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The influence of persuasion in opinion formation and polarization

C. E. La Rocca[1], L. A. Braunstein[1,2] and F. Vazquez[3]

[1] *Instituto de Investigaciones Físicas de Mar del Plata, UNMDP-CONICET - 7600, Mar del Plata, Argentina*
[2] *Center for Polymer Studies, Physics Department, Boston University - Boston, MA 02215, USA*
[3] *Instituto de Física de Líquidos y Sistemas Biológicos, UNLP-CONICET - 1900, La Plata, Argentina*

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Abstract – We present a model that explores the influence of persuasion in a population of agents with positive and negative opinion orientations. The opinion of each agent is represented by an integer number k that expresses its level of agreement on a given issue, from totally against $k = -M$ to totally in favor $k = M$. Same-orientation agents persuade each other with probability p, becoming more extreme, while opposite-orientation agents become more moderate as they reach a compromise with probability q. The population initially evolves to (a) a polarized state for $r = p/q > 1$, where opinions' distribution is peaked at the extreme values $k = \pm M$, or (b) a centralized state for $r < 1$, with most opinions around $k = \pm 1$. When $r \gg 1$, polarization lasts for a time that diverges as $r^M \ln N$, where N is the population's size. Finally, an extremist consensus ($k = M$ or $-M$) is reached in a time that scales as r^{-1} for $r \ll 1$.

Introduction. – Many empirical investigations show the importance of social influence in the formation of people's opinions. For instance, it is argued that two interacting partners may exert social pressure to change their attitudes to conform each other [1]. Some physics models have incorporated this particular social mechanism by means of a *compromise process* [2–5]. In these models, opinions are represented by a real number between two extreme values, and pair of individuals interact only if their opinion difference is smaller than a given threshold. Individuals resolve the conflict by reaching a compromise, in which both opinions are changed in the same amount to reduce their difference. A less explored mechanism of social interactions is the *persuasive arguments exchange* [6–9]. As observed by Myers [6] in group discussion experiments, when two individuals talk, they do not only state their opinions, but they also discuss about the arguments that support their opinions. Then, if they already hold the same opinion orientation, they could intensify their opinions by persuading each other with new arguments or reasons, becoming more extreme in their believes. This mechanism was proposed by Lau and Murnighan [8] after the works by Myers [6] and Isenberg [7], and recently explored by Mäs *et al.* [9] using a computational model.

In this letter, we introduce a simple model that explores the competition between the compromise and persuasive-argument mechanisms in a population of N interacting agents. The state of each agent is represented by an integer number k ($-M \leq k \leq M$ and $k \neq 0$), where the sign of k indicates its opinion orientation, like for instance to be in favor (positive) or against (negative) marijuana legalization, and the absolute value $|k|$ measures its opinion intensity or strength. Thus, $k = M$ ($-M$) correspond to extremists which are strongly in favor (against) of legalization, while $k = 1$ and -1 represent moderates. In a time step, two agents with states j and k are picked at random to interact. Then, their states are updated according to two elemental processes (see fig. 1).

i) *Compromise*: if they have opposite orientations, their intensities decrease in one unit with probability q:

– If $j < 0$ and $k > 0 \Rightarrow (j,k) \rightarrow (j^r, k^l)$ with prob. q.

– If $j > 0$ and $k < 0 \Rightarrow (j,k) \rightarrow (j^l, k^r)$ with prob. q.

If $j = \pm 1$ and $k = \mp 1$, one switches orientation at random:

$$(\pm 1, \mp 1) \rightarrow \begin{cases} (1,1) & \text{with prob. } q/2, \\ (-1,-1) & \text{with prob. } q/2. \end{cases}$$

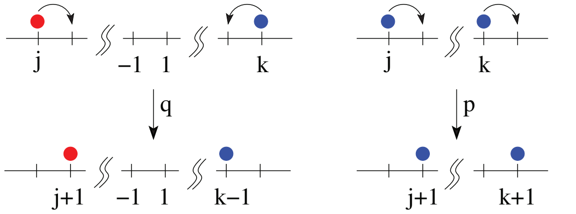

Fig. 1: (Colour on-line) Two main processes of the model. Left: compromise: two interacting agents with opposite opinion orientation become more moderate. Right: persuasion: two interacting agents with the same orientation become more extremists.

ii) *Persuasion*: if they have the same orientation, their intensities increase by one unit with probability p:

– If $j < 0$ and $k < 0 \Rightarrow (j, k) \rightarrow (j^l, k^l)$ with prob. p.

