Spontaneous recovery in dynamic networks

Antonio Majdandzic

Boston University

***Advisor*: H. E. Stanley**

Collaborators:

B. Podobnik

S. Havlin

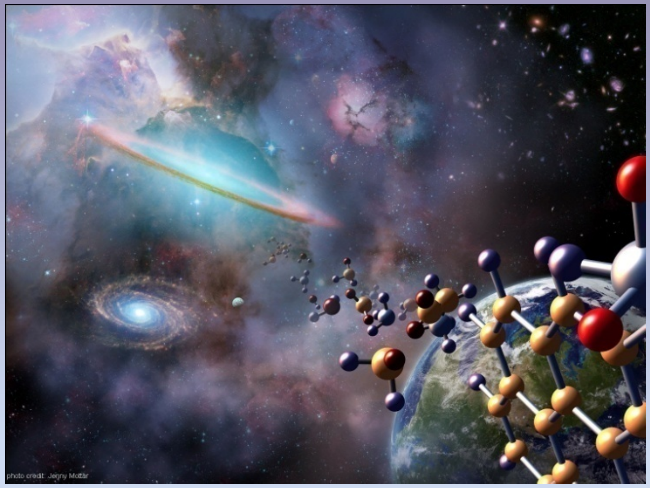
S. V. Buldyrev

D. Kenett

Outline

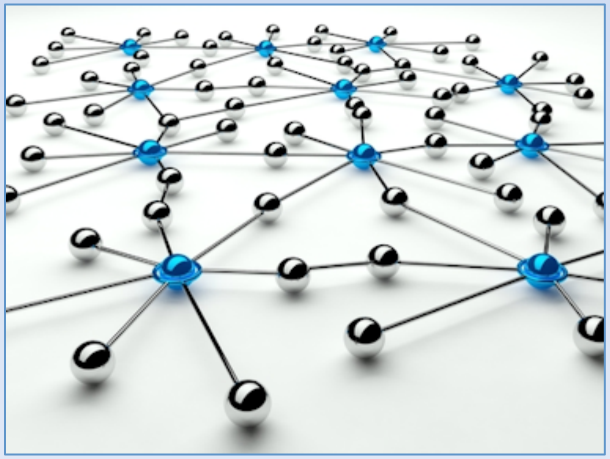
- Motivation
- Model
- Numerical results and theory
- Real networks: empirical support
- Discussion and conclusion

Interactions and connections



-elementary particles
-atoms
-molecules
-planets, stars, galaxies...

NETWORKS:
Interactions between individual units in :
-society
-biology
-finance
-infrastructure & traffic



Many, many models on the question: How do networks fail?

Motivation

Recovery?
1. We can repair it by hand
2. It recovers spontaneously

In many real-world phenomena such as
-**traffic jams** suddenly easing
-people **waking from a coma**
-sudden **market crashes** in finance

after it fails, the network is seemingly being able to **become spontaneously active again**.

→The process often occurs repeatedly: ***collapse, recovery, collapse, recovery,...***

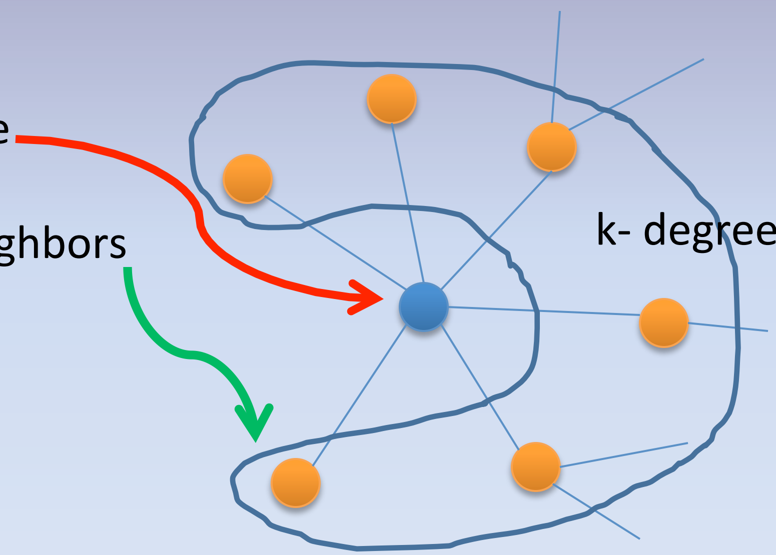
MODEL

We have a network: each node can be **active** or **failed**.

