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Reducing congestion on complex networks by dynamic relaxation processes

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Abstract

We study the effects of relaxational dynamics on the congestion pressure in general transport networks. We show that the congestion pressure is reduced in scale-free networks if a relaxation mechanism is utilized, while this is in general not the case for non-scale-free graphs such as random graphs. We also present evidence supporting the idea that the emergence of scale-free networks arise from optimization mechanisms to balance the load of the networks nodes.

Keywords: Complex networks; Gradient networks; Transport networks

Information transfer is one of the cornerstones of the modern world. It takes place in communication processes as old as spoken language and as modern as the world wide web (WWW). All forms of communication can, at the end, be thought of as the transfer of data units or packets. Complex networks have emerged as one of the basic models to represent the medium for data transfer on the Internet, airports, roads, power grids, etc. These transportation networks carry large amounts of load, such as data packets, passengers, vehicles, power, etc., that can produce traffic congestion and reduce efficiency of the flow. As a consequence, there is a great deal of interest in determining efficient conditions for the transport flow due to its commercial and technological applications.

During the last decades, these kind of problems motivated the study of Erdős–Rényi (ER) random graphs [1] which have a Poisson degree distribution. However, in the last few years it was realized that most real-world networks such as the Internet, the WWW, biological and social networks share many common topological and dynamical features that cannot be explained with ER graphs. Instead, many of these systems are well represented by scale-free networks (SF) [2] characterized by a power-law degree distribution $P(k) \sim k^{-\lambda}$, in the domain $m<k<\kappa$ where m and κ are the lower and the upper degree cutoff, respectively. Therefore, a natural question arises: why do complex systems auto organize into SF networks?

Assuming a simple transport model based on potential gradients, Toroczkai and Bassler recently suggested [3] that real-world networks may evolve to SF topology in order to ensure efficiency in the flow. Inspired on the idea that gradients drive transport, in their model flow is induced by the local gradient of a non-degenerate

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random scalar h distributed on each single node of the network. The scalar can represent the processors load in distributed calculation [4], the queue of packet routing on the Internet [5] or the virtual time horizons of the processors in parallel discrete events in parallel computing [6]. The gradient network is defined on the original substrate network as the collection of directed links pointing from each node to whichever of its nearest neighbors or itself has the lowest value of the scalar h. It can be shown that all non-degenerate gradient networks are trees, that is, they have no loops (except for self-loops) [7]. In the gradient network each node has a unique outgoing link and ℓ incoming links characterized by the distribution $R(\ell)$. The efficiency to transmit the flow can be quantified considering the congestion pressure or jamming coefficient $J = R(0)$ which represents the fraction of nodes that do not receive flow from their neighbors. At those nodes, the flow is jammed since they locally have the maximum load or queue and cannot process more jobs.

In Ref. [3] it was found that Erdős–Rényi networks jam, that is $J \to 1$ as the network grows to a larger size N. However, for Barabási–Albert (BA) SF networks for which $\lambda = 3$ [8], the jamming coefficient approach some finite value smaller than one. This fact was suggested in Ref. [3] as a kind of selection principle which favors SF against ER networks. Since real networks emerge most frequently with $2 < \lambda < 3$, one might expect that SF networks with λ in this range will be more efficient than the ones with $\lambda = 3$ with a lower jamming coefficient. In this work, we find that this intuitive assertion is not true. We show that this occurs on considering a static model (SM) where the scalar h does not evolve with time. In this case SF networks have increasing jamming coefficients for decreasing λ. However, the SM picture is a highly idealized approximation. In real-world systems we would expect that some dynamic process to take place which make evolve the scalar field h in order to balance the load of processors or to synchronize the queue of servers along the network. As a consequence, an enhancement of the efficiency in the transport properties of the network, i.e., a reduction of the jamming coefficient J is expected. In this work, we represent the mechanism of load balance or queue synchronization with the well-known model of surface growth known as random deposition with surface relaxation to the minimum [9]. In this model, the main observable is the roughness, that is, the mean square deviation of the surface heights. These surface heights are mapped here onto the scalar h_i over the network nodes. In this way, the mapped roughness $w_h = \{\langle h^2 \rangle - h^2\}$ represents how unbalanced are the loads of the processors or how unsynchronized are the queue of servers. Here we denote with $\langle . \rangle, \{.\}$, averaging over the network and realizations, respectively.

To construct an ER network of size N we select two nodes at random and connect them with probability p disallowing multiple edges between two nodes. We generate SF graphs of size N using the configuration model [10] ensuring that all the nodes are connected, and avoiding multiple edges between nodes, and self-loops.

