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Cascading failure and recovery of spatially interdependent networks

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Abstract. Many real networks, such as infrastructure networks, are interdependent and their structure is influenced by spatial constraints. However, the existence of spatial constraints and dependency links makes the system more vulnerable to an initial random failure. In this paper, we model the interdependent network with the strength of embedding length ζ. We propose an effective recovery strategy which recovers the boundary of the failed nodes during the cascading of failures with probability γ. We find that without changing the transition type, both the range and the duration of the cascading failure can be decreased by increasing γ. Our model could be used to improve the robustness of real-world network systems.

Keywords: network reconstruction, critical phenomena of socio-economic systems, network dynamics

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Contents

1. Introduction

The cascading of failures, is a process that evolves on top of systems, such as infrastructures, triggered by an initial failure or overload that propagates eventually destroying the functionality of the full system. The last few years this process has been studied in Networks of Networks such as Interdependent networks composed by internal connectivity links inside each network and interdependent links between them. It was shown that under an initial failure $1 - p$ of the nodes in one network, the cascade of failures propagates between the networks step by step. At each time step, depending on the value of p the system decreases its resilience and at a critical threshold p_c the system fully collapse, i.e. at this critical value the system overcomes a first order phase transition in which the size of the functional giant component (GC) suddenly jump from a finite value to zero. The cascading of failures in interdependent complex random regular (RR), Erdős–Rényi (ER) networks and scale-free (SF) networks have been extensively studied via simulations and theoretically using the percolation framework [1–13]. In order to decrease the catastrophic effects of the cascade of failures, many researchers studied how to improve the resilience of complex networks, modeling mitigation and recovering strategies [24–28]. Motter proposed a selective removal strategy of nodes that is applied after the initial failure [24]. Majdandzic *et al* studied the spontaneous recovery in single and interacting networks based on a mean-field approach [25, 26]. Valdez *et al* [30] proposed a spontaneous recovery model with competition between different kinds of failures using a degree based approach. Stippinger *et al* [27] introduced a healing process, to bridge non-functioning nodes enhancing the network resilience. Di Muro *et al* [2] implemented a recovery strategy for interdependent networks under the cascading of failures in which the nodes in the border of the functional GC are restored. Hu *et al* [28] proposed a recovery strategy in a single infrastructure network under localized attacks and applied it to real-world highway network. Böttcher *et al* [31] studied the spontaneous failure and recovery in the single spatially embedded and random networks which mapped the dynamics to a generalized contact process.

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Their research suggests that the spatially embedded system with short characteristic link lengths whose dynamics are described by their model are substantially more robust against abrupt failures.

All these studies were performed in complex networks. However, many complex systems in the real world, such as transportation, infrastructure and the neural networks are often organized under the form of the networks in which the length of the links between nodes is spatially constrained. Isolated spatial networks such as the lattice based model [14], the random geometric graph [15], the Waxman model [16], the spatial Watt–Strogatz model [17] and the spatial growths model [18] have being proposed in the past. In the lattice based model links are more likely to exist between nearby nodes than between distant nodes. As a consequence interdependent infrastructure networks can be represented as multiplex networks, in which each network or layer has connectivity links among nearby nodes. In lattice networks, the effect of spatially on the robustness of a multiplex network embedded in a 2-dimensional space, in which links in each layer are variable but constrained length, was studied in [19–23]. It is found that weak coupling leads to an abrupt collapse whereas the non-embedded networks undergo a smooth continuous transition, which indicate the extreme vulnerability of the spatially multiplex network [19]. The authors also find that when the length of the dependency link is longer than a certain critical value, abrupt, discontinuous transitions take place, while at and below this critical value the transition is continuous, indicating that the risk of an abrupt collapse can be eliminated if the typical link length is shorter than the critical one [23]. On the other hand, few strategies have being proposed to restrain the cascading of failures in interdependent lattice networks, however the long range links between nodes were not taken into account. Increasing the probability of long-range links in the network, using a strength of spatially embedding, will reduce the robustness of the spatially multiplex networks [29] under an initial random failure and thus it is important to implement recovery strategies in order to avoid the collapse of the system.

In this work, we study a recovery strategy that is initiated at the beginning of the cascade of failures in a spatially interdependent networks. We find that our recovery strategy is effective for all the values of spatially embedding strength ζ. By recovering the failed nodes during the cascading of failures and connecting them to the giant components with probability, γ, the critical percolation threshold of the network decreases, increasing the robustness of the system.