We suppose there are **TWO possible reasons for the nodes' failures**: INTERNAL and EXTERNAL.

INTERNAL failure: intrinsic reasons inside a node

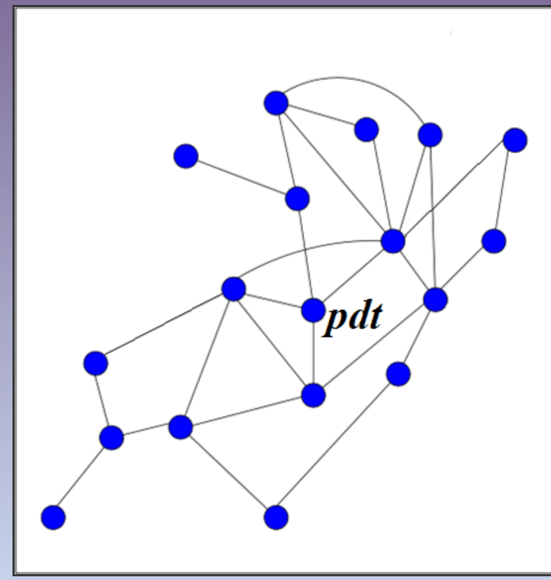
EXTERNAL failure: damage "imported" from neighbors



RECOVERY: A node can also **recover** from each kind of failure; suppose there is some **characteristic time of recovery** from each kind of failure.

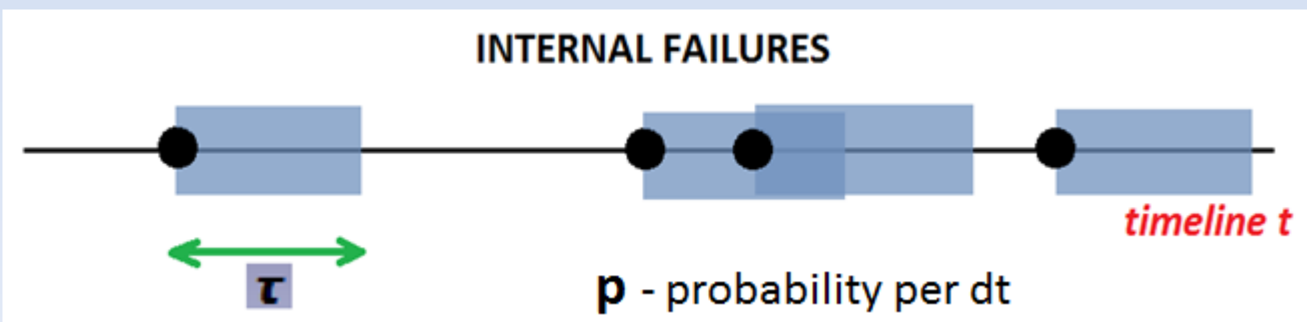
INTERNAL FAILURES

p- rate of internal failures (per unit time, for each node). During interval *dt*, there is probability **p***dt* that the node fails.