– If $j > 0$ and $k > 0 \Rightarrow (j, k) \rightarrow (j^r, k^r)$ with prob. p.

Here k^r and k^l denote the right and left neighboring states of k, respectively, defined as

$$k^r = \begin{cases} 1, & \text{for } k = -1, \\ M, & \text{for } k = M, \\ k+1, & \text{otherwise,} \end{cases} \qquad k^l = \begin{cases} -1, & \text{for } k = 1, \\ -M, & \text{for } k = -M, \\ k-1, & \text{otherwise.} \end{cases}$$

With this dynamics, opinions are constrained to the interval $[-M, M]$ and the neutral opinion $k = 0$ is excluded. We find that the population's opinion settles in a centralized state when the compromise process dominates ($q > p$), and in a polarized state when persuasion dominates ($p > q$). These states are not stable, and the system ultimately reaches extremist consensus. We solve the equations for the dynamics in the stationary state, and also in the strong and small persuasion limits, and find that the mean extremist consensus time is non-monotonic in the ratio p/q.

We note that similar mechanisms to the compromise process i) are found in nonlinear and multiple-state voter models with a reinforcement rule [10–15], in which agents switch orientation (opinion's sign) only after receiving multiple inputs of agents with the opposite orientation. Besides, persuasion was used in recent works [16,17] as a degree of a person's self-conviction, where in addition to the influence from others, a person takes into account its own opinion when making a decision. Also, persuasion between opposite-orientation agents was recently studied in [15]. However, we understand that the mechanism of strengthening of opinions due to same-orientation interactions has not been investigated within an interacting particle model.

Dynamics. – We study the dynamics of the system by looking at the time evolution of the number of agents in the different opinion states. We denote by $x_k(t)$ the fraction of agents in state k at time t. Initially, states are uniformly distributed, thus $x_k(t = 0) \simeq 1/2M$. Figure 2 shows results from Monte Carlo (MC) simulations for $M = 5$ and a population of size $N = 10^9$. Given that qualitative results depend on the ratio $r \equiv p/q$ that relates the persuasion and compromise time scales, we show two representative cases, one with $r = 3$ (fig. 2(a)) and the other with $r = 1/3$ (fig. 2(b)). We observe that densities x_k reach a nearly constant value (plateau) that depends on k, but eventually all x_k decay to zero, except x_M that goes to 1, corresponding to a consensus in the extremist state M. The two extremists consensus $x_{\pm M} = 1$ are absorbing states of the system, thus they are the only possible final states in the long run. The length of the plateau increases with the system size as $\ln N$ (not shown), a typical time scale that appears in models with intermediate states [10,14]. We shall see that this particular scaling is also a consequence of the discrete nature of the system when a small initial asymmetry is introduced [14].

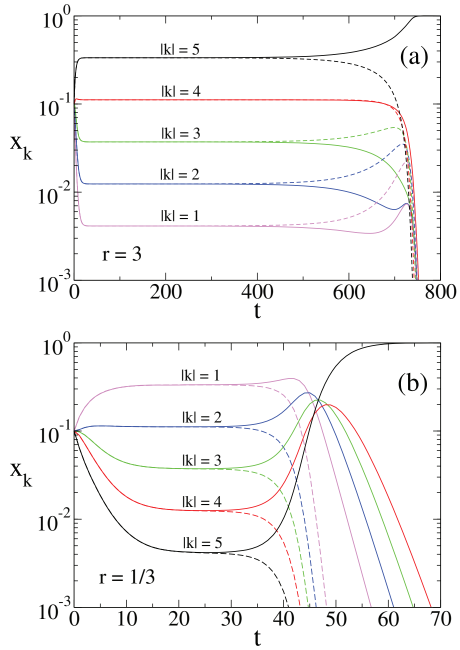

Fig. 2: (Colour on-line) Time evolution of the fraction of agents in different opinion states, for maximum opinion intensity $M = 5$ in a population of $N = 10^9$ agents. (a) $x_k(t)$ for $p = 3/4$ and $q = 1/4$. (b) $x_k(t)$ for $p = 1/4$ and $q = 3/4$. Solid (dashed) curves correspond to positive (negative) opinions. A logarithmic scale was used in the y-axis to clearly see all plateaus together.