2. Model

2.1. The spatially interdependent network

In this paper, we consider two single spatially interdependent networks, denoted by A and B. Each single network model is a variant of the Waxman model [16]. This model is similar to the model in [29], and the modeling process can be summarized as follow:

(i) We assign $N = L^2$ nodes, where each node is in the coordinate (x_i, y_i) with $i = 1, \ldots, L$.

(ii) We randomly choose a node and compute the length of the link which starts from this node according to the length distribution $P(l) \sim e^{-l/\zeta}$ where ζ is the strength of the spatially embedding length. Since the distance between each pair of nodes i and j is $d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$, we find the set of nodes, in the single network, $\Omega = \{(x_2, y_2) \,|\, \min(|l - d_{12}|), (x_2, y_2) \in [1 \ldots, L]\}$ i.e. the distance between (x_1, y_1) and (x_2, y_2) closer to l. Then we randomly choose a node with coordinates $(x_2, y_2) \in \Omega$ and build a link between them.

(iii) We repeat step 2., until the number of the links reaches the desired number $N \langle k \rangle /2$, where N is the system size and $\langle k \rangle$ is the average degree of the single network.

(iv) We build the dependency links between nodes at the same position in different networks.

In figure 1 we show an example of the spatially interdependent network for $L = 10$.

2.2. Cascading failure and recovery procedure

The cascading failure process of the system is initiated by the random failure of a fraction $1 - p$ of nodes in network A. The nodes that do not belong to the GC_A fail and as a consequence the interdependent nodes in the other network fail together with the finite components. Then the failure propagates at each time step of the cascade until the GC's of both networks reach the steady state in which both GC's have the same size [24]. Next we present the recovery strategy [5].

(i) The initial failure is triggered by removing a fraction $1 - p$ of nodes from network A. Due to the dependency links, nodes with the same position in network B also fail. The failed nodes of each network are denoted as $Fail_A\,(t = 0)$ and $Fail_B\,(t = 0)$, where $t = 0$ is the initial time step.

(ii) For each pair of interdependent nodes that belong to $Fail_A\,(t = 0)$ and $Fail_B\,(t = 0)$, and were connected to the GC of each network $GC_A\,(t = 0)$ and $GC_B\,(t = 0)$, we recover both nodes with probability γ. Note that the recovery restores the failed nodes together with the links which connect the remaining functional nodes.

(iii) The failure propagates through the connectivity link in network A. All the nodes which do not belong to the GC of network A will fail, and these failed nodes will produce the failure of their interdependent nodes in network B. We update the nodes in $Fail_A$, $Fail_B$, the GC_A and GC_B, and remove their connectivity in this step.

(iv) As in step 2, we recover the pair of failed nodes which connected to the GC_A and GC_B at the initial state with probability γ and update the GC_A and GC_B.

(v) The nodes in network B fail if they do not belong to the GC_B, then the nodes in network A become dysfunctional if their interdependent node in network B fails. We update $Fail_A$, $Fail_B$, GC_A and GC_B.

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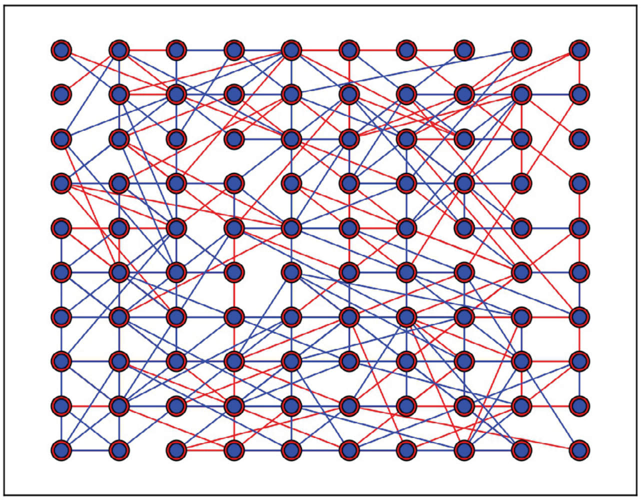


Figure 1. The spatially interdependent network for $L = 10$. The red nodes and edges belong to network A and the blue nodes and edges to network B. The nodes of each layer at the same position are connected by dependency links.

(vi) We recover with probability γ the pair of interdependent failed nodes connected to the GC_A and GC_B.

(vii) We repeat step 3-6 until the steady state is finally reached.