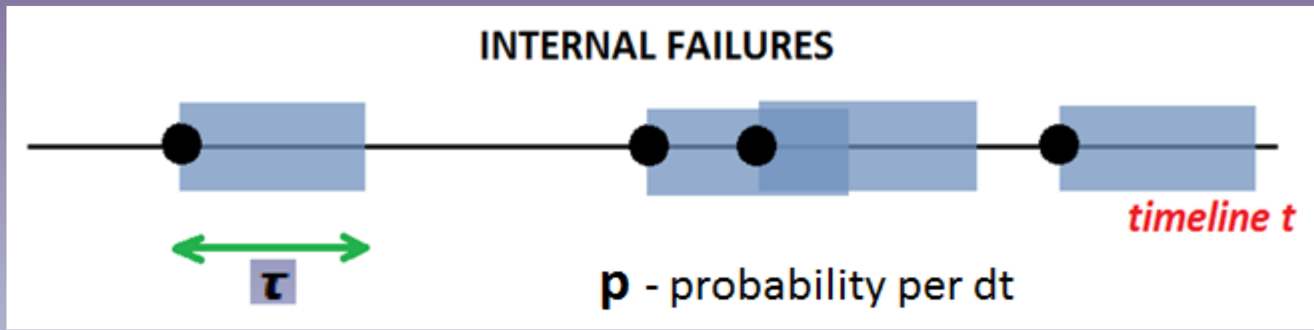


A node *recovers from* an *internal failure after a time period* ***τ*** *.*

INTERNAL FAILURES - independent process on each node



LEFT: Observing one node during time.



A relevant quantity is the **fraction of time during which the node is internally failed**. Lets call this quantity **p***, 0<p*<1.

This would make a nice problem for a course in statistics.
We just give the result:

$$\boxed{p^* \equiv 1 - \exp(-p\tau)}$$

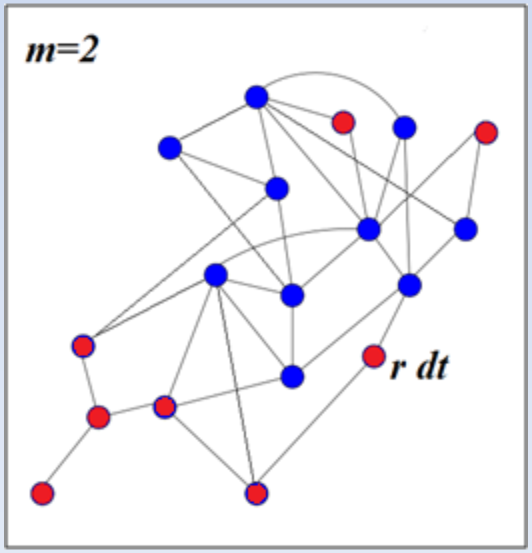
→It turns out we will need only a single parameter, **p***, to describe internal failures.

Let's remember p*!

EXTERNAL FAILURES – if the neighborhood of a node is too damaged

a) "HEALTHY" neighborhood **(def: more than *m* active neighbors, where *m* is a fixed <u>treshold parameter</u>)**: there is no risk of externally- induced failures

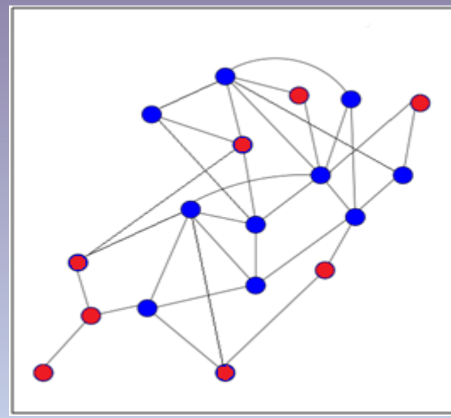
b) CRITICALLY DAMAGED neghborhood **(def: less than or equal to *m* active neighbors):** there is a probability ***r dt*** that the node will experience externally-induced failure during *dt*.



r- external failure probability (per unit time, **<u>for nodes with critically damaged neighborhood</u>**)

