One highlight of the reported results is the ability to tune this critical rotation rate.

The experiment by Wright *et al.* represents an advance in the emergent field of atomtronics. Modern electronics rely on the transport and interaction of electrical charges, primarily in semiconductors, whereas atomtronic devices rely on neutral atoms, whose characteristics include an internal structure, tunable interactions and long coherence times[6]. In the future it may be possible to fabricate atomtronic devices analogous to batteries, diodes and transistors, as well as fundamental logic gates[7].

Atom SQUIDs have the potential to improve the sensitivity of rotation-measuring devices up to ten orders of magnitude compared with today's photon-based precision inertial navigation technology[8]. The work of Wright *et al.* represents a step towards the realization of a new generation of inertial sensing devices that have an ultracold atomic gas at their heart. ❐

*Mark Edwards is in the Department of Physics, Georgia Southern University, Statesboro, Georgia 30460-8031, USA.*
*e-mail: edwards@georgiasouthern.edu*

### References

1. Pitaevskii, L. & Stringari, S. *Bose-Einstein Condensation* (Oxford Univ. Press, 2003).
2. Ramanathan, A. *et. al. Phys. Rev. Lett.* **106,** 130401 (2011).
3. Clarke, J. & Braginski, A. I. *The SQUID Handbook* Vol. 1 & 2 (Wiley-VCH, 2004).
4. Wright, K. C., Blakestad, R. B., Lobb, C. J., Phillips, W. D. & Campbell, G. K. *Phys. Rev. Lett.* **110,** 025302 (2013).
5. Henderson, K., Ryu, C., MacCormick, C. & Boshier, M. G. *New J. Phys.* **11,** 043030 (2009).
6. Pepino, R. A., Cooper, J., Anderson, D. Z. & Holland, M. J. *Phys. Rev. Lett.* **103,** 140405 (2009).
7. Seaman, B. T., Krämer, M., Anderson, D. Z. & Holland, M. J. *Phys. Rev. A* **75,** 023615 (2007).
8. Gustavson, T. L., Bouyer, P. & Kasevich, M. A. *Phys. Rev. Lett.* **78,** 2046 (1997).

---

NONLINEAR DYNAMICS

# New tricks for big kicks

Techniques for understanding how a system responds to an infinitesimal perturbation are well developed — but what happens when the kick gets stronger? Insight into the topology of phase space may now provide the answer.

Avi Gozolchiani and Shlomo Havlin

In the winter of 1961, meteorologist Edward Lorenz was hard at work putting his computer through the paces of a weather-prediction simulation. Pressed for time, Lorenz decided to rerun his resource-hungry program from an intermediate time point, dispensing with the need to start from scratch. He dutifully copied the initial conditions from his printout into his simulation, and was astonished to find that the new weather pattern quickly diverged from its predecessor — meaning that two phase-space trajectories with seemingly identical initial conditions had deviated from one another exponentially[1].

We all know the punchline to this story, which forms part of a larger history of phase space[2] and in many ways marks the beginning of our modern understanding of chaos theory. In this endeavour, linearization techniques for probing infinitesimal perturbations have proven particularly useful, largely due to their inherent simplicity. Now, writing in *Nature Physics*, Peter Menck and colleagues urge us to pay closer attention to a more general scenario, in which perturbations are not necessarily so small[3].

The rate of deviation of two close trajectories, quantified by the Lyapunov exponent, can be calculated analytically by linearizing a set of dynamical equations. But in the case where linearization yields a negative Lyapunov exponent, trajectories stick together, falling towards a stable fixed point instead of exponentially diverging. The larger the magnitude of the negative Lyapunov exponent, the more linearly stable its fixed point is. In such situations, the proverbial flapping of a butterfly's wings in Brazil will not have much of an effect on weather patterns in Texas. More specifically, a trajectory **d** initiated at some distance from a stable fixed point **p** will approach it only if the initial state **p** + **d** is within the basin of attraction of the fixed point.