The structure of the population at the quasistationary state or plateau shows interesting properties, as can be seen in fig. 3 where we plot x_k for a given time in the plateau. The distribution of opinions depends on the ratio r, which controls the relative frequency of persuasion and compromise events. When $r > 1$, the persuasion process dominates over compromise, driving the states of agents towards the extreme opinions $k = \pm M$. This induces opinion polarization, where x_k is symmetric and peaked at the opposite extreme values (see fig. 3(a)). Instead, for $r < 1$ compromise events occur more often than persuasive encounters, thus most opinions accumulate around the moderate values $k = \pm 1$, inducing a centralized opinion state where x_k has a maximum value at center states (see fig. 3(b)).

Stationary states. – To gain an insight about these observations, we write and analyze a set of ordinary differential equations for the time evolution of x_k. Here we consider for simplicity the large-N limit, where demographic noise coming from system size fluctuations is neglected. Then, the densities of positive states evolve according to the following set of equations:

$$\frac{dx_1}{dt} = 2(x_{-1}q - x_1 p)\sigma_+ + 2q(x_2 - x_1)\sigma_-, \tag{1a}$$

$$\frac{dx_k}{dt} = 2p\,(x_{k-1} - x_k)\,\sigma_+ + 2q\,(x_{k+1} - x_k)\,\sigma_-, \quad \text{for } 2 \le k \le M-1, \tag{1b}$$

$$\frac{dx_M}{dt} = 2p\,x_{M-1}\sigma_+ - 2q x_M \sigma_-, \tag{1c}$$

where $\sigma_+ = \sum_{k=1}^{M} x_k$ and $\sigma_- = \sum_{k=-1}^{-M} x_k$ are the total densities of positives and negatives states, respectively, which satisfy the density conservation constraint $\sigma_+ + \sigma_- = 1$. Equations for negative-state densities are obtained from eqs. (1) by the transformations $k \leftrightarrow -k$ and $\sigma_+ \leftrightarrow \sigma_-$. The gain and loss terms in the rate equations account for the different processes. The first term describes persuasive interactions between two positive agents, while the second term accounts for the compromise between positive and negative agents. In addition, the gain term $2q\,x_{-1}\,\sigma_+$ in eq. (1a) corresponding to $-1 \to 1$ transitions, describes the negative to positive flux of states, while the absence of the loss term $-2p\,x_M\sigma_+$ and the gain term $2q\,x_{M+1}\sigma_-$ in eq. (1c) reflect the fact that there is no state flux through the $k = M$ boundary.

The properties of the quasistationary distributions of fig. 3 can be obtained by studying the stationary solutions of eqs. (1). The two trivial solutions $x_M = 1$ and $x_{-M} = 1$ correspond to the M and $-M$ extremists consensus, respectively, where all agents end up with the same maximum opinion intensity. These are stable fixed points in the space of densities. But there is also a non-trivial solution that corresponds to a balanced mix of positive and negative agents, as the ones in fig. 3. Setting $\frac{dx_k}{dt} = 0$ and $\sigma_+ = \sigma_- = 1/2$ in eqs. (1) we obtain a linear system of algebraic equations that can be solved by iteration. The solutions are $x_k^s = x_1^s\, r^{k-1}$ for $1 \le k \le M$ and $x_k^s = x_{-1}^s\, r^{-k-1}$ for $-M \le k \le -1$, with $r = p/q$. Using the normalization condition $1/2 = \sigma_+ = \sum_{k=1}^{M} x_k = \frac{x_1(1-r^M)}{(1-r)}$ and $1/2 = \sigma_- = \sum_{k=-1}^{-M} x_k = \frac{x_{-1}(1-r^M)}{(1-r)}$, we obtain the values

$$x_1^s = x_{-1}^s = \frac{1-r}{2(1-r^M)}. \tag{2}$$

Finally, densities at the quasistationary mixed state are

$$x_k^s = \frac{1}{2}\left(\frac{1-r}{1-r^M}\right) r^{|k|-1} \quad \text{for} \quad -M \le k \le M. \tag{3}$$

