the interbank network of obligations with a bipartite network of banks and assets. Though it may be seen as a limitation of the model that the direct network of obligations is not incorporated into the model, the benefit is that the model requires only more readily available
balance sheet data and makes no assumptions about interbank obligations. More, most studies agree that contagion caused through interbank exposures is rare [60].

Studies of risk contagion using changes in bank asset values have received less attention than credit exposure networks. A financial shock that contributes to the bankruptcy of a bank in a complex network will cause the bank to sell its assets. If the financial market’s ability to absorb these sales is less than perfect, the market prices of the assets that the bankrupted bank sells will decrease. Other banks that own similar assets could also fail because of loss in asset value and increased inability to meet liability obligations. This imposes further downward pressure on asset values and contributes to further asset devaluation in the market. Damage in the banking network continues to spread, and the result is a cascading of risk propagation throughout the system, as described in [63] and [64].

Using this coupled bank-asset network model, it is possible to test the influence of each particular asset or group of assets on the overall financial system. This model has been shown to provide critical information that can determine which banks are vulnerable to failure and offer policy suggestions, such as requiring mandatory reduction in exposure to a shocked asset or closely monitoring the exposed bank to prevent failure. The model shows that sharp transitions can occur in the coupled bank-asset system and that the network can switch between two distinct regions, stable and unstable, which means that the banking system can either survive and be healthy or collapse. Because it is important that policy makers keep the world economic system stable, we suggest that our model for systemic risk propagation might also be applicable to other complex financial systems, such as, for example, modeling how sovereign debt value deterioration affects the global banking system or how the depreciation or appreciation of certain currencies affect the world economy.

We present a dynamic version of the model in [1]. The model begins by collecting bank asset value data from balance sheets. All bank assets are grouped into some number of asset classes, so we have total value in the system for each bank and each asset. We begin by shocking an asset class which reduces the value of that asset on each bank’s balance sheet. This reduces the total asset value of the bank. If that reduced value causes the insolvency
of some number of banks, it triggers a fire sale of assets, which reduces the value of the assets being sold. This may once again trigger further insolvencies, and so on.

We study the banking system of Venezuela from 2005 to 2013 as a case study of the applicability of the model. Although in [1], the model was applied using just the data from one moment at the end of 2007 and used to predict failures, our analysis is applied to over eight years of monthly data. We run stress tests on each data set over a range of parameters and can track how the system’s sensitivity to these parameters changes on monthly basis.

The dynamical bank-asset bipartite network model (DBNM-BA) provides a first tool of “Risk Management Version 3.0” [44], which allows one to rate the risk of different assets alongside the stability of financial institutions in a dynamical fashion.

3.2 Data

We use statistical information from the Superintendence of the Institutions of the Banking Sector, or SUDEBAN, its monthly statistics, publication, newsletters and press releases, as well as its annual reports. The information is presented in national currency units, Bolivars, after the conversion process of 2008. Using the SUDEBAN information, we built bipartite networks for each month of the 16 years under study. We identified the banking subsectors in each period (commercial banking, universal banking, investment, savings and loan, mortgage, leasing, money market funds, microfinance and development banking) and based their systemic weight on asset levels. From the balance sheet of each bank we have identified the assets items (cash and equivalents, credit portfolio and securities), breaking each down to consider its systemic relevance. Later, we focus in detail on the loan portfolio by credit destination, namely: consumption (credit cards, vehicles), commercial, agricultural, micro-entrepreneurs, mortgage, tourism, and manufacturing. From that we derived the impact of the legal transformations in the credit portfolio composition. For the period of 2005—2013, we also analyzed the securities held by the different banks, specified as: private securities, treasury bonds, treasury notes, bonds and obligations of the public national debt, bonds and obligations issued by the Central Bank of Venezuela (BCV) and agricultural bonds.
Table 3.1: Asset and Bank Types

The analysis was done with the interest of specifying the kinds of assets that warrant the intermediation’s activity in the country. The credit and investment portfolio composition depicted the underlying structure of the system during the whole period, allowing us to show its evolution. A summary of the bank and asset types investigated is presented in Table 3.1.

The data used was derived from three datasets provided by SUDEBAN:

1) PUBLICATION BALANCE (Balance de Publicación, BP files, 1999-2013) Report Title: Banking System. Publication General Balance (Sistema Bancario. Balance General de Publicación) From here we extracted: Total Assets, total Liabilities and
29

total Equity. Aggregates assets value (Cash, Total Credits, Total securities)

2) PRESS REPORTS (Boletines de Prensa, BPR files, 2005-2013) Report Title: Inv-
vestment in Securities by type by bank (inversiones en Títulos Valores por tipo, segn
banco) From here we extracted security details by bank: Treasury Notes, Treasury
Bonds, Private Securities, National Debt Bonds

3) MONTHLY BULLETIN (Boletines Mensuales, BM files, 1999-2013) Report Title:
Credit Portfolio by Credit Destiny, by bank (Cartera de Créditos por Destino del
Credito, segn Banco) From here we extracted all the credit details by bank: Com-
mercial Credit, Credit Cards, Vehicle Credit, Agricultural Credit, Tourism Credit,
Manufacturing Credit, Mortgage Credit, Microfinance

The evolution of the total size and percentage make-up of each asset is provided in
Figure 3.1.

3.2.1 Interpolated data

There were some months where credit data were missing for certain banks, so to maintain
series continuity we interpolated. For example, in July 1999, we were missing credit data
for all mortgage banks, savings and loans, and leasing companies. In each case of missing
data where it was clear that the bank in question did exist in a given month, i.e. we had
data for the bank before and after the missing data points, we used a geometric mean to
fill in the missing points. For example, if Bank A was missing data for August 2005, then
for each missing data point, we replaced the null value with the geometric mean of the July
2005 and September 2005 data for each data series. Table B.1 details the list of missing
data that we interpolated.

3.3 Model

In bipartite networks, there are two types of nodes, in this case: banks and asset classes,
and links can only exist between the two different types of nodes. So in this network, banks
are linked to each type of asset that they hold on their balance sheet in a given month. Banks are never directly linked to other banks and assets never to other assets.