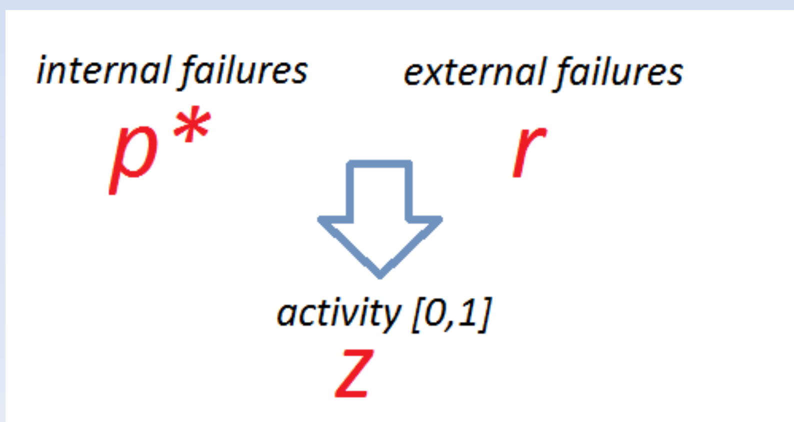
*A node recovers from an external failure after time **τ '**. We set τ '=1 for simplicity.*

Network evolution: combination of internal and external failures, and recoveries.

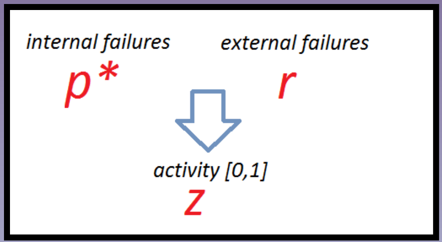


Network is best described with **a fraction of active nodes,** $0<\mathbf{Z}<1$.

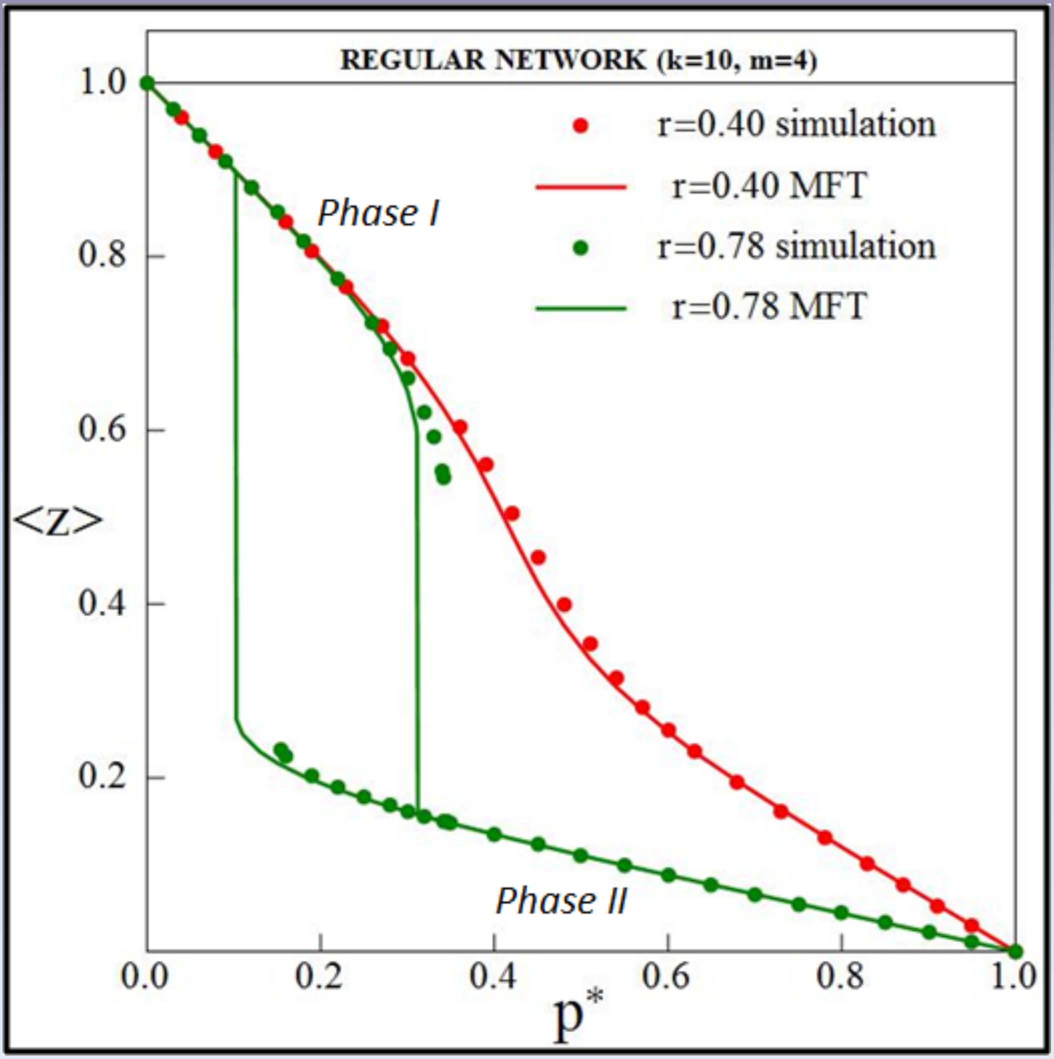
Only 3 quantities to remember:



Model simulation [Random regular networks]



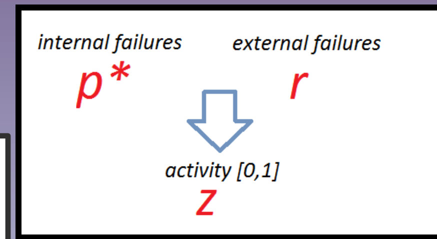
<z>- average fraction of active nodes (during time it fluctuates)



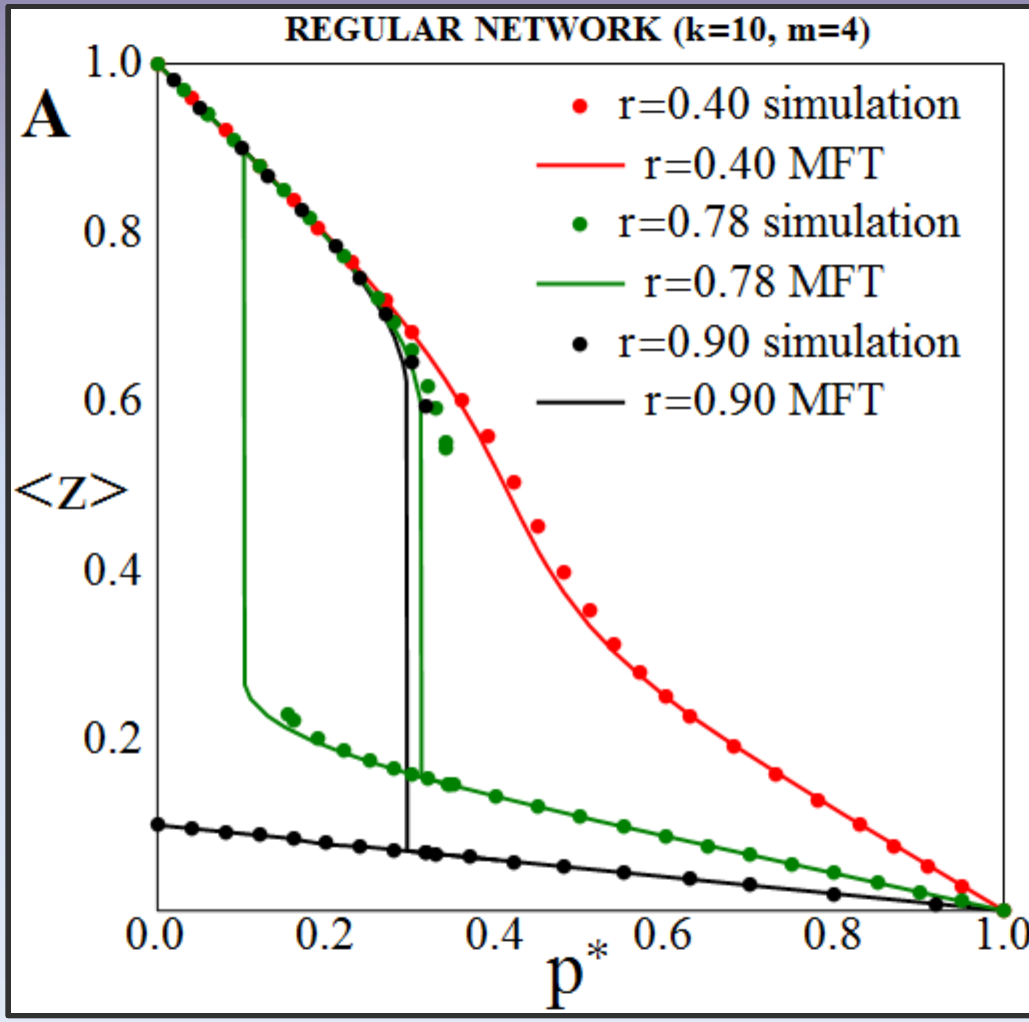
We fix r, and measure how <z> changes as we change p*.

