

EPL, **118** (2017) 18001 doi: 10.1209/0295-5075/118/18001 www.epljournal.org

Promoting information spreading by using contact memory

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received 19 March 2017; accepted 9 May 2017 published online 25 May 2017

PACS 89.75.Hc – Networks and genealogical trees
PACS 87.19.X- – Diseases
PACS 87.23.Ge – Dynamics of social systems

Abstract – Promoting information spreading is a booming research topic in network science community. However, the existing studies about promoting information spreading seldom took into account the human memory, which plays an important role in the spreading dynamics. In this letter we propose a non-Markovian information spreading model on complex networks, in which every informed node contacts a neighbor by using the memory of neighbor's accumulated contact numbers in the past. We systematically study the information spreading dynamics on uncorrelated configuration networks and a group of 22 real-world networks, and find an effective contact strategy of promoting information spreading, *i.e.*, the informed nodes preferentially contact neighbors with a small number of accumulated contacts. According to the effective contact strategy, the high-degree nodes are more likely to be chosen as the contacted neighbors in the early stage of the spreading, while in the late stage of the dynamics, the nodes with small degrees are preferentially contacted. We also propose a mean-field theory to describe our model, which qualitatively agrees well with the stochastic simulations on both artificial and real-world networks.

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Introduction. - A wide range of propagation phenomena in the real world, such as the spreading of information, rumor, disease and behavior, can be described by spreading dynamics on complex networks [1-6]. The problem of how to enhance or promote the spreading has attracted much attention in the last few years in many disciplines [7]. Promoting spreading speed and understanding its effects in the outbreak size of the reached nodes are two important features to study in both theoretical and empirical aspects. Theoretically existing studies found that promoting the spreading dynamics could induce distinct critical phenomena with different outbreak thresholds and critical exponents [8]. Practically speaking, promoting the spreading dynamics could shed some lights on the propagation of information [9–11], marketing management [12,13], disease spreading [14–18], etc.

Many strategies for promoting spreading dynamics have been proposed, such as choosing influential seeds [19,20] and designing effective contact strategies [21,22]. Kitsak et al. found that selecting nodes with high k-shells as spreading sources can effectively enhance the size of the spreading in most cases [20], however the k-shell index cannot reflect the importance of nodes in the core-like group in which the nodes are connected very locally [23]. Yang et al. proposed a contact strategy based on the degree of neighbors nodes to promote the information spreading [21]. They found that the information spreading can be greatly promoted in uncorrelated networks if the small-degree neighbors are preferentially contacted. In addition, if the reached nodes, denoted as informed nodes, preferentially contact nodes with few informed neighbors, the information spreading could be further promoted [22].

In the real world, the activities of humans exhibit the characteristic of having memory [24,25]. This

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characteristic has significant impacts on the spreading dynamics of information, epidemic, and behavior [26–28]. For instance, human's memory produces a larger prevalence in the exponential decay time of new informed nodes than processes without memory [29]. Wang et al. [27] found that memory affects the growth pattern of the final outbreak size in the dynamics of social contagion. However, previous works about promoting information spreading always neglected the memory of individuals. In this letter, we propose a non-Markovian information spreading model, in which each individual (node) keeps memory of the number of accumulated contacts (NAC) with informed neighbors in the past. We assume that every informed node contacts a neighbor based on the values of neighbors' NAC. To describe our model, we develop a novel meanfield theory. Our theoretical predictions are in good qualitative agreement with the stochastic simulations on both artificial networks and 22 real-world networks. Through theoretical analysis and extensive numerical simulations, we find that preferentially contacting neighbors with small NAC is an efficient strategy to promote the spreading. This strategy markedly promotes the information spreading, since it increases the number of effective contacts with susceptible nodes. With our effective strategy, we find that the informed nodes are more likely to contact highdegree nodes in the early stage, while small degree nodes are preferentially contacted in the late stage. As a result, our strategy unifies the probability that nodes of different degrees will be contacted enhancing the information spreading remarkably.