In order to make a non-degenerated scalar field h, we start with an initial condition where it is composed of random numbers taken from an uniform distribution in the interval $[0, 1]$. At this point the SM gradient network is constructed. Then we make the scalars h evolve obeying the following dynamic model (DM) of random deposition with surface relaxation [9]. At every time step a node i of the substrate is chosen at random with uniform probability $1/N$ and becomes a candidate for growth. Then, h_i changes through $h_i \to h_i + 1$, if $h_i < h_j$ for every j which is the nearest neighbor of the node i. On the other hand, if h_i is not a minimum, the node j with minimum h is incremented by one. After several time steps, the roughness w_h reaches a steady state where the load balance or the queue synchronization cannot be improved. At this point we measure the jamming coefficient J.

In Fig. 1(a) we show both J as a function of N in the SM and DM for ER networks generated with $p = 0.1$. We observe that the dynamic process does not alter the performance of an ER network at all. The behavior for SF networks with $m = 2$ in the SM and DM is displayed in Figs. 1(b) and (c), respectively. We observe that in the SM, for large fixed N, J decreases with increasing λ. Since it is well known that most of real networks have small values of λ between 2 and 3 [2] we can conclude that the SM gradient network does not support the emergence of real SF networks as an optimization mechanism to reduce the congestion. On the other hand, when the flow is governed by a surface relaxation process, the opposite happens. The jamming coefficient J in the DM is an increasing function of λ. Therefore, dynamics on the substrate network seems to support the emergence of real-world SF networks with $\lambda < 3$ as a natural mechanism to enhance the efficiency in the flow. In Figs. 1(d) and (e), we plot J as function of $1/\ln N$ for SF networks in the SM and DM. For big values of N the data show a linear behavior. We estimated J_∞ as the limit of J when $N \to \infty$ by the intercept of the linear fit of the curves. We can notice the increment in the flow efficiency produced by the relaxation process. For the

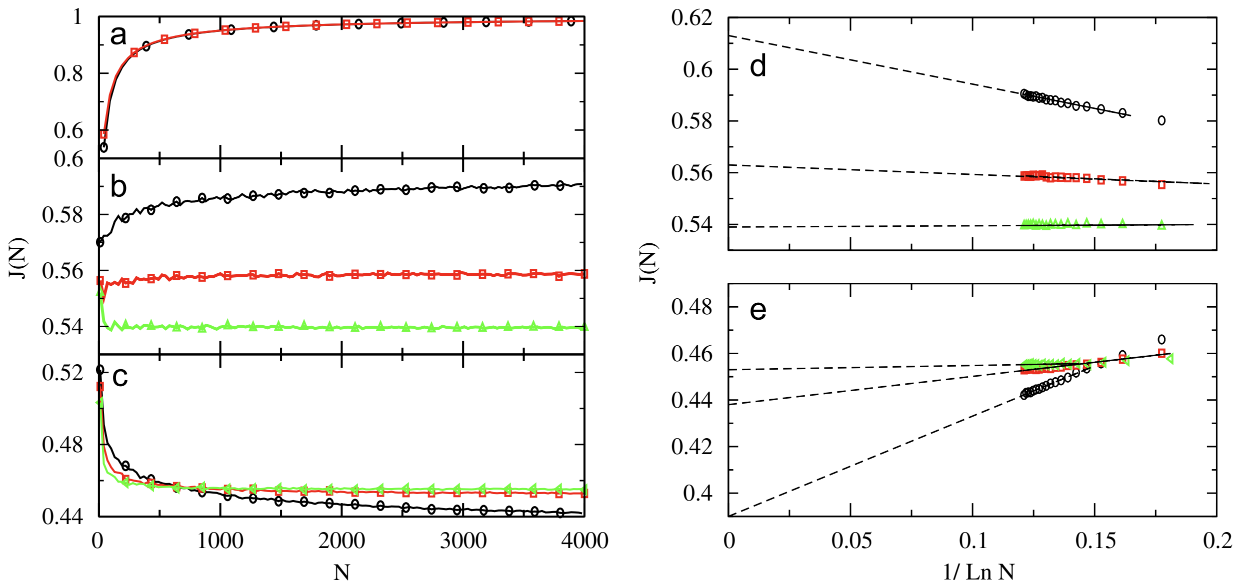

Fig. 1. Plot of the jamming coefficient J. Left panel, as a function of N. (a) ER networks in the SM (○) and in the DM (□). (b) SF networks for the SM. (c) SF networks in the DM. Right panels, J as a function of $1/\ln N$. (d) SF networks in the SM. (e) SF networks in the DM. The dashed lines show a linear fit and the intercept corresponds to J_∞. Symbols in (b)–(e) represent ○, $\lambda = 2.5$; □, $\lambda = 3.0$ and △, $\lambda = 3.5$.