The asset portfolios of banks contain such asset categories as commercial loans, residential mortgages, and short and long-term investments. We model banks according to how they construct their asset portfolios. For each bank, we make use of its balance sheet data to find its position on different non-overlapping asset categories, e.g., bank $i$ owns amounts $B_{i,0}, B_{i,1}, ..., B_{i,N_{asset}}$ of each asset, respectively. The total asset value $B_i \equiv \sum B_{i,j}$ and
<table>
<thead>
<tr>
<th>Date</th>
<th>Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec. 1998</td>
<td>Banco Popular y de los Andes (BH), Confederado</td>
</tr>
<tr>
<td>Jul. 1999</td>
<td>Unido, Banesco (BH), Inverbanco, Venezolano, Corporacion Hipotecario, Union (EAF), Sofitasa (EAF), Sogecredito, Arrendaven, Fivca, Corpoindustria, La Venezolana, La Vivienda, Oriente, Casa Propia, Central, Del Centro, Mi Casa, La Primogenita, La Margarita, Valencia, Merenap, Corp Leasing, Prosperar, Del Sur, Provivienda, Caja Familia, Fondo Comun</td>
</tr>
<tr>
<td>Nov.–Dec. 1999</td>
<td>Arrendaven, Corpoindustria, Sofitasa (EAF), Sogecredito, Union (EAF)</td>
</tr>
<tr>
<td>Dec. 1999</td>
<td>Caja Familia, Casa Propia, Central, Del Centro, Del Sur, Fondo Comun, La Margarita, La Primera, La Primogenita, La Venezolana, Merenap, Mi Casa, Oriente, Prosperar, Provivienda, Valencia</td>
</tr>
<tr>
<td>Aug.–Nov. 2003</td>
<td>Anfico, Banesco (BH), Baninvest, Banplus, Banvalor, Casa Propia, Federal (BI), Federal (FMM), Financorp, Fivca (BI), Inverbanco, Mi Casa, Participaciones Vencred, Provivienda, Sofioccidente</td>
</tr>
<tr>
<td>Mar. 2004</td>
<td>Banplus, Casa Propia, Mi Casa</td>
</tr>
<tr>
<td>Nov. 2004</td>
<td>Banplus, Casa Propia, Mi Casa</td>
</tr>
<tr>
<td>Apr.–May 2005</td>
<td>Anfico, Arrendaven, Banesco (BH), Baninvest, Banplus, Banvalor, Casa Propia, Federal (BI), Federal (FMM), Financorp, Fivca (BI), Inverbanco, Mi Casa, Participaciones Vencred, Provivienda, Sofioccidente</td>
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Table 3.2: List of Interpolation of Balance Sheet Data for Banks, with Dates of Missing Data

total liability value \( L_i \) of a bank \( i \) are obtained from the investigated dataset. The weight of each asset \( m \) in the overall asset portfolio of a bank \( i \) is then defined as \( w_{i,m} \equiv B_{i,m}/B_i \).

From the perspective of the asset categories, we define the total market value of an asset \( m \) as \( A_m \equiv \sum_i B_{i,m} \). Thus the market share of bank \( i \) in asset \( m \) is \( s_{i,m} \equiv B_{i,m}/A_m \).

Furthermore, we define two additional parameters for the individual assets. We calculate
the relative size of the asset, $\beta$, defined as:

$$\beta_m = \frac{A_m}{\sum_m A_m},$$  \hspace{1cm} (3.1)

and we define the level of concentration/distribution of a given asset, using the Herfindahl-Hirschman Index (HHI) \cite{65}. If $A_m$ is the total value of asset class $m$ and $B_{i,m}$ is the value of asset $m$ on the balance sheet of bank $i$, then

$$\text{HHI}_m = \sum_i \left( \frac{B_{i,m}}{A_m} \right)^2.$$  \hspace{1cm} (3.2)

The HHI measures the degree to which a given asset class is distributed across the banks in the system. It reaches a maximum of 1 when the asset is entirely concentrated within one bank and a minimum of $1/N$ where the asset is evenly spread across all N banks in the system.

The model begins by introducing a shock to one of the given asset classes within a given month. The parameter, $p$, determines the fraction of the asset class remaining after the shock. So $p \in [0, 1]$ is an exogenous parameter to the banking system that cannot be controlled. If we begin by shocking asset class $m$ and $A_{m,\tau=0}$ is its total value, where $\tau$ represents the iteration of the model, then the initial shock reduces its value as follows,

$$A_{m,\tau=1} = p A_{m,\tau=0}.$$  \hspace{1cm} (3.3)

So a value of $p = 0.7$, would mean that after the first step of the model, the total value of the specified asset across the system would be reduced to 70% of its original value, or in other words it is a 30% shock to the asset. A smaller $p$ corresponds to a larger shock.

In the next step of the model, any bank that holds some of that shocked asset on its balance sheet will have that asset decreased by the same percentage. So if $B_{i,m}$ represents the value of asset class $m$ on the balance sheet of bank $i$, then the value of $B_{i,m}$ is reduced similarly,

$$B_{i,m,1} = p B_{i,m,0} = B_{i,m,0} \frac{A_{m,1}}{A_{m,0}}.$$  \hspace{1cm} (3.4)
Figure 3.2: Bank-asset coupled network model with banks as one node type and assets as the other node type. Link between a bank and an asset exists if the bank has the asset on its balance sheet. Upper panel: illustration of bank-node and asset-node. $B_{i,m}$ is the amount of asset $m$ that bank $i$ owns. Thus, a bank $i$ with total asset value $B_i$ has $w_{i,m}$ fraction of its total asset value in asset $m$. $s_{i,m}$ is the fraction of asset $m$ that the bank holds out. Lower panel: illustration of the cascading failure process. The rectangles represent the assets and the circles represent the banks. From left to right, initially, an asset suffers loss in value which causes all the related banks’ total assets to shrink. When a bank’s remaining asset value is below certain threshold (e.g. the bank’s total liability), the bank fails. Failure of the bank elicits disposal of bank assets which further affects the market value of the assets. This adversely affects other banks that hold this asset and the total value of their assets may drop below the threshold which may result in further bank failures. This cascading failure process propagates back and forth between banks and assets until no more banks fail. After [1]
This reduction in assets for bank $i$ reduces its equity accordingly. If after the initial shock, no bank has their equity reduced to zero or below, the algorithm stops and all banks survive the impact of the external shock. However, if any bank’s equity is reduced to zero or below, then that bank node fails and any asset classes that it holds on its balance sheet (that it is linked to in the network) will suffer a corresponding devaluation and the cascading failure algorithm will continue. This is where the endogenous parameter, $\alpha \in [0, 1]$, which is related to the structure of the system, comes into play. If bank $i$ fails and has $B_{i,m}$ of asset $m$, then,