For some values of r we have hysteresis.

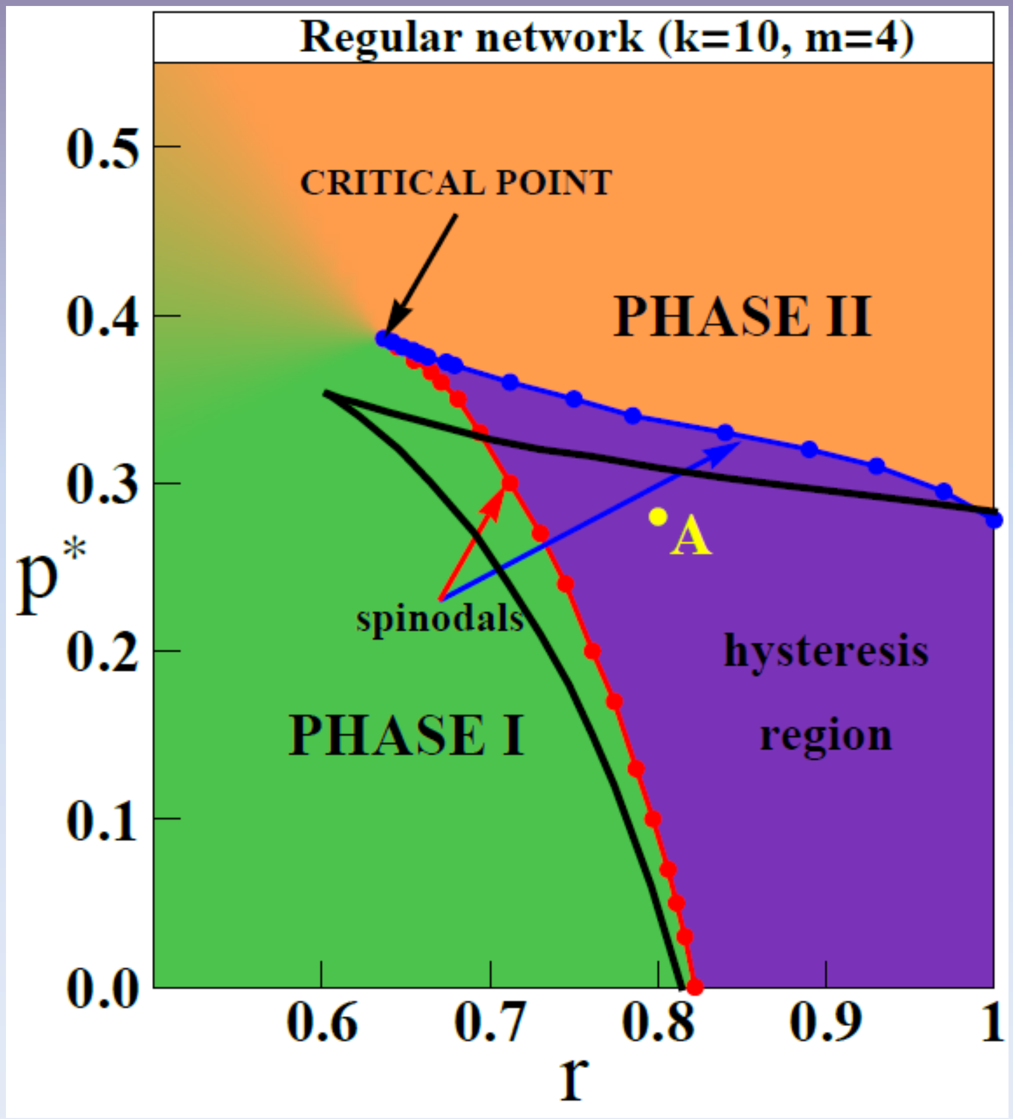
Simulation results



<z>- dynamical average of the fraction of active nodes



Phase diagram



Blue line: critical line (spinodal) for the abrupt transition I→ II

Red line: critical line (spinodal) for the abrupt transition II→I

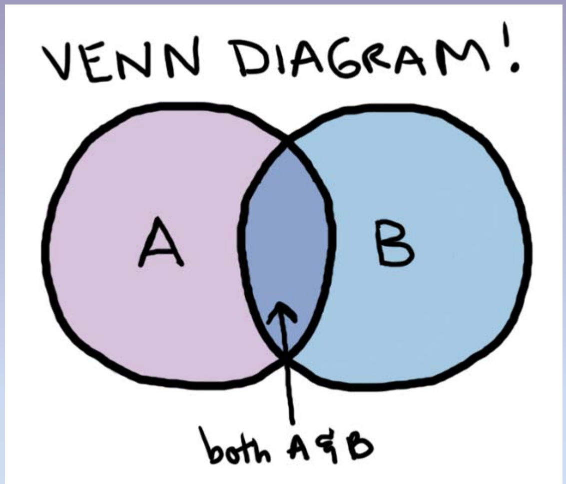
In the hysteresis region both phases exist, depending on the initial conditions or the memory/past of the system.

THEORY (short overview)

Denote the events of failures as
A = {internal failure} ,
B = {external failure}.

The probability **Z**k that a randomly-chosen node of degree k has failed is:

$$1 - z_k = P(A) + P(B) - P(A \cap B)$$



Assume that internal and external failures are approx. independent events, then

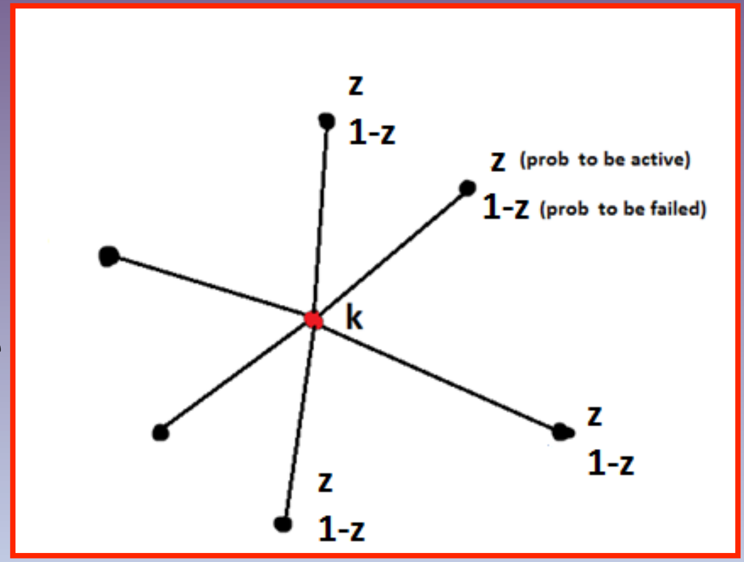
$$1 - z_k = P(A) + P(B) - P(A)\, P(B)$$

P(A) is just p*, and P(B) can be calculated using a mean field theory and combinatorics.

Basic idea for P(B):

In the **mean field theory**, every neighbor has probability **z to be active and 1-z to be failed** (no matter what degrees these nodes might have -"average neighbor").

Using combinatorics P(B) can be expressed as a function of the **mean field z** and node degree k.