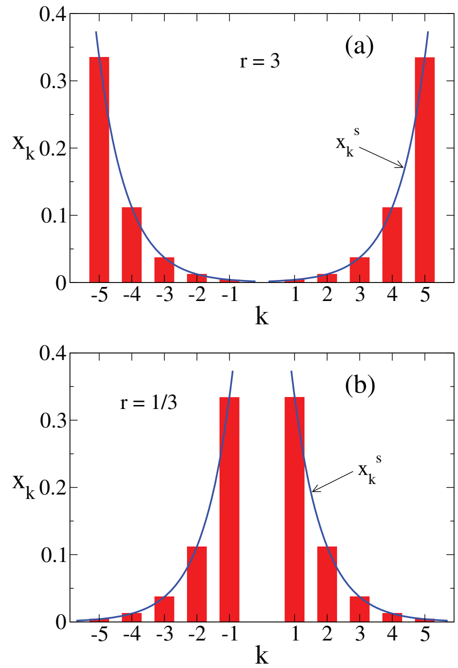

Fig. 3: (Colour on-line) Distribution of opinions' densities at the quasistationary mixed state of fig. 2, and for the same parameter values. (a) x_k at time $t = 200$. (b) x_k at $t = 20$. Solid lines correspond to expression (3).

In fig. 3 we observe that expression (3) in solid lines gives a good mathematical description of the opinions' distributions from MC simulations, in a population of agents whose opinions are polarized ($r > 1$) or centralized ($r < 1$).

To study the stability of these states we have integrated eqs. (1) numerically for $M = 5$ and two values of r. The time evolution is very similar to the one depicted in fig. 2. We mimic the initial state of MC simulations by taking $x_k(t = 0) = 1/2M + \epsilon$, where $|\epsilon| = N^{-1/2}$ corresponds to a stochastic size fluctuation with respect to the uniform state. All densities quickly reach a nearly constant value in time, corresponding to the mixed solution x_k^s of eq. (3), and stay very close to this attractor for a time that scales as $\ln N$, to finally reach either fixed point $x_{\pm M} = 1$. The attractor x_k^s corresponds to a saddle point of the dynamics —starting from the exact uniform state $x_k(t = 0) = 1/2M$ (or any symmetric case $x_k = x_{-k}$) causes the system to hit x_k^s, and stay there. But any small initial asymmetry, for

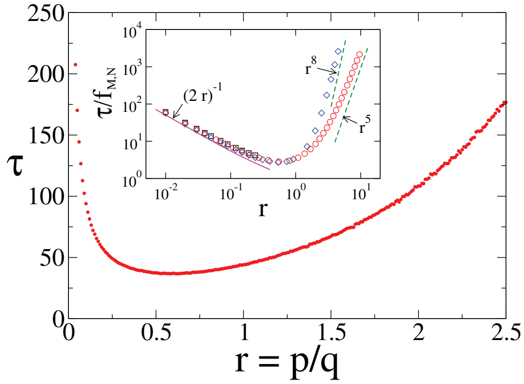

Fig. 4: (Colour on-line) Main: mean extremist consensus time τ *vs.* persuasion strength $r = p/(1-p)$ for $M = 5$ and $N = 1000$. Inset: rescaled time τ *vs.* r in log-log scale for $M = 3$ (squares), $M = 5$ (circles) and $M = 8$ (diamonds). The solid line corresponds to the approximation (9) in the $r \ll 1$ limit, while dashed lines denote the asymptotic behavior r^M in the $r \gg 1$ limit.

instance in the positive opinion, makes the system stay in the vicinity of x_k^s for a finite time, and eventually escape and hit the positive extremist consensus state $x_M = 1$. The time spent near the saddle point is related to the time to reach a consensus in orientation (all states with the same sign) and, as we show in the next section, is non-monotonic in r.

Convergence times. – In fig. 4 we plot the mean time τ to reach the final extremist consensus $x_{\pm M} = 1$ as a function of r for $M = 5$, obtained from MC simulations. As qualitative results only depend on r we took $q = 1-p$, thus $r = p/(1-p)$ varies from 0 to ∞ as p goes from 0 to 1. Therefore, r can be seen as the relative strength of persuasion, as compared to compromise. We observe that τ is non-monotonic in r, and has a minimum value around $r \simeq 0.6$. This means that the population reaches the fastest consensus when interactions between agents of the same orientation have a probability of success p similar to that of opposite-orientation agents q. Instead, mostly chatting with same-opinion partners (large r) reinforce initial believes, leading to a polarized state that last for very long times. Besides, only interacting with opposite-opinion partners (small r) first induces a centralized consensus, which is unstable, and then the population is slowly driven to the final extremist consensus.