$$A_{m,\tau+1} = A_{m,\tau} - \alpha B_{i,m,\tau}. \quad (3.5)$$

So if $\alpha = 0$, then the total value of an asset is not affected by the failure of a bank that owns that asset and there will be no cascading of failures. If $\alpha = 1$, then it is as if the assets of the defaulted bank have no value and the total value of those asset classes is reduced by the entire value on the defaulted bank’s balance sheet.

This reduction in the value of the asset classes will again cause the reduction at the bank level for any bank holding any of the devalued assets as such,

$$B_{i,m,\tau} = B_{i,m,0} \frac{A_{m,\tau}}{A_{m,0}}. \quad (3.6)$$

This reduction in assets may again reduce a bank’s equity to zero or below, thus triggering more bank failures, which will further devalue asset classes and so on. The process, which is visualized in Fig. 3.2 continues until the asset class devaluation no longer triggers any new bankruptcies. The primary observable at the end of the run is $\chi$, the fraction of surviving banks.

As an example, let’s assume a shock of $p=0.7$ to credit cards, that reduces 30% of their value causes one bank, Bank A, to have its equity reduced below zero. Let’s also assume that Bank A only has commercial credit, mortgage loans, treasury notes and public national debt, in addition to credit cards, on its balance sheet. These asset classes will be reduced in value by $\alpha$ times the value of each of these asset classes on Bank A’s balance sheet. So if
\( \alpha = 0.1 \), then the total value of each of these five asset classes would be reduced by 10% of the respective values on Bank A’s balance sheet. If more than one bank were to fail, then the reduction of each total asset class would be 10% of the sum of the respective assets on all the failed banks’ balance sheets.

We observed the behavior of the model for various values of the parameters \( \alpha \) and \( p \), across all months and while separately performing the initial shock on each of the 16 asset classes. In addition to observing \( \chi \) as an output of the model, noting that in most runs we see either most of the banks surviving or fewer than 20\% surviving, we therefore set a critical threshold of \( \chi = 0.2 \) and for fixed \( \alpha \) or \( p \), found the corresponding \( p_{\text{crit}} \) or \( \alpha_{\text{crit}} \) (varying each in 0.01 increments) that resulted in a \( \chi \) just below the 0.2 threshold for initial shocks to each of asset classes. We performed this analysis for each month of data and observed the changes in \( \alpha_{\text{crit}} \) and \( p_{\text{crit}} \) over time. The importance of these parameters is that they are intrinsically related to the asset distribution in the network structure of the system, given a surviving threshold. In the DBNM-BA, we focus on the time evolution of the critical parameters, \( p_{\text{crit}} \) and \( \alpha_{\text{crit}} \). Following the definitions above, the two parameters can be defined as following:

\[
 p_{\text{crit}}(\alpha) = p|\chi(p, \alpha) \leq 0.20 \& \chi(p + 0.01, \alpha) > 0.20, \tag{3.7}
\]

and

\[
 \alpha_{\text{crit}}(p) = \alpha|\chi(p, \alpha) \leq 0.20 \& \chi(p, \alpha - 0.01) > 0.20, \tag{3.8}
\]

where \( \chi \) is calculated given an asset class to be initially shocked and a date from which the data is taken. The fraction of surviving banks may be greater than 20\% for all values of \( \alpha \) between 0 and 1, in which case \( \alpha_{\text{crit}} \) is by definition set to 1.

A summary of the key parameters of the DBNM-BA is presented in Table C.1. One of the most important features of the model is that it shows the differences of the impact of the shock of the assets in the system in different moments. So at a particular time a small shock of a particular asset is needed to generate a cascading failure while at another time it needs to be much larger to generate an impact. Another relevant feature of the
model is that impacts of assets not only depends on its weight on the system but on their specific distribution among banking institutions in the different moments. Thus the model allows us to see systemic features not assessed by traditional measures, which is valuable for supervisory agencies.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{m,\tau}$</td>
<td>Total value of asset $m$ at iteration $\tau$</td>
</tr>
<tr>
<td>$B_i$</td>
<td>Total value of all assets owned by bank $i$</td>
</tr>
<tr>
<td>$B_{i,m,\tau}$</td>
<td>Value of asset $m$ owned by bank $i$ at iteration $\tau$</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of banks</td>
</tr>
<tr>
<td>$p$</td>
<td>Parameter representing the shock level $(1 - p)$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Parameter representing the spreading effect of a shock to other asset values</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Fraction of banks surviving the cascading failure model</td>
</tr>
<tr>
<td>$\alpha_{\text{crit}}$</td>
<td>Smallest $\alpha$ given a $p$ for which $\chi &lt; 0.20$</td>
</tr>
<tr>
<td>$\beta_m$</td>
<td>Relative size of asset $m$ with respect to all assets</td>
</tr>
<tr>
<td>$\text{HHI}_m$</td>
<td>Diversification of asset $m$ among banks</td>
</tr>
</tbody>
</table>

Table 3.3: List of model parameters and measurements

### 3.4 Results

As a first step, the Venezuelan financial system is represented using the bank-asset bipartite network. We began using the three types of aggregated assets (cash, credit and securities) and created networks visualization for each month (see Fig. 3.3). These graphs made it easier to observe the relative significance of the different sub-sectors in the banking system during the period under study. They show clearly that the system shifted from a specialized one, with different types of institutions, to a system in which primarily universal banks and commercial banking remain (including those promoted by the public sector). We can also see the decrease in number of institutions in the system over the given period. Likewise the graphs showed the greater weight that credit assets has had in the system, although
in the period 2003–2004 the weight of securities was higher. The networks visualization allows showing specific bank, type of institution, kind of asset and relative size of the asset, all in the same graph. Moreover, its periodic concatenation allows showing clearly transformations in time. As we use a bipartite network model, the lines that we see in these visualizations represent connections between banks and the asset types they hold in their portfolios. There are no direct connections among banks nor assets.