Model. – We propose a generalized susceptibleinformed model [1] to describe the information spreading. In this model, each node can either be in the susceptible or informed state. Initially, we randomly choose a small fraction of nodes in the informed state, while the remaining nodes are in the susceptible state. At each time step t, each informed node i contacts one of its neighbors j with a probability $w_{ij}(t)$ (to be defined later). If node j is susceptible, it will become informed with a transmission probability λ . During the spreading process, we adopt the synchronous updating rule, *i.e.*, all nodes will update their states synchronously at each time step [30]. The dynamical process evolves until time T, at which we compute the density of informed nodes $\rho(T)$. The change of this magnitude with the parameters will allow us to evaluate the efficiency of our strategy [31].

An effective information spreading strategy should increase the effective contacts between informed and susceptible nodes [32]. To achieve this goal our strategy assumes that every node remembers the number of accumulated contacts (NAC) with informed neighbors in the past. An informed node i contacts a neighbor j at time twith probability

$$w_{ij}(t) = \frac{[n_j(t) + 1]^{\alpha}}{\sum_{v \in \Gamma(i)} [n_v(t) + 1]^{\alpha}},$$
(1)



Fig. 1: (Color online) An illustration of the information spreading process on complex networks. (a) At time t, the informed node 5 will contact a neighbor and transmit the information to it if the contacted neighbor is in the susceptible state. (b) Individual 5 contacts neighbor $j \in \{1, 2, 3, 4\}$ with probability $w_{5j}(t) = [n_j(t) + 1]^{\alpha} / \sum_{v=1}^{4} [n_v(t) + 1]^{\alpha}$, where $n_j(t)$ is the NAC of node j. For example, node 5 contacts node 1 with probability $w_{51}(t)$. (c) Assuming that node 5 contacts node 1 at time t successfully, node 1 will get the information and change to the informed state with probability λ .

where $n_j(t)$ is the value of NAC of node j at time t, and Γ_i is the neighbor set of node i (see footnote¹). The parameter α determines the tendency of node i to contact a neighbor j with small or large value of $n_j(t)$. For the case of $\alpha > 0$, neighbors with larger $n_j(t)$ are preferentially contacted, and for the case of $\alpha < 0$, the opposite situation occurs. When $\alpha = 0$, all neighbors are randomly chosen and we recover the classical susceptible-informed model. As every node remembers its NAC before time t, the information spreading process is a non-Markovian process. In fig. 1 we show a schematic of the information spreading dynamics.

Results. –

Theoretical analysis. To describe our model of information spreading dynamics, we develop a mean-field theory. We denote as $s_i(t)$ and $\rho_i(t)$ the probabilities that node *i* is susceptible and informed at time *t*, respectively. Since each node can only be in the susceptible or informed state, we have $s_i(t) = 1 - \rho_i(t)$. A susceptible node *j* will become informed at time *t* if it fulfills two conditions simultaneously: 1) being contacted by an informed neighbor *i* with probability $w_{ij}(t)$, and 2) being informed successfully with probability λ . The probability of node *j* to make a transition to the informed state by neighbors at time *t* is [33]

$$\Phi_j(t) = 1 - \prod_{i=1}^{N} [1 - \lambda A_{ij} w_{ij}(t) \rho_i(t)], \qquad (2)$$

where N is the system size and A_{ij} is the element of the adjacency matrix of a given network. If there is an edge between nodes i and j, $A_{ij} = 1$; otherwise, $A_{ij} = 0$. Thus,

¹In the equation, we set $n_j(t) + 1$ to avoid that the denominator is zero.

the evolution of the probability of the informed node j is

$$\rho_j(t+1) = \rho_j(t) + s_j(t)\Phi_j(t).$$
(3)

In order to obtain the value of $w_{ij}(t)$, we compute $n_j(t)$ which is given by

$$n_j(t+1) = n_j(t) + \Theta_j(t), \qquad (4)$$

where

$$\Theta_j(t) = \sum_{i=1}^N A_{ij} w_{ij}(t) \rho_i(t)$$
(5)

is the increase of node j's NAC at time t. From eqs. (2)–(5), we obtain the evolution equations for the information spreading dynamics. At time t, the density of informed nodes is given by

$$\rho(t) = \frac{1}{N} \sum_{j=1}^{N} \rho_j(t),$$
(6)

and the density of susceptible nodes is $s(t) = 1 - \rho(t)$.

Stochastic simulations. We perform extensive numerical simulations on both artificial and real-world networks. All the obtained results are averaged over 10^3 independent realizations of seeds on a fixed network. Initially, we randomly select 5% nodes as informed seeds. The information transmission probability is set as $\lambda = 0.1$. Notice that other values of $\lambda > 0$ do not qualitatively affect our results.