An insight about the non-monotonic behavior of τ can be obtained by means of eqs. (1). For a simpler analysis of the equations and a better understanding of the previous results, it proves convenient to split the evolution of the system into two distinct stages: a first stage with an associated time scale τ_1, in which all agents adopt the same opinion orientation (all states are either positive or negative), and a second stage where the system reaches extremist consensus, characterized by a time scale τ_2. Therefore, the convergence time can be written as $\tau = \tau_1 + \tau_2$. The nonlinearity of eqs. (1) makes it hard to find a complete solution, but it is possible to obtain approximate expressions for τ in the two limiting cases of very strong and very weak persuasion.

Small persuasion limit $r \ll 1$. In this limit, the second stage is much longer than the first stage ($\tau_2 \gg \tau_1$), and we can approximate $\tau \simeq \tau_2$. This is because once all agents' states become positive (negative) they are slowly driven by persuasion events —which happen with a very small probability $p = r/(1+r)$— to the consensus state $x_M = 1$ ($x_{-M} = 1$), thus the system spends most of the time in the second stage. To estimate τ_2 we assume, without loss of generality, that the system starts at time $t = 0$ from a configuration in which all states are positive ($x_k(t=0) = 0 \quad \forall\, k < 0$). This initial condition implies that states remain positive since only persuasive events can take place, and thus $\sigma_+(t) = 1$ and $\sigma_-(t) = 0$ for $t \geq 0$. Then, eqs. (1) become linear

$$\begin{aligned} \frac{\mathrm{d}x_1}{\mathrm{d}t'} &= -x_1, \\ \frac{\mathrm{d}x_k}{\mathrm{d}t'} &= x_{k-1} - x_k, \quad \text{for} \quad 2 \leq k \leq M-1, \\ \frac{\mathrm{d}x_M}{\mathrm{d}t'} &= x_{M-1}, \end{aligned} \tag{4}$$

where we have introduced the rescaled time $t' \equiv 2\,p\,t$. In the Laplace space, eqs. (4) are reduced to the following system of coupled algebraic equations:

$$\begin{aligned} s\,x_1(s) - x_1(0) &= -x_1(s), \\ s\,x_k(s) - x_k(0) &= x_{k-1}(s) - x_k(s), \quad 2 \leq k \leq M-1, \\ s\,x_M(s) - x_M(0) &= x_{M-1}(s), \end{aligned}$$

with solutions

$$\begin{aligned} x_k(s) &= \sum_{n=0}^{k-1} \frac{x_{k-n}(0)}{(s+1)^{n+1}}, \quad \text{for} \quad 1 \leq k \leq M-1, \\ x_M(s) &= \frac{x_M(0)}{s} + \frac{1}{s} \sum_{n=0}^{M-2} \frac{x_{k-n}(0)}{(s+1)^{n+1}}. \end{aligned}$$

Transforming back to the original space and replacing t' by $2\,p\,t$ we finally obtain

$$\begin{aligned} x_k(t) &= e^{-2p\,t} \sum_{n=0}^{k-1} \frac{(2p\,t)^n\, x_{k-n}(0)}{n!} \quad 1 \leq k \leq M-1, \\ x_M(t) &= 1 - e^{-2p\,t} \sum_{k=1}^{M-1} \sum_{n=0}^{k-1} \frac{(2p\,t)^n\, x_{k-n}(0)}{n!}. \end{aligned} \tag{5}$$

The above solutions are valid for all values of r, but we explore here their behavior in the $r \ll 1$ limit. In this case we expect an initial distribution of states peaked at $k = 1$, that is, $x_1(0) \simeq 1$ and $x_k(0) \simeq 0$ for $2 \leq k \leq M$. This is because the strong bias towards the center during the first stage keeps most states close to $k = 1$. Then, eq. (5) becomes

$$x_M(t) \simeq 1 - e^{-2p\,t} \sum_{k=0}^{M-2} \frac{(2p\,t)^k}{k!}, \qquad (6)$$

which shows that x_M approaches 1 quasiexponentially fast with time. Having $x_M > 1 - 1/N$ at a time $t = \tau_2$ is equivalent to an extremist consensus in the discrete system of N agents, since this corresponds to have a number of agents in state M larger than $N - 1$. Therefore, from eq. (6) τ_2 obeys the following relation:

$$N\, e^{-2p\,\tau_2} \sum_{k=0}^{M-2} \frac{(2p\,\tau_2)^k}{k!} = 1. \qquad (7)$$