Next, the asset classes were separated into two categories, credit and securities, and created two respective sets of network visualizations. From either set of figures, it is clear that the assets tend to be concentrated in a few of the given asset classes. Credit networks showed the relevance of commercial credit during the whole period, even diminished since 2005, as credit disaggregation grew by legal requirements for mandatory credit to specified sectors. The securities networks showed, during the period 2005–2013, the growing influence of national public debt instruments while diminishing that of private bonds and of those issued by the BCV. As well as with aggregated assets, these two groups of networks showed the transformations of the system month by month.

Having identified the structure transformation, the following step was to test the strength of the banking system by initiating a shock to each of the 16 asset classes and simulating the resulting aftershocks across the banking system. We did this from July 2005 through December 2013, period in which we have complete credit and securities data for all the banks in the system at each moment. We tracked 9 different classes of credit and 7 different classes of securities over that time period for each bank.

3.4.1 Surviving banks, shock level and contagion effect

The three main parameters of the model, as discussed above, are $p$ (external shock level), $\alpha$ (level of asset contagion), and $\chi$ (fraction of surviving banks). We thus begin the analysis by focusing on a given month, and investigating the relationship between these three parameters, for different individual assets. This comparison provides the means to identify how a shock to a given asset sets off the spreading of damage to the entire system (see
Figure 3.3: Banking network structure for December 2000 and December 2013 with aggregate assets. Visualization made using Cytoscape®. Blue circles represent asset types (cash, credit and securities) and squares represent banks (Red: commercial banks, Green: investment banks, Aquamarine: leasing companies, Yellow: mortgage banks, Purple: universal banks, Light blue: savings and loan, Orange: money market funds). The plots show the two different structures of the system in the two moments. The first shows a specialized system with different kinds of institutions. The second plot shows a universal banking system with fewer banks. The lines connect different banks to the assets in their portfolios. In both moments credit is the largest asset in the aggregated portfolios. In 2013 we can see an increase in the relative weight of securities in the aggregated portfolios of the banks.

In Fig. 3.4, we plot 3D surfaces, that show the fraction of surviving banks for different levels of $p$ and $\alpha$, for three types of assets: vehicle credit, commercial credit and BCV bonds. The analysis is done for data from December 2005 and from December 2013. These surfaces indicate the importance of both the relative size of the initial shock $(1 - p)$ and the relative magnitude of the feedback aftershocks $(\alpha)$ for each type of asset in a given moment.

When the initial shocked asset class is one of the smaller asset classes, note that we often see flat surfaces with $\chi = 1$. This indicates no bank holds a position in that asset class greater than its equity. However, for most asset classes, particularly the larger ones,
we see a great sensitivity to both $p$ and $\alpha$. We generally see two regimes in the $p$-$\alpha$ phase space: one where the fraction of survived banks at the end of the model is well over half and one where it is generally below 20%. Thus it appears that there are critical values of $\alpha$ as a function of $p$ and vice versa which separate these two regimes and we will want to observe how these critical values change over time. In the case of BCV bonds, as seen in Figs. 3.4(c) and 3.4(f), we note that these critical values change quite drastically between 2005 and 2013.

Figure 3.4: Fraction of surviving banks ($\chi$) as a function of the fraction of shocked asset remaining ($p$) and the impact of bankruptcies on asset prices ($\alpha$) for three different shocked assets, each for December 2005 and December 2013. (a/d) Vehicle credit is too small to cause bankruptcies for any value of $p$ or $\alpha$ on the given dates. (b/e) Commercial credit is large enough that catastrophic bankruptcies occur for $p \leq 0.80$ for all but the smallest values of $\alpha$. (c/f) In 2005, shocking BCV bonds causes systemic failure for all but the smallest values of $\alpha$ and $1 - p$. In 2013, only BCV bond shocks with the largest values of $\alpha$ and $1 - p$ cause the system to collapse. Color coded from black to yellow, with a range of $[0,1]$, which represents the fraction of surviving banks under the shocks.
3.4.2 Asset Size Versus Surviving Banks

Following the recent financial crisis, one point of debate has been the issue of *too big to fail*. Thus, the question arises whether the damage observed in the model is resulting from the size of the shocked asset. Thus, we investigated the relationship between the relative size of the shocked asset class, $\beta$, and the fraction of surviving banks, $\chi$, for given $\alpha$ and $p$ levels. In Fig. 3.5, we present an example for the case of $p = 0.60$ and $\alpha = 0.1$ (panels (a) and (c)) and $\alpha = 0.2$ (panels (b) and (d)). Points are plotted for each month and each type of asset class getting the initial shock. In Figs. 3.5(a) and 3.5(b), the points are color-coded by the year for which the model was run. We can see that for lower levels of $\alpha$ there is an approximate linear relationship between $\beta$ and $\chi$ in the range $0.05 < \beta < 0.20$. Increasing $\alpha$ to 0.20, we see an abrupt change in $\chi$ around $\beta = 0.1$. There exists a wide range of $\beta$ ($0.1 < \beta < 0.3$) for which the system collapse independent of the value of $\beta$. This shows that not only the relative weight of the asset is relevant, but also the way in which it is distributed through the structure of the system. Thus, the bank-asset network structure provides systemic risk based on details that are not captured or apprehended with traditional tools. For the model runs in which fewer than 20% of the banks survive, we see there was a tendency in the earlier years, for greater concentration of a given asset type. Simultaneously, it is possible to observe that for assets of the same weight in the system the surviving percentage of banks was greater in the initial period of analysis. See ?? for more examples.