After summing over all k-s, all z_k sum up to z, the result is a self consistency equation of the form:

$$\boxed{\mathbf{z} = f[\mathbf{z}(p^*, r)]}$$

Depending on p* and r we have either:

- 1 solution (pure phase)
- 3 solutions (2 physical sol., corresponding to the hysteresis region)

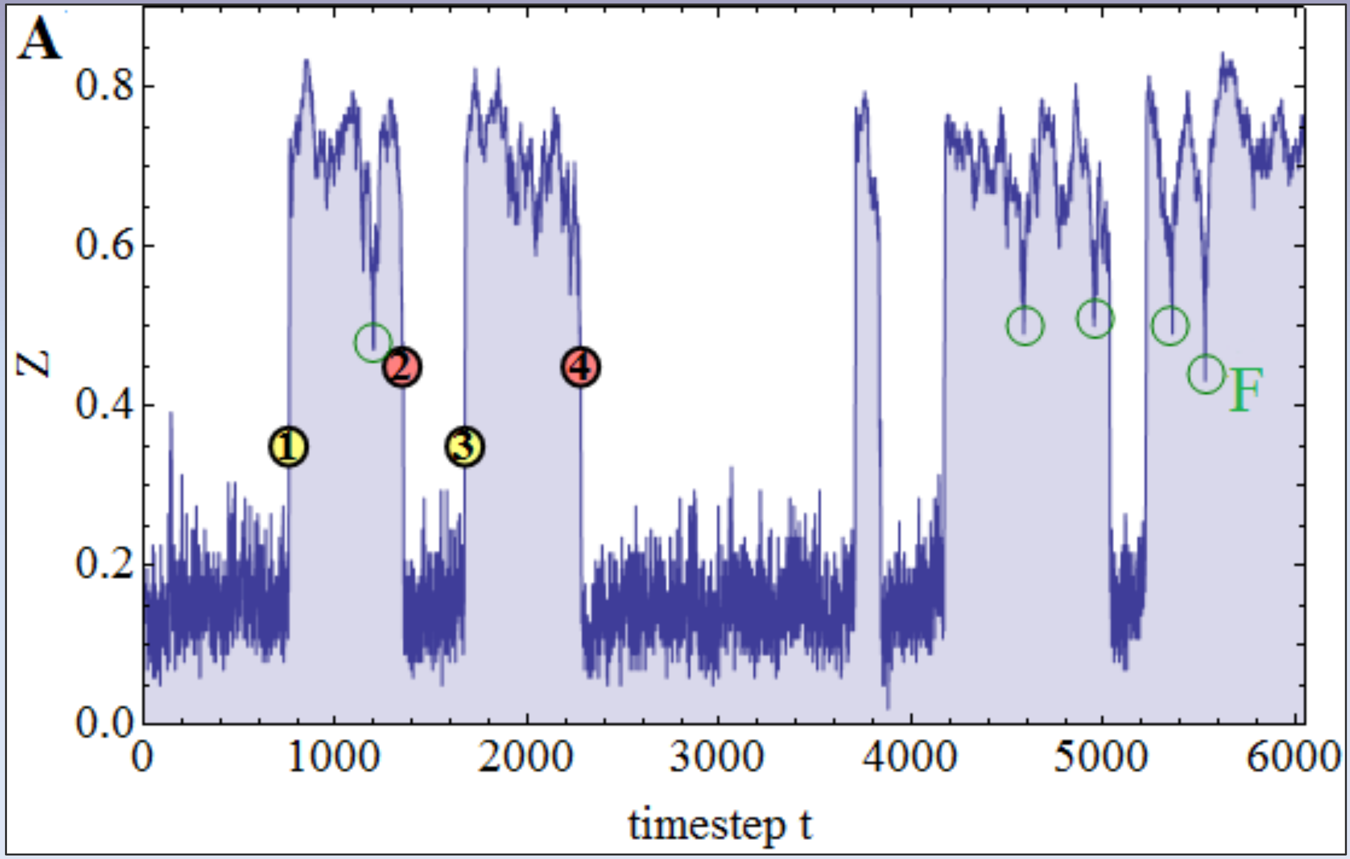
Finite size effects: qualitatively different physics