We first study the information spreading dynamics on artificial networks. We built the artificial networks using the uncorrelated configuration model [34,35] with power-law degree distributions $P(k) \sim k^{-\gamma}$ with $k_{min} \leq k \leq \sqrt{N}$, where k_{min} is the smaller value of the degree and γ is the degree exponent. In all our simulations the network size is $N = 10^4$ and $k_{min} = 4$ in order to have a high average degree and, therefore, a relative high number of contacts.

In fig. 2 we show the results on artificial networks with different degree exponents. For a given value of T, we find that $\rho(T)$ decreases with α , as shown in figs. 2(a) and (c). Specifically, if every informed node i preferentially contacts a neighbor j with small value of $n_i(t)$, *i.e.*, $\alpha < 0$, more nodes will be informed. On the other hand, if neighbors with large NAC are preferentially contacted, *i.e.*, $\alpha > 0$, there will be few nodes to be informed. From the figures we can see that for T = 100 almost all nodes are in the informed state when $\alpha \leq -1$, however, there are few informed nodes when $\alpha > 1$. We can explain the above phenomena as follows. The larger the value of $n_i(t)$, the larger the probability that node i has been informed. As a result, an informed node *i* should contact a neighbor with small value of NAC to increase the number of effective contacts (*i.e.*, contact with a susceptible neighbor), and further promotes the information spreading. In figs. 2(b) and (d) we show the time evolution of $\rho(t)$ for $\gamma = 2.1$



Fig. 2: (Color online) Information spreading on artificial networks with different degree exponents. The fractions of informed nodes at time T = 50,100 and 200 as a function of α for $\gamma = 2.1$ (a) and $\gamma = 3.0$ (c). Time evolution of $\rho(t)$ in log-log scale with $\alpha = -2.0, 0.0$ and 1.0 vs. t for $\gamma = 2.1$ (b) and $\gamma = 3.0$ (d). The symbols represent the simulation results, and the lines are the theoretical results from eq. (6).

and 3.0, respectively. We find that for small values of α , $\rho(t)$ is large, nevertheless when $\alpha = 1.0$ the information spreads slowly and it is hard to expand the information to the whole network. Our theoretical predictions agree with the stochastic simulations in most cases. The deviations between the theoretical and the numerical results arise from disregarding the strong dynamical correlations among the states of neighbors in the theory [1,2]. From the figures we can also see that the phenomena for $\gamma = 2.1$ and $\gamma = 3.0$ are similar, which indicates that the heterogeneity of the degree distribution does not qualitatively affect the results.

In order to explain our above results we compute the average values of NAC of nodes with degree $k \langle n_k(T) \rangle$ at time T. From figs. 3(a) and (c), we can observe that at the early stage of the information spreading dynamics (*i.e.*, T = 10), $\langle n_k(T) \rangle$ increases linearly with k for all the values of α , *i.e.*, high-degree nodes are more likely to be contacted by informed neighbors than low-degree nodes. Note that $\langle n_k(T) \rangle$ is slightly smaller for $\alpha = -2.0$ than for $\alpha = 1.0$. At a later stage of the information spreading dynamics (*i.e.*, T = 100), we find some interesting phenomena as shown in figs. 3(b) and (d). For the case of $\alpha = 0.0$, $\langle n_k(T) \rangle$ still increases linearly with k. When $\alpha = 1.0$, the values of $\langle n_k(T) \rangle$ of the largest degree nodes are more than 200 times larger than the ones for nodes with minimum degree, and the number of the effective contacts are decreased in this case. However, for the case of $\alpha = -2.0$, almost all the nodes have the same values of $\langle n_k(T) \rangle$. This indicates that, compared to the case of $\alpha = 1.0$, in the latter case informed nodes are more likely to contact neighbors with small degrees. Since nodes with small degrees have small probabilities to be informed, preferentially contacting them increases the number of effective contacts. As a result, nodes with different



Fig. 3: (Color online) The average number of accumulated contacts $\langle n_k(T) \rangle$ for nodes with degree k at time T = 10 (top) and T = 100 (bottom). In (a) and (b), the degree exponent is $\gamma = 2.1$. In (c) and (d), we set $\gamma = 3.0$.