Figs. 3.5(c) and 3.5(d) presents the points color-coded by the asset initially shocked. We observe that different asset classes have different ranges of relative size. However, it is interesting to note, that different asset classes seem to show different critical values for $\beta$, though always within the range $0.1 < \beta < 0.2$. This further demonstrates the importance of $\alpha$ when the shock to the asset is on the order of 20% or greater. The smaller the shock to the asset, the more linear the relationship by $\chi$ and $\beta$. 
Figure 3.5: The plots show the relationship between $\beta$ and $\chi$. Fig. 3.5(a) and Fig. 3.5(b) show points color-coded by the year for which the model was run. Fig. 3.5(c) and Fig. 3.5(d) show points color-coded by the asset which was initially shocked. Fig. 3.5(a) and Fig. 3.5(c) show the relationship for $\alpha = 0.10$ and $p = 0.60$, Fig. 3.5(b) and Fig. 3.5(d) for $\alpha = 0.20$ and $p = 0.60$.

### 3.4.3 External Shock and Contagion Sensitivity

As discussed above, the DBNM-BA provides the means to rate the risk of the different assets held by the components of the financial system. Here, we focus on the $\alpha$ parameter, which measures the extent of contagion that results from a given asset. We set a critical threshold of $\chi = 0.2$ (20% of banks survive) and for a given $p$ (or $\alpha$) find the minimum $\alpha$ (or maximum $p$) that results in fewer than 20% of the banks surviving. Defined this way, we are able to simulate asset fire sales, and assign a value to each asset, according to the extent of damage it can cause to the system. Thus, throughout the rest of this section, we will focus on $\alpha_{\text{crit}}$, however, the results presented below can alternatively be presented for
<table>
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<th>$\alpha = 0.0$</th>
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Figure 3.6: Relationship between share of assets ($\beta$) and fraction of surviving banks ($\chi$) for different shock levels ($p$) and spreading effect ($\alpha$). The points are color-coded by the year for which the model was run.

the case of $p_{crit}$.

In Fig. 3.8(a) we present results obtained for the scenario of $p = 0.80$ (an initial shock of 20% to each of the respective assets) and track over time the critical value of $\alpha$ for which just under 20% of the banks survive the cascading failure algorithm. The plot shows that larger shocked assets, in general, show a lower $\alpha_{crit}$ than smaller shocked assets. It also reveals volatile behavior of $\alpha_{crit}$ in time. We see frequent large jumps in $\alpha_{crit}$ indicating that month-to-month changes within the system can result in drastically different levels of fragility to similar shock events. The value of $\alpha_{crit}$ reflects the macro-prudential risk of the
Figure 3.7: Relationship between share of assets ($\beta$) and fraction of surviving banks ($\chi$) for different shock levels ($p$) and spreading effect ($\alpha$). The points are color-coded by the asset which was shocked.

asset, and reflects the level of damage resulting from the network structure, and is thus a network effect.

In Fig. 3.8(b) we also tracked the systemic size of the assets ($\beta$) and in general, the higher $\beta$ values correspond to lower $\alpha_{\text{crit}}$ values. However we can see two small assets, mortgage loans and vehicle credits, that during 2009–2010 saw a significant drop in $\alpha_{\text{crit}}$ even their systemic size had only very small growth. Also at the beginning of 2009 there was a moment in which the size of public national debt was the same as that of vehicle credits though $\alpha_{\text{crit}}$ was higher for the latter. These details allow us to infer that the relative size
of the asset is not the only factor to consider.

Figure 3.8: (a) The behavior of $\alpha_{cr}$ in time for certain shocked asset classes. For $p = 0.80$ (an initial shock of 20% to each of the respective assets), we track over time the critical value of $\alpha$ for which just under 20% of the banks survive the cascading failure process. We see high volatility in $\alpha_{cr}$ indicating that monthly changes can produce different levels of fragility. (b) The size of the asset class relative to the entire system ($\beta$) over the same time period for the same asset classes.
We are further interested in how $\alpha_{\text{crit}}$ may change in time with respect to the HHI for the initial shocked asset and $\beta$. Both the HHI and $\beta$ reflect characteristics of the individual asset embedded in the system, and thus can be considered a macro-prudential feature to assess risk factors. In Fig. 3.9(a) we present the case of an asset which has a low weight in the average portfolio of the banks. It is important to note that its HHI is low, mainly from 2007–2010, a period in which its $\alpha_{\text{crit}}$ was also very low, which means that a large negative shock—even in the value of a small asset which is distributed among institutions—can be easily disseminated in the system and generate a cascading failure. In this case, the model is able to uncover information that generally speaking we may not find with traditional measures, showing a weakness in the structure of the system. On the other hand if we check another asset, such as commercial credit in Fig. 3.9(b), we see an example where $\alpha_{\text{crit}}$ and HHI tend to move against each other indicating that the more an asset is concentrated in a smaller number of banks, the smaller $\alpha_{\text{crit}}$ is, indicating that the system is more sensitive to cascading failures.

Figure 3.9: Fig. 3.9(a) presents the case of vehicle credit, which has always had a small $\beta$. It is important to note that its HHI is lower from 2007–2010, and during that period the $\alpha_{\text{crit}}$ was also very low, which means that a large negative shock in the value of that asset, with a less homogeneous distribution among institutions, can be easily disseminated in the system and generate a cascading failure. Fig. 3.9(b), shows the case of shocked commercial credit (high $\beta$) whose $\alpha_{\text{crit}}$ and HHI tend to move against each other indicating that the more concentrated a shocked asset is, the more sensitive the system is to cascading failures.
As presented in Fig. 3.9, we observed that for a given shock level, there is a different relationship between the size of the asset, $\beta$, and its $\alpha_{\text{crit}}$ value, as a function of time. Thus, we ask whether it is possible to quantify this relationship for all assets. To this end, we calculate the correlation between $\alpha_{\text{crit}}$ and the $\beta$ across a range of shock sizes and for shocking each of the asset classes. In Fig. 3.10 we present these correlation values, using a heatmap graphic. We find that there is a strong tendency for $\alpha_{\text{crit}}$ and $\beta$ to be anti-correlated for large shock levels. Only for the case of small shocks it is possible to observe a lack of correlation.