A schematic of the cascading of failures with the recovery procedure is shown in figure 2.

3. Results and analysis

We generate the spatially interdependent square network with $N = 100 \times 100$, $\langle k \rangle = 3$ and 4 respectively.

The simulation results are obtained by the Monte Carlo simulations with at least 10 realizations. We compute the number of iteration steps (NOI) needed to reach the steady state which indicates the time scale of the process, and the order parameter P_∞ which is the relative size of the GC of the networks at the end of the process.

In order to visualize the dynamical process of the cascading of failures and recovery in figure 3, we plot the network structure at NOI = 1, 3, 6 and 8 for p above the threshold. From the plot we can see that the GC of the networks reaches a stable state.

Figure 4 shows the order parameter P_∞ and NOI as a function of p and γ, when $\gamma > 0$, $\langle k \rangle = 3$ and $\zeta = 1$. We can see that when the recovery strategy is taken, the network still undergoes a second order transition. But when the recovery probability γ improved, the percolation threshold p_c decreases from 0.86 to 0.76, and the NOI at p_c from 8 to 6. Thus our recovery strategy improves the robustness of the spatially interdependent networks affected by the random failure, decreases both the range and the duration of the cascading failure.

Beside the embedding strength, the average degree of the network will also influence the effect of the recovery strategy. As shown in figures 5 and 6, as $\langle k \rangle$ increased to 4,

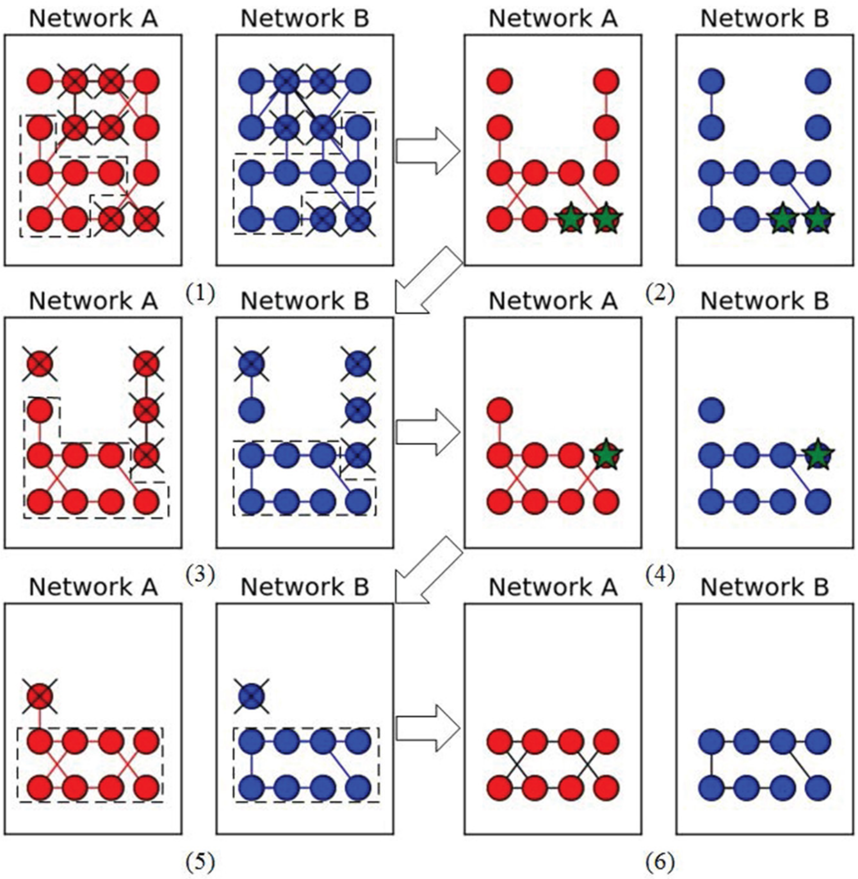


Figure 2. The cascading failure process with recovery. The failure node in each step is denoted by an X mark and the recovery node is denoted with the green pentagram. The nodes which belong to the GC of each network are surrounded by dashed line.

