Comment on "Percolation Transitions Are Not Always Sharpened by Making Networks Interdependent"

In a recent Letter, Son *et al.* [1] studied the percolation transition of fully interdependent diluted lattices. They claim that in this case the order parameter exponent β is larger than in ordinary percolation (OP). From this they reach their main conclusion that interdependent networks can be more robust (less exposed to sudden transition) compared to single networks. They claim that the percolation transition in coupled two-dimensional (2D) diluted lattices with $q < 1$, where q is a measure for the dilution, is characterized by a different set of exponents than in OP and thus belongs to different universality class. Specifically, instead of the known values of OP ($q = 1$): $\beta = 5/36$, $\nu = 4/3$, and $D_f = 91/48$, they claim to find $\beta = 0.172$, $\nu = 1.19$, and $D_f = 1.85$ for any $q < 1$. They based their findings on numerical simulations of lattice sizes up to $L \times L = 512 \times 512$.

In this Comment, we present more extensive simulations suggesting that the results and conclusion of [1] are not valid. Our results suggest that the percolation transition in fully interdependent diluted lattices ($q < 1$) belongs to the same universality class as OP ($q = 1$) and characterized by the same critical exponents. Thus, their main conclusion (reflected also in the title) that dependency may increase robustness is not proven. Indeed, it is expected that dilution of a lattice ($q < 1$) should not change the topology and therefore will not change the universality class. We study here larger systems than [1] up to $N = L \times L = 3000 \times 3000$ (10^4 realizations) and approach closer to the critical threshold where universal scaling is expected.

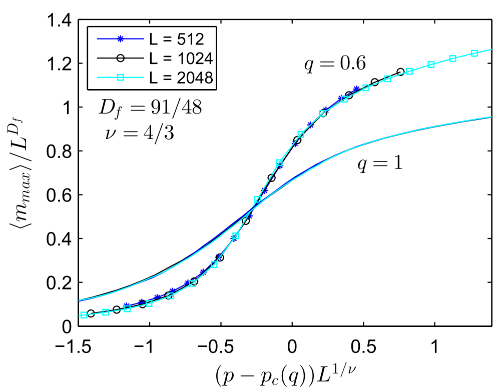

FIG. 1 (color online). Data collapse for $\langle m_{\max}\rangle / L^{D_f}$ against $(p - p_c)L^{1/\nu}$, for 2D lattices. Each set of three curves for different L corresponds to one value of q. For this plot $\nu = 4/3$ and $D_f = 91/48$ were used. The values of p_c are 0.9609, and 0.5927 for $q = 0.6$ and $q = 1$, respectively. Note also that this figure shows that for $q = 0.6$ the transition is sharper compared to $q = 1$ in contrast to the claim in [1].

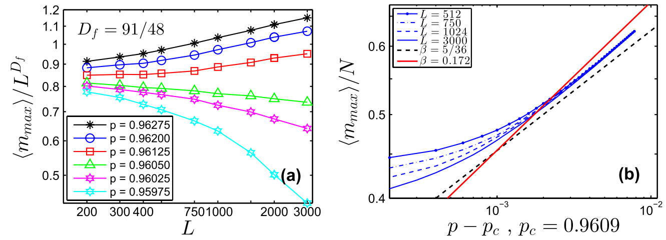

FIG. 2 (color online). (a) Log-log plot of $\langle m_{\max}\rangle / L^{D_f}$ against L, for 2D interdependent percolation at $q = 0.6$, at fixed values of p. At the critical point ($p_c \simeq 0.9609$) we expect a horizontal line. The value of the fractal dimension is $D_f = 91/48$ as in OP. (b) Log-log plot of $\langle m_{\max}\rangle / N$ against $p - p_c$, for 2D interdependent percolation with $q = 0.6$. It is clear that the exponent $\beta = 5/36$ (dashed) fits better than $\beta = 0.172$ (red) claimed in [1].

Figure 1 shows the finite size scaling ansatz [2,3]

$$\langle m_{\max}\rangle = L^{D_f} f((p - p_c)L^{1/\nu}),$$

where ν is the correlation length exponent, and $D_f = d - \beta/\nu$ is the fractal dimension of the incipient infinite cluster. According to this ansatz, we expect a data collapse if we plot $\langle m_{\max}\rangle / L^{D_f}$ against $(p - p_c)L^{1/\nu}$ near p_c. Indeed, a data collapse was obtained in Fig. 1 for $q = 0.6$ with exactly the same exponents as for $q = 1$. This indicates that the case of interdependent diluted lattices belongs to the same universality class as OP. Once D_f and ν are known, we obtain the value of $\beta = \nu(d - D_f) = (4/3)(2 - 91/48) = 5/36$.

Figure 2(a) shows that for $q = 0.6$, $\langle m_{\max}\rangle \sim L^{D_f}$ for $p_c \simeq 0.9609$, with $D_f = 91/48$ while Fig. 2 shows that $\langle m_{\max}\rangle \sim (p - p_c)^{\beta}$ in the limit $L \to \infty$, with $\beta = 5/36$. Both exponents are the same as for OP.

Note that we agree that the model presented in [1] has a continuous percolation transition. This model corresponds to the case of the zero dependency length as shown later in [4].

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