Figure 3.10: Heat map of $\alpha_{\text{crit}}$ and the $\beta$ correlation for each asset type and various shock levels. Color represents the strength of the correlation, ranging from red for positive values, to blue for negative values.

3.4.4 Non-Surviving Banks versus Solvency Index

In addition to studying the effect of the assets on the stability of the banking system, we also investigated the bank nodes of the network. To this end, a series of tests was performed to find the order in which banks underwent the simulated process of failure, and we considered its relationship with traditional measures to estimate banks solvency, such as the debt-to-equity ratio (total liabilities/total equity), which is used to evaluate the
long term robustness of a firm. It must be noted that the debt-to-equity ratio assesses the strength of a banking institution, while the DBNM-BA is aimed at assessing the strength of the banking system. However, both elements are relevant to elevate the fragility of the banking sector.

Figure 3.11: Heat map showing the average cascading failure steps for all banks, shocking all the assets with $p = 0.70$ and a contagion effect of $\alpha = 0.10$, from 2005–2013. Red indicates a bank failing earlier in the model. Green indicates that the bank survived the cascading failure process. White indicates that the bank did not exist at that specific moment in time.

We find that the order of bank failures depends on the asset shocked, and that the model provides details of the strength beyond the state of the individual institution, which results
Figure 3.12: Heat map showing the average cascading failure steps for all systems banks, shocking all the assets with $p = 0.70$ and a contagion effect of $\alpha = 0.20$, from 2005–2013. Red indicates a bank failing earlier in the model. Green indicates that the bank survived the cascading failure process. White indicates that the bank did not exist at that specific moment in time.

From the whole network of institutions and assets of the system. The order of bank failure for all assets, given a shock level ($p$) and a spreading effect ($\alpha$), is calculated. Next, these results are aggregated, representing the average failure order of each bank after a shock to its assets. This procedure was performed for all the institutions and for each month of the period 2005–2013. Simultaneously, the debt-to-equity ratio was also calculated for all the institutions and for each month of the same period.
Figure 3.13: Heat map showing the debt-to-equity ratio for each bank, from 2005–2013. Its heat maps color code goes from red to green. Red indicates the higher debt equity ratio. Green indicates the lower debt-to-equity ratio. White indicates that the bank did not exist at that specific moment in time.

Figs. ?? show the results of the average cascading failure steps for each institution in two states: (1) \( p = 0.70 \) and \( \alpha = 0.10 \) and (2) for \( p = 0.70 \) and \( \alpha = 0.20 \). Fig. 3.13 shows the debt-equity ratio. We can see that Figs. 3.11 and 3.13 are more or less similar, while 3.12 shows a more fragile situation of the system. These results reinforce the capability of the model to show the sensitivity of the system due to the interdependence of the agents of the system. Traditional measures are able to capture important features of the units of
the system. As soon as the connectivity is considered and the contagion effect is possible, traditional measures cannot assess the systemic effect, and so forth, underestimate the risk.
Chapter 4

Conclusion

With the increasing frequency and scope of financial crises and the realization that financial contagion can spread through the highly interconnected global financial network via the contagion effect, understanding the dynamics of such spreading has become of paramount importance. During the last crisis, the world experienced the impact of the reduction of value of a specific kind of asset, which was included in many portfolios and generated a systemic contagion, ultimately resulting in a global recession. The impact was felt across the board, regardless of size, leverage or any other traditional microprudential measures of risk. Institutions succumbed under the negative impact of the diminishing value of assets, which caused fire sales and ultimately a disruption of financial markets. Even though financial institutions are under supervision, the systemic impact was not foreseen by regulators.

In this highly complex environment, financial and banking supervision has to be thought of as a systemic task, focusing on the health of the nodes (the banks and financial institutions involved) and on the connections among those nodes (different kind of links as flows of funds, loans, assets owned, etc.) to unravel the structure of the system under surveillance. This indicates the need to include the shadow banking institutions along with the traditional banking institutions because of their important role in the financial system and multiple links and connections. Simultaneously, we must remember the system is dynamic and so more than analyzing static balance sheets is required to understand the evolution and transformation of the system and its strengths and weakness at different times.
With this in mind, the work contained herein proposes modeling frameworks that are able to view the entire system holistically rather than just the individual firms that make up the system and that track systemic changes of a banking system rather than just looking at point-in-time measures of risk. The models are all applied to publicly available empirical data, avoiding as much as possible theoretical biases and data restrictions. However, data limitations are great and assumptions are always required to produce a model based on public data. Going forward, the well-being of individual banks, and more importantly the banking system as a whole, heavily depends on the transparency of the banks with regards to their balance sheets and contractual obligations.

The proposed models focus on the network of banks, linked by either their credit obligations or shared portfolio structures. These represent two different classes of networks. The first belongs to the class of structural networks and the second to functional networks.

The credit obligation network models provide a tool for monitoring the tight web of counterparty exposures in which the linkages can cause exponential increases in counterparty risk which cannot be seen in traditional counterparty risk measures. As a case study for this model, we investigated the US banking system from as of December 2013, through the FR Y-15 survey required of the 33 largest banks in the Unites States. The central role of the US financial system within the global financial system makes it a key network for study. The great difficulty in finding publicly available credit exposure data among US banks made the existence of the FR-Y 15 report integral to the study. While the predicted failures of 4 of these 33 financial institutions with out any exogenous shock should be worrisome, we cannot say whether the assumptions required to build the model are realistic or not and should temper our fears somewhat.

The DBNM-BA model provides a novel macroprudential stress testing tool for the functional level, in the case where the exposure positions is the only available information. On the structural level, the contractual obligations would map out the network of claims and liabilities between institutions, and these types of networks have been extensively investigated in different countries [29, 67–71]. The ability of banks to fulfill these promises of course
depends on the shocks to assets and asset classes. A general multi-level stress-testing framework would combine both functional and structural networks, and the dependencies between them. This would be made possible using the recently breakthroughs in the formalism of interdependent networks [72], where only first steps have been made in its applications to the financial system [73, 74]. As a case study for this model, we investigated the Venezuelan banking system from 1998 to 2013, because it is a period with several legal transformations that had impact on its structure. The DBNM-BA showed the impact of these legal transformations in the asset portfolio of all the units of the system in time. In this sense, the model yielded expected results.