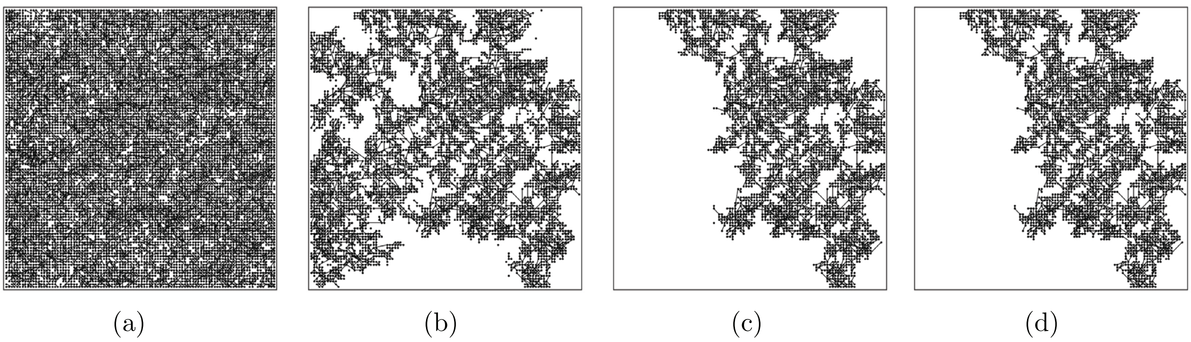
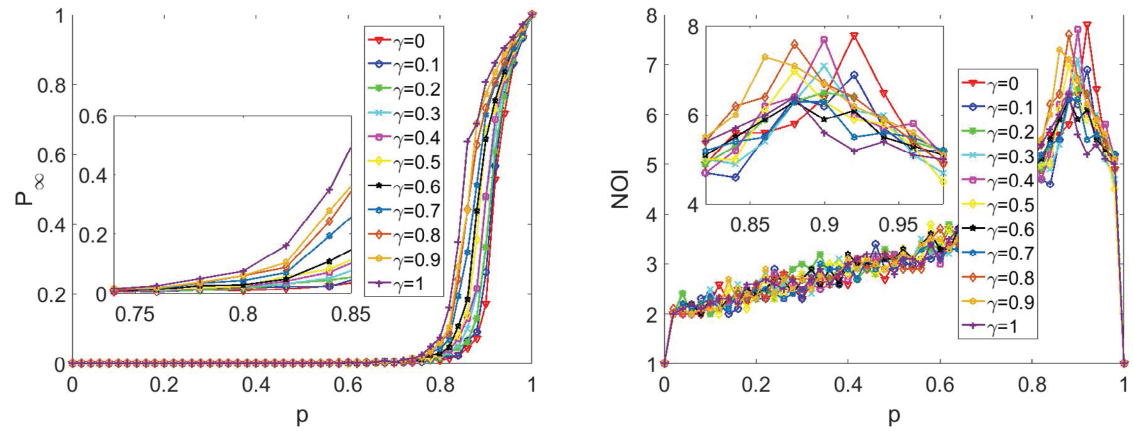


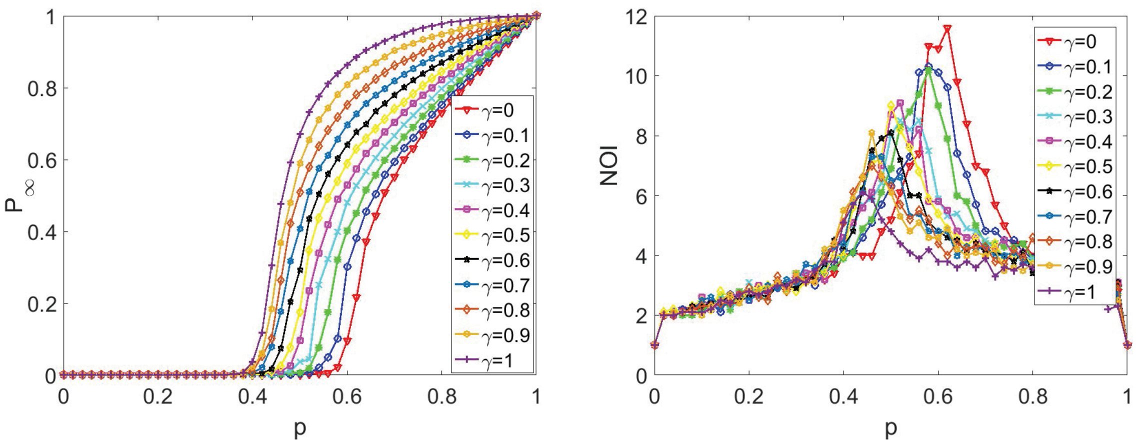
Figure 3. The network structure at NOI $=1$ (a), 3 (b), 6 (c) and 8 (d). Functional nodes are colored in black and dysfunctional nodes in white. Here $L=100, \langle k\rangle=3, \zeta=1, p=0.83, \gamma=0.5$.