A bifurcation diagram quantifying the available stable fixed points — on the basis of a set of dynamical equations — is an important tool in climate sciences, where multistability is common. There is a multitude of numerical methods dedicated to finding the fixed points in large multidimensional systems[4]. Complementary methods, based on the statistical properties of integrated time series, either from models or from measurements, are commonly used to assess the proximity of a system to bifurcation points[5,6]. But both the numerical methods and statistical time-series analysis implicitly assume that externally applied stochastic perturbations remain small with respect to the basin of attraction. However, a real external perturbation **d** may kick a system far away from the basin of a stable point even if **p** is the most linearly stable fixed point for the specific dynamics in question.

This provides ample motivation for studying basins of stability — and in particular, the volume of these basins. Menck *et al.*[3] chose real-world neural networks and power grids as key systems for which the notion of basin-volume assessment is particularly helpful. It turns out that engineering considerations may have led to a peculiar property shared by these two systems: both evolution and engineers seem to have unconsciously chosen a design with similar specific pathway lengths and a comparable level of acquaintance between common neighbours.

These two properties form a 2D metric that covers the span of topologies between fully organized lattices and completely random network models[7]. Menck *et al.*[3] found that neural networks of macaques, cats and *Caenorhabditis elegans*, together with the power grids of four countries, happen to be close to each other in terms of this metric. The surprising similarity of these diverse systems in the realm of theoretically possible network topologies seems to rest with their need for supporting synchronized dynamics.

Drawing on previously derived conditions for the linear stability of synchronized states[8], Menck *et al.*[3] compared linear stability with a new global stability concept based on the basin's volume, for a multitude of network topologies. To accomplish this task, they used a Monte Carlo network generator with a control parameter spanning a range of topologies, from lattices to completely random Erdös–Rényi networks. Each node in their network was a 3D nonlinear oscillator coupled to other nodes. The authors randomly chose initial conditions **p** + **d** from a partial subset of the full phase space and counted the trajectories that ended up in a synchronized dynamics (describing one example of a Monte Carlo rejection method[9]).

The study demonstrated that, in the range of possible topologies, the widest synchronization basin of stability for the network of oscillators was situated far away in phase space from the most linearly stable configuration. That is, the shortest paths in the network were actually longer in the optimal configuration, as defined by the basin volume, and acquaintance between shared neighbours of two adjacent nodes was found to be more common than expected from linear considerations. The experimental data from different networks that rely on synchronized dynamics were shown to lie close to the optimum chosen by the basin-stability model (although it should be noted that the details of single-node dynamics differ from the model).

The upshot of the study by Menck *et al.*[3] is that we can now revise our understanding of systems with multiple stable points, for which we already have full bifurcation diagrams and Lyapunov exponents. Models for neural and cell networks, ocean circulation and chemical oscillators are just a few examples of those that stand to benefit from this new insight into basin stability beyond linearization. Using the authors' Monte Carlo rejection method to estimate the volume of the stable basin may even shake up some of our previous ideas — particularly those involving the chance that an almighty kick will induce an abrupt transition to an alternative stable state[10]. ❒

*Avi Gozolchiani and Shlomo Havlin are in the Department of Physics, Bar Ilan University, Ramat Gan 59200, Israel.*
*e-mail: havlin@ophir.ph.biu.ac.il*

## References

1. Lorenz, E. N. *J. Atmos. Sci.* **20,** 130–141 (1963).
2. Nolte, D. D. *Physics Today* **63,** 33–38 (2010).
3. Menck, P. J., Heitzig, J., Marwan, N. & Kurths, J. *Nature Phys.* **9,** 89–92 (2013).
4. Dijkstra, H. *Nonlinear Physical Oceanography* (Springer, 2005).
5. Lenton, T. M. *Nature Clim. Change* **1,** 201–209 (2011).
6. Kuehn, C. *Physica D* **240,** 1020–1035 (2011).
7. Li, D., Kosmidis, K., Bunde, A. & Havlin, S. *Nature Phys.* **7,** 481–484 (2011).
8. Pecora, L. M. & Carroll, T. L. *Phys. Rev. Lett.* **80,** 2109–2112 (1998).
9. Von Neumann, J. *J. Res. Natl Bur. Stand.* **12,** 36–38 (1951).
10. Scheffer, M. *Critical Transitions in Nature and Society* (Princeton Univ. Press, 2009).

Published online: 6 January 2013