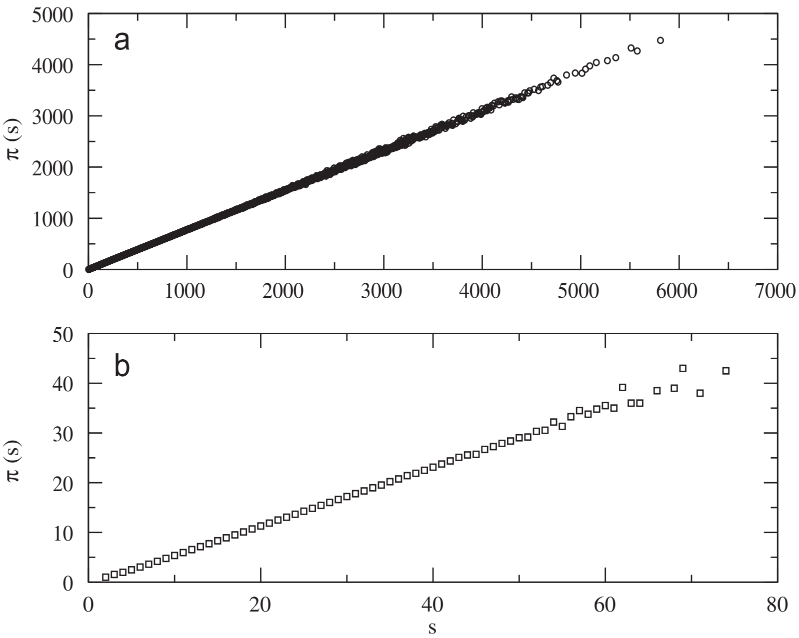

Fig. 2. Plot of $\pi(s)$ as function of s for $\lambda = 2.5$ and $m = 2$ in (a) the SM and (b) the DM.

case of $\lambda = 2.5$, for example, the J_∞ corresponding to the SM is 40% lower than that of the ER network. Meanwhile, the relaxation dynamics reduces J_∞ in 60% that of the ER network. As in the DM J_∞ is smaller than in the SM case, it is expected that the clusters perimeter $\pi(s)$ of mass s on the gradient network will be smaller in the first case because only those nodes on the cluster perimeter contribute to $R(0)$. In Fig. 2, we plot $\pi(s)$ as a function of s for a SF network with $\lambda = 2.5$. We can see that in SM there are larger clusters than in the

DM and for a given value of s, $\pi(s)$ is bigger in the SM state than in the DM one. In measurements not presented here, we also observe that the diameter of the cluster in the DM is smaller than in the SM one. All this indicates a transition on the topology of the gradient network from big trees with a star-like structure (where there are many nodes in the perimeter with a small diameter) in the SM to small trees with a more elongated structure (where there are less nodes in the perimeter with a bigger diameter) in the DM state. The details on the topology of the gradient network go beyond the aim of this paper and will be published elsewhere.

In summary, in this work we show that surface growth with relaxation process reduce congestion in SF networks, meanwhile do not affect ER networks. We also present evidence supporting the idea that the emergence of SF networks with $\lambda < 3$ arise from optimization mechanisms to balance the load of the networks nodes.

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References

[1] P. Erdős, A. Rényi, Publ. Math. (Debreccen) 6 (1959) 290.
[2] R. Albert, A.-L. Barabási, Rev. Mod. Phys. 74 (2002) 47.
[3] Z. Toroczkai, K. Bassler, Nature 428 (2004) 716.
[4] Y. Rabani, A. Sinclair, R. Wanka, in: Proceedings of the 39th Symposium on Foundations of Computer Science (FOCS), 1998, p. 694.
[5] S. Valverde, R.V. Sole, Eur. Phys. J. B 38 (2004) 245.
[6] G. Korniss, Z. Torocczkai, M.A. Novotny, P.A. Rikvold, Phys. Rev. Lett. 84 (2000) 1351.
[7] Z. Toroczkai, B. Kozma, K.E. Bassler, N.W. Hengartner, G. Korniss, cond-mat/0408262.
[8] A.-L. Barabási, R. Albert, Science 286 (1999) 509.
[9] A.L. -Barabási, H.E. Stanley, Fractal Concepts in Surface Growth, Cambridge University Press, Cambridge, 1995.
[10] M. Molloy, B. Reed, Random Struct. Algorithms 6 (1995) 161.