To evaluate the stability of the system, we applied a series of shocks to the system to reveal intrinsic weaknesses at different times. It should be noted that the system displayed an important variation that did not appear to follow any specific trend. Quite the opposite, the sensitivity of the system to initial conditions (structural distribution of the assets among banks) is important. It is also worth noting that some assets of insignificant systemic weight in some periods were able to cause important damage to the whole system even under small levels of shocks. The concentration of the assets in particular units of the system, as well as their distribution in it, were also elements of high relevance.

The proposed model provides a dynamical stress test modeling framework. Once the critical values are associated for each asset for a given month, we repeated the analysis for the next month. In this way, it is possible to define a dynamic, or time-evolving, model and track how the values of the different parameters, specifically the critical ones, are changing in time and evolving on a month by month basis. This provides the means of tracking changes in these critical values, which can be used as a signal in a decision support system or early warning system for regulators and policy makers.

In conclusion, our models were able to reveal functional and structural strengths and weakness of a banking system, giving supervisory agents and the banks themselves important new information about its stability. These models have much room for growth and can be extended in many possible ways. The credit obligation network model can be ex-
tended to other financial institutions and other countries’ financial systems wherever data is available. The DBNM-BA model can provide a general tool for policy and decision makers to monitor and regulate the financial system. This work provides new tools to test and assess different economic scenarios and elaborate actions to be addressed by policy makers. The stress scenarios and insights resulting from this work further provide early alert signs of weakness of the economic and financial system, identifying vulnerabilities of the system as a whole. During or following a crisis, this model also provides the means to evaluate nodal points that promote the recovery of a system; for example, policy makers will have the capability to calculate to which nodes and to what extent actions should be applied to recover the system. Finally, these models can be complemented using the multilayer network approach when considering the banking system as part of a more complex system, including the global financial system and the real economy as a whole.
Appendices
Appendix A

Cross-entropy Minimization Algorithm

A.1 Explanation of the Method

This method finds the most interconnected obligation matrix between banks. The inputs are the total interbank obligations \( L \) and the total interbank exposures \( A \) of each bank. The output is the most interconnected total interbank obligation matrix \( U \) where element \( U_{i,j} \) represents the obligation of bank \( i \) to bank \( j \) and conversely, the exposure of bank \( j \) to bank \( i \). The result represents a fully connected bank network where the obligations between banks are as balanced as possible within the constraints.

The sums of the rows of \( U \) should be equal to \( L \), the total interbank obligations for each bank. The sums of the columns of \( U \) should be equal to \( A \), the total interbank exposures for each bank. In order for the matrix to have a solution, the sum of the row sums must be equal to the sum of the column sums (the sums of the elements must the same no matter the order that they are added) and so either \( L \) or \( A \) must be rescaled. By convention, we rescale \( A \rightarrow A' \) such that the \( \sum_i A'_i = \sum_i L_i \). There are \( N^2 \) elements in \( U \) and \( 3N - 1 \) linear equality constraints. There are \( N \) constraints for the zeros on the diagonal as no bank can have any obligations or exposures to itself. There are \( 2N - 1 \) constraints sums of the rows and columns \( (N \) for each of the row sums and \( N \) for each of the column sums, minus one for the rescaling.) Thus, for a network of any significant number of banks, there will be infinitely many solutions that satisfy the constraints.
We choose the most interconnected matrix using methods described in [75], [76] and [30]. This matrix is the one that minimizes the Kullback-Leibler divergence (also known as cross-entropy) with the prior matrix \( U_{ij}^0 = L_i A_j^0 \) when \( i \neq j \) and is equal to zero on the diagonal, when \( i = j \). The Kullback-Leibler divergence between matrices \( U \) and \( U_0 \) is defined as follows,

\[
D_{KL}(U, U_0) = \sum_{ij} U_{ij} \frac{U_{ij}}{U_{ij}^0}.
\]  

(A.1)

The matrix that satisfies the constraints and minimizes the Kullback-Leibler divergence is found through Bregman’s balancing method as described in [32] shown in the following section.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>Number of banks in the system</td>
</tr>
<tr>
<td>( L )</td>
<td>Interbank total obligation vector</td>
</tr>
<tr>
<td>( A )</td>
<td>Interbank total exposure vector</td>
</tr>
<tr>
<td>( U )</td>
<td>Interbank liability matrix</td>
</tr>
</tbody>
</table>

Table A.1: List of model parameters and measurements

### A.2 R Code

```r
balance <- function(L,A) {
  if (length(L) != length(A)) stop("Vector length incompatibility")
  N <- length(L)
  A <- sum(L)/sum(A) * A
  U <- L %o% A / sum(L)
  U <- U - diag(diag(U))
  U2 <- diag(N)
  while (e_distance(U,U2) > 1e-20) {
    U2 <- U
    for (i in 1:N) {
      U[i,] <- U[i,] * L[i] / sum(U[i,])
      U[,i] <- U[,i] * A[i] / sum(U[,i])
    }
  }
}
```

```
e_distance <- function(U1,U2) {
  if (!all(dim(U1)==dim(U2))) stop("Matrices must be the same size")
  N1 <- length(U1[1,])
  N2 <- length(U1[,1])
  e_d <- 0
  for (i in 1:N1) {
    for (j in 1:N2) {
      e_d <- e_d + (U1[i,j] - U2[i,j])^2
    }
  }
  return(e_d)
}
```
Appendix B

Payment Clearing Vector Algorithm

B.1 Explanation of the Method

Once an interbank obligation matrix is known, we need a method for finding the payment vector that is cleared when the maximum possible payments are made in proportion to the obligations. A framework for such a method is laid out in [27]. We begin with the known interbank obligation matrix $U$. From that we can calculate the proportional obligation matrix $⇧$, as follows:

$$⇧_{ij} = \begin{cases} 
U_{ij} & \text{if } i \neq j, \\
0 & \text{if } i = j.
\end{cases} \quad (B.1)$$