Fig. 4: (Color online) The time evolution properties of the information spreading on theoretical networks. $\langle n_S(t) \rangle$ ((a) and (e)), $p_S(t)$ ((b) and (f)), $\langle k_I(t) \rangle$ ((c) and (g)), and $D_I(t)$ ((d) and (h)) vs. t. In the left column, we set $\gamma = 2.1$. In the right column, we set $\gamma = 3.0$. The information transmission probability is $\lambda = 0.1$.

degrees almost have uniform probabilities of being contacted and the information spreading can be accelerated significantly.

In fig. 4, we study the time evolution properties of the information spreading. In figs. 4(a) and (e), we show the time evolution of the average number of accumulated contacts $\langle n_S(t) \rangle$ for the effective contacts (*i.e.*, the newly



Fig. 5: (Color online) Information spreading on real-world networks. The fractions of informed nodes at time T = 50,100 and 200 for Advogato (a) and Hamsterster friendships networks (c). The time evolution of $\rho(t)$ with $\alpha = -2.0, 0.0$ and 1.0 vs. t for Advogato (c) and Hamsterster friendships networks (d). The theoretical analysis results are obtained from eq. (6). We set the information transmission probability as $\lambda = 0.1$.

contacted susceptible nodes) for $\gamma = 2.1$ and 3.0. For the case of $\alpha = -2.0$, most of the nodes with a small number of contacts are informed at the early stage, while the remaining few nodes with a large number of contacts are informed at the late stage. However, for the case of $\alpha = 1.0$, nodes with a small number of contacts are hard to be informed. We also find that the ratio of the number of effective contacts to all contacts at time t, *i.e.*, $p_S(t)$, is very high in the early stage when $\alpha = -2.0$ (see figs. 4(b) and (f)). To clarify the types of informed nodes at different stages of the process, we study the average degree $\langle k_I(t) \rangle$ and the degree diversity $D_I(t)$ [22] of the newly informed nodes at time t. Here $D_I(t)$ is defined as

$$D_I(t) = \frac{1}{\sum_k \left[\frac{I_k(t) - I_k(t-1)}{I(t) - I(t-1)}\right]^2},$$
(7)

where I(t) is the number of newly informed nodes at time t, and $I_k(t)$ is the number of those with degree k. The large values of $D_I(t)$ indicate that the newly informed nodes have heterogeneous degrees. From figs. 4(c) and (g), we can see that for the case of $\alpha = -2.0$, $\langle k_I(t) \rangle$ is very large at the early stage, and then decreases at the later stage. $D_I(t)$ shows the same tendency as seen in figs. 4(d) and (h). Thus, when $\alpha = -2.0$ high-degree nodes are more likely to be informed in the early stage, while in the late stage low-degree nodes are often informed. When $\alpha = 1.0$ both $\langle k_I(t) \rangle$ and $D_I(t)$ are large in the whole spreading process, which means that nodes with low degrees are hard to be informed. These results corroborate our finding that the heterogeneity of degree distribution does not qualitatively affect the above-stated results.

Finally, we study our suggested information spreading dynamics on 22 real-world networks, which includes metabolic networks, infrastructure networks,

Table 1: Statistical characteristics of the 22 real-world networks. The statistical characteristics including the network size $(N$),
number of edges (E), maximum degree (k_{max}) , first and second moments of the degree distribution $(\langle k \rangle)$ and second moment	ts
$(\langle k^2 \rangle)$, degree-degree correlations (r), clustering (c), and modularity (Q).	