the recovery strategy is more efficient. This is due to the fact that the recovered nodes are selected from the failed nodes which were connect to the GC. A degree distribution with increasing broadness increases the connectivity of the node and as a consequence the networks become more resilient. From the figures we can also see that increasing



(a) (b)

Figure 4. The cascading failure procedure with recovery. For average degree $\langle k\rangle = 3$ and $\zeta = 1$. (a): Order parameter P_∞ as a function of p and γ, (b): the number of iteration steps NOI as a function of p and γ.

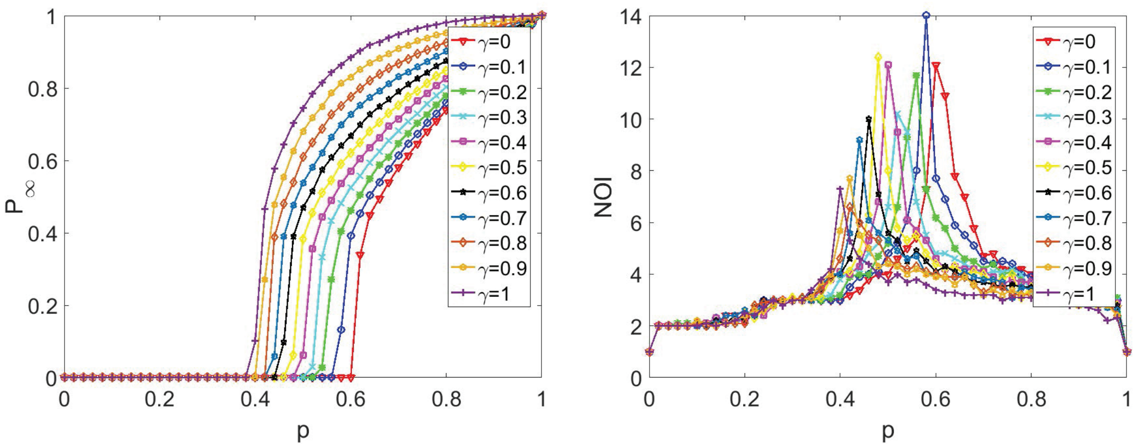


(a) (b)

Figure 5. The cascading failure procedure with recovery. The average degree of this network $\langle k\rangle = 4$, and the strength of spatially embedding $\zeta = 3$. (a): the order parameter P_∞ as a function of p and γ. (b): the number of iteration steps NOI as a function of p for different values of γ.

ζ the efficiency of our strategy improves. In fact, the probability of an edge $P(l)$ will become a constant for different edge length l when $\zeta \to \infty$. So the increase of ζ will make the network more like an interdependent ER network, hence improve the resilient of the network. Therefore, as ζ and $\langle k\rangle$ increases, a failed node increases the probability to be connected to the GC of functional nodes and will lead to an increasing recovery effect.

Moreover, the structure of real spatially networks is neither the lattice network or the ER random network. As shown in [29], ζ, the strength of the spatially embedding of EU power grid, has $\zeta \approx 6$ and Japan local railway network has $\zeta \approx 12$, remaining thus in an intermediate value between zero and infinity. Therefore, as shown in figure 5, our



(a) (b)

Figure 6. The cascading failure procedure with recovery. The average degree of this network $\langle k \rangle = 4$, and the strength of spatially embedding $\zeta = 20$. (a): the order parameter P_∞ varying with p and γ. (b): the number of iteration steps NOI varying with p and γ.

recovery strategy is very effective in improving the robustness of the interdependent network, by decreasing the percolation threshold p_c form 0.56 to 0.38, and the NOI at p_c from 12 to 6. Thus our recovery strategy can be successfully implemented in a real-world network.

4. Conclusion

In summary, we investigated the modeling process of a spatially interdependent network. Based on this network model, we proposed the in-process recovery strategy during the cascading failure procedure. The simulation is made to illustrate that we can lower the percolation threshold p_c and improve the network robustness against random failure by adjusting the recovery probability γ. By analyzing the parameter characteristics of the real-world networks, we describe the implementation of the recovery strategy. Our future work will consider how the load and capacity features influence the cascading failure procedure of this network and provide an efficient recovery strategy.

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