Then, $\tau_2 = f_{M,N}/2p$, where $f_{M,N}$ is a solution of

$$Ne^{-f} \sum_{k=0}^{M-2} f^k/k! - 1 = 0, \qquad (8)$$

a non-trivial function of M and N. Finally, replacing back $p = r/(1+r)$ we arrive to following expression for τ

$$\tau \simeq \tau_2 \simeq \frac{(1+r) f_{M,N}}{2r}. \qquad (9)$$

In the inset of fig. 4 we show the curves τ *vs.* r from MC simulations in a system of size $N = 1000$, and rescaled by the functions $f \simeq 9.233, 13.062$ and 18.062, for $M = 3, 5$ and 8, respectively. These values of f were obtained by numerically solving eq. (8), given that a closed expression for f in terms of M and N is very hard to obtain. The collapse of the three curves confirm the scaling given by eq. (9), which also captures the $r \to 0$ asymptotic behavior r^{-1} observed from simulations.

Large persuasion $r \gg 1$ limit. In this case, the first stage takes much longer than the second stage, and thus $\tau \simeq \tau_1$. The system quickly becomes polarized by the driving bias towards the extreme states $k = \pm M$, and stays polarized for very long times, given that the flux of particles from one side to the other is limited by the very small compromise probability $q = 1/(1+r)$. To estimate τ_1, it proves useful to work with the magnetization m, defined as the difference between the fraction of positive and negative states

$$m(t) \equiv \sigma_+(t) - \sigma_-(t) = 2 \sum_{k=1}^{M} x_k(t) - 1. \qquad (10)$$

From eqs. (1), the magnetization evolves according to

$$\frac{\mathrm{d}m}{\mathrm{d}t} = 4\,q\,(x_{-1}\sigma_+ - x_1\sigma_-), \qquad (11)$$

or, using the relations $\sigma_\pm = (1 \pm m)/2$, is

$$\frac{\mathrm{d}m}{\mathrm{d}t} = 2\,q\,[x_{-1}(1+m) - x_1(1-m)]\,. \qquad (12)$$

Equation (11) can also be obtained by noting that m only changes after a compromise event that involves states 1 or -1. The first term accounts for $-1 \to 1$ transitions due to compromises between agents with states -1 and $k > 0$, which happen at a rate $2\,x_{-1}\,\sigma_+$, increasing m by $2/N$. The second term stems for the reverse transition $1 \to -1$, where m decreases. Equation (12) is not closed because $x_{\pm 1}$ depend on $x_{\pm 2}$, which in turn depend on $x_{\pm 3}$ and so on, as we observe from eqs. (1). However, we can still close the equation by finding approximate expressions for $x_{\pm 1}$ in terms of m, as we detail below. As we showed before, the distribution of opinions at the quasistationary mixed state follows the exponential relation $x^s_{\pm k} = x^s_{\pm 1}\, r^{k-1}$ $(1 \le k \le M)$. Monte Carlo simulations show that the distribution remains exponential during the first stage, $x_{\pm k}(t) = x_{\pm 1}(t)\,\alpha_\pm^{k-1}(t)$, where $\alpha_\pm(t)$ are time-dependent variables. Interestingly, we have numerically checked that $\alpha_\pm(t)$ are almost constant over time, and only a significant change is observed at the very end of the stage. Therefore, they can be considered as slow variables, as compared to m, and taken as constants and equal to their initial values $\alpha_\pm(t) \simeq \alpha_\pm(0)$. Thus, we can write

$$\sigma_\pm = \frac{1 \pm m}{2} \simeq x_{\pm 1}(t) \sum_{k=1}^{M} \alpha_\pm^{k-1}(0) = x_{\pm 1}(t) \left[\frac{1-\alpha_\pm^M(0)}{1-\alpha_\pm(0)}\right],$$

from where

$$x_{\pm 1}(t) \simeq \frac{[1-\alpha_\pm(0)]}{2[1-\alpha_\pm^M(0)]}[1 \pm m(t)]. \qquad (13)$$