Category	Networks	Statistical characteristics of networks							
		N	E	k_{max}	$\langle k \rangle$	$\langle k^2 \rangle$	r	С	Q
Social	Advogato [36]	5042	39227	803	15.56	1284	-0.096	0.092	0.337
	Hamsterster friendships [37]	1788	12476	272	13.955	635.61	-0.089	0.09	0.396
	Hamsterster full [37]	2000	16098	273	16.098	704.71	0.023	0.23	0.45
Metabolic	Caenorhabditis elegans [38]	453	2025	237	8.9404	358.49	-0.226	0.124	0.401
	Reactome [39]	5973	145778	855	48.812	6995.1	0.241	0.606	0.719
	Human protein (Figeys) [40]	2217	6418	314	5.79	324.93	-0.332	0.008	0.472
	Human protein (Vidal) [41]	2783	6007	129	4.317	68.103	-0.137	0.035	0.615
Infrastructure	Air traffic control [37]	1226	2408	34	3.928	28.899	-0.015	0.064	0.686
	OpenFlights [42]	2905	15645	242	10.771	601.45	0.049	0.255	0.581
	Power $[43]$	4941	6594	19	2.669	10.333	0.003	0.08	0.932
Collaboration	CA-Hep [44]	8638	24806	65	5.744	74.601	0.239	0.482	0.752
	Netsci [45]	379	914	34	4.823	38.686	-0.082	0.741	0.846
Citation	DBLP [46]	12495	49563	709	7.933	347.28	-0.046	0.062	0.538
HumanContact	Jazz musicians [47]	198	2742	100	27.697	1070.2	0.02	0.52	0.439
	Adolescent health [48]	2539	10455	27	8.236	86.414	0.251	0.142	0.597
Computer	Route views [49]	6474	12572	1458	3.884	640.08	-0.182	0.01	0.612
	Route [50]	5022	6258	106	2.492	34.181	-0.138	0.012	0.898
Lexical	King James [37]	1707	9059	364	10.614	441.85	-0.052	0.162	0.461
OnlineContact	Pretty Good Privacy [51]	10680	24340	206	4.558	86.287	0.239	0.266	0.877
	Email [52]	1133	5451	71	9.622	179.18	0.078	0.22	0.565
	Emailcontact [20]	12625	20362	576	3.226	356.36	-0.387	0.109	0.674
	Blog $[53]$	3982	6803	189	3.417	47.145	-0.133	0.284	0.853



Fig. 6: (Color online) Comparison of the theoretical and numerical predictions of the information spreading on 22 realworld networks. Different colors indicate different values of α . The analysis results are obtained from eq. (6). The results are obtained at the end time T = 100. We set $\lambda = 0.1$.

collaboration networks and citation networks, etc. For simplicity, the directed networks are treated as undirected ones and the weighted networks are treated as unweighted ones. table 1 displays their statistical characteristics in details.

Figure 5 shows the information spreading on two representative networks. Strikingly, we find that the results on real-world networks are qualitatively similar to the ones found in artificial networks as shown in fig. 2. Specifically, if the informed nodes preferentially contact neighbors j with small $n_j(t)$, the information spreading will be markedly facilitated. On the contrary, the information spread is slower if nodes with large NAC are favored, and it is hard to spread the information to the network.

We further verify the effectiveness of our suggested mean-field theory on the real-world networks at T = 100for different values of α , as shown in fig. 6. We can see that our theoretical predictions qualitatively agree with the numerical simulations, although the theoretical predictions are slightly larger than the simulation results. The deviations between theoretical and numerical predictions derive from the strong dynamical correlations among the states of neighbors [1,2].

Discussions. – In this letter, we proposed a novel non-Markovian information spreading model by introducing the individuals' memory, which assumes that every node remembers the number of accumulated contacts (NAC) with informed neighbors in the past, and an informed node contacts a neighbor with different biases values of NAC. To describe the non-Markovian spreading dynamics, we developed a novel mean-field theory. Through extensive numerical simulations on artificial networks and

real-world networks, we found that the memory characteristic markedly affects the spreading dynamics. If the informed nodes preferentially contact neighbors with low NAC, the information spreading is markedly promoted due to the increasing of the number of effective contacts between susceptible and informed nodes. In the effective contact strategy, the high-degree nodes are more likely to be contacted in the early stage, while the low-degree nodes are preferentially contacted in the late stage. As a result, nodes with different degrees almost have uniform probabilities of being contacted and the information spreading is accelerated significantly. Importantly, the effectiveness of the promoting strategy weakly depends on the structures of networks. The agreement between our theoretical predictions and numerical results is verified on artificial networks as well as the real-world networks. Our effective strategy to promote the information spreading on complex networks, could be used for other spreading dynamics, such as technical innovations, healthy behaviors, and new products adoption. Our theoretical method of non-Markovian spreading model can be extended to the study of other social dynamics.

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This work was supported by the National Natural Science Foundation of China (Grant No. 61673086). LAB thanks UNMDP and FONCyT, Pict 0429/2013 for financial support.

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