Every bank will attempt to make payments equal to its obligations to each other bank. However, banks with interbank obligations that exceed their interbank exposures, may not be able to make full payments if they do not have enough equity to cover the difference. We make the assumptions that whatever payments a bank does make, those payments will be in proportion to their obligations. Thus $\sum_j \Pi_{ij}^T p_j$ will equal the incoming payments to bank $i$, where $\Pi^T$ is equal to the transpose of $\Pi$ and $p_i$ are the payments made by each bank. So if $e_i$ is the equity of bank $i$ before payments are made, the equity of bank $i$ after payments is equal to
Eisenberg and Noe show in [27] that there exists a unique payment clearing vector, $p$, such that

$$e_i + \sum_j \Pi_{ij}^T p_j - p_i. \quad \text{(B.2)}$$

We begin with the assumption that all banks make payments in full. If some banks are not able to make full payments, i.e. the value in B.2 is less than zero, then we solve for $p$, knowing that those banks make payments equal to their equity plus their incoming payments. If solving for that new payment vector results in any further bank failures, then we again resolve for $p$ with that knowledge. We continue until we solve for a payment vector in which no bank make payments resulting in a new negative equity and thus have the payment clearing vector. The algorithm can take no more than $N$ steps as the slowest possible solution is one in which one new bank fails at each step.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Number of banks in the system</td>
</tr>
<tr>
<td>$L$</td>
<td>Interbank total obligation vector</td>
</tr>
<tr>
<td>$U$</td>
<td>Interbank liability matrix</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Proportional interbank liability matrix</td>
</tr>
<tr>
<td>$p$</td>
<td>Payment clearing vector</td>
</tr>
<tr>
<td>$e$</td>
<td>Total bank equity vector</td>
</tr>
</tbody>
</table>

Table B.1: List of model parameters and measurements

**B.2 \ R Code**

```r
pcv <- function(p0,U,e) {
```
if (length(p0) != length(e)) stop("Vector length incompatibility")
if (length(e) != nrow(U)) stop("Vector/matrix size incompatibility")
N <- length(p0)
def <- rep(0, N)
Pi <- U/rowSums(U)
p <- p0
for (k in 1:N) {
  e_new <- e + t(Pi) %*% p - p0
  Lambda = diag(rep(0, N))
  for (i in 1:N) {
    if (e_new[i] < 0) {
      Lambda[i, i] <- 1
      if (def[i] == 0) def[i] <- k
    }
  }
  M <- diag(N) - Lambda %*% t(Pi) %*% Lambda
  v <- Lambda %*% t(Pi) %*% (diag(N) - Lambda) %*% p0 + e
  + (diag(N) - Lambda) %*% p0
  p <- solve(M, v)
}
return(cbind(p, def))
Appendix C

Dynamical Bank-Asset Bipartite Network Model Algorithm

C.1 Technical Description of the Algorithm

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{m,\tau}$</td>
<td>Total value of asset class $m$ at iteration $\tau$</td>
</tr>
<tr>
<td>$B_{i,\tau}$</td>
<td>Total value of all assets owned by bank $i$ at iteration $\tau$</td>
</tr>
<tr>
<td>$B_{i,m,\tau}$</td>
<td>Value of asset class $m$ owned by bank $i$ at iteration $\tau$</td>
</tr>
<tr>
<td>$p$</td>
<td>Parameter representing the shock level $(1 - p)$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Parameter representing the spreading effect of a shock to other asset values</td>
</tr>
</tbody>
</table>

Table C.1: List of model parameters and measurements

Step 1. Select data

Choose the month of the dataset to evaluate, which asset to shock ($m'$) and values for $p \in [0, 1]$ & $\alpha \in [0, 1]$.

Step 2. $B_{i,m,0} \leftarrow$ value of asset $m$ on balance sheet of bank $i \quad \forall i, m$

$L_i \leftarrow$ value of all liabilities on balance sheet of bank $i \quad \forall i$

Record the value of each asset class and total liabilities on the balance sheet of each bank from our chosen dataset.
Step 3. \( B_{i,0} \leftarrow \sum_m B_{i,m,0} \quad \forall i, \quad A_{m,0} \leftarrow \sum_i B_{i,m,0} \quad \forall m \)

Calculate both the value of all assets for each bank and the total value of each asset class across all banks.

Step 4. \( A_{m',1} \leftarrow pA_{m',0}, \quad B_{i,m',1} \leftarrow pB_{i,m',0} \quad \forall i \)

Shock the chosen asset class \((m')\) both at the bank level and the asset class itself.

Step 5. \( B_{i,1} \leftarrow \sum_m B_{i,m,1} \quad \forall i, \quad \tau \leftarrow 1 \)

Recalculate the total assets of each bank after the shock to asset \(m'\).

Step 6. If \( B_{i,\tau} > L_i \quad \forall i \), then end, else proceed to Step 6.

If the assets of each bank are still greater than their liabilities, then there are no bankruptcies in the model and the algorithm stops. Otherwise, the algorithm continues.

Step 7. \( A_{m,\tau+1} \leftarrow A_{m,\tau} - \alpha B_{i,m,\tau} \quad \forall m, i \mid B_{i,\tau} \leq L_i \)

Each bank whose total assets dropped to or below the total liabilities is considered bankrupt and each asset class owned by those banks is devalued by an amount that scales both with the value owned by the failed bank and the parameter \(\alpha\).

Step 8. \( B_{i,m,\tau+1} \leftarrow B_{i,m,0} \frac{A_{m,\tau+1}}{A_{m,0}} \quad \forall i, m, \quad \tau \leftarrow \tau + 1 \)

Rescale the value of each asset class owned by each bank to the new total value of each asset class.

Step 9. Return to Step 5.

Once again recalculate the new total assets of each bank and then check for new bankruptcies.
Bibliography


[74] Rick Bookstaber and Dror Y Kenett. The multilayer structure of the financial system. preprint.


Curriculum Vitae

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EDUCATION

• 2017, Ph.D. Physics, Boston University, Boston, MA, USA
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