Given that the quasistationary state is reached in a fast time scale that is $\mathcal{O}(1)$ (see fig. 2(a)), we neglect this short transient and assume that the initial condition corresponds to the stationary solution eq. (3). Therefore, from eq. (13), the initial variables $\alpha_\pm(0)$ obey

$$\frac{[1-\alpha_\pm(0)]}{2[1-\alpha_\pm^M(0)]} \simeq \frac{x_1^s}{1 \pm m_0}, \qquad (14)$$

where $m_0 = m(0)$ is the initial magnetization, and $x_1^s = (1-r)/2(1-r^M)$ is the state-1 density at the quasistationary state (eq. (2)). Note that starting from the perfectly symmetric mixed state gives $m_0 = 0$, and thus $\alpha_\pm(0) = r$. From eqs. (13) and (14) we get

$$x_{\pm 1}(t) \simeq \frac{x_1^s}{1 \pm m_0}[1 \pm m(t)]. \qquad (15)$$

Plugging this expression for $x_{\pm 1}$ into eq. (12) leads to

$$\frac{\mathrm{d}m(t)}{\mathrm{d}t} \simeq \frac{4\,q\,x_1^s\,m_0}{1-m_0^2}[1 - m(t)^2]. \qquad (16)$$

The integration of eq. (16) gives

$$m(t) \simeq \frac{(1+m_0)e^{At} - (1-m_0)e^{-At}}{(1+m_0)e^{At} + (1-m_0)e^{-At}}, \qquad (17)$$

where

$$A \equiv \frac{4\,q\,x_1^s\,m_0}{1-m_0^2} = \frac{2(1-r)m_0}{(1+r)(1-r^M)(1-m_0^2)} \qquad (18)$$

is the prefactor of eq. (16). Expression (17) captures the qualitative behavior of the magnetization, which approaches to $|m|=1$ as

$$|m(t)| \simeq 1 - \frac{2(1-|m_0|)}{(1+|m_0|)} e^{-2|A|t}. \qquad (19)$$

Within this framework of rate equations, the first stage ends at a time τ_1 when $|m|$ equals $1-1/N$, that is, when less than one particle remains in one of the two sides. From eq. (19) we obtain

$$\tau \simeq \tau_1 \simeq \frac{(1-m_0^2)(1+r)(1-r^M)}{4\,|m_0|\,(1-r)} \ln\left[\frac{2N(1-|m_0|)}{1+|m_0|}\right]. \qquad (20)$$

The scaling $\tau \sim r^M$ gives the right asymptotic behavior for $r \gg 1$ (inset of fig. 4).

Summary and discussion. – In summary, we proposed and studied a model that incorporates two mechanisms for the formation of opinions —compromise and persuasion. Compromise interactions between individuals tend to moderate their opinions, while persuasive contacts lead to extreme positions. When compromise events are more frequent than persuasive events, opinions are grouped around moderate values, leading to a centralized state of opinions. In the opposite case, if persuasion events dominate over compromise events, opinions are driven towards extreme positive and negative values, inducing polarization. The centralized and polarized states are unstable, and consensus in either positive or negative extreme opinions is eventually achieved. For a symmetric initial distribution of opinions, these final extremist states are equiprobable, but any asymmetry in the initial condition that favors a given opinion orientation makes the population reach consensus in the extreme state of the favored orientation. The mean extremist consensus time τ is non-monotonic in the ratio $r=p/q$ between the probabilities of successful persuasive and compromise events, and has a minimum when p and q are of the same order of magnitude. In the small ($r \ll 1$) and large ($r \gg 1$) persuasion limit, the consensus time scales as $\tau \sim r^{-1}$ and $\tau \sim r^M \ln N$, respectively, with the maximum intensity M and population size N.

In the studied model, individuals reinforce their opinions by talking to other partners with the same opinion orientation. It would be worthwhile to explore some extensions that include a reinforcement mechanism between individuals with opposite orientations. Related to that, it was recently found that a rejection rule between very dissimilar individuals enhances polarization [18]. It might also be interesting to explore the model when interactions are no longer all-to-all, but rather take place in other topologies like square lattices or complex networks. In lattices we expect the formation of domains composed by same-orientation partners, with a coarsening dynamics driven by surface tension, as it happens in models with intermediate states [10,12]. The connectivity of complex networks could play an important role, by enhancing the propagation and ultimate dominance of an extreme opinion [19]. The effects of more complex topologies on the opinion dynamics, for instance those containing community structure, are more difficult to predict, and thus warrants future work. Finally, our model could be validated by posting a popular question on a social networking website like Facebook, MySpace or YouTube, and asking users to rate their level of agreement/disagreement about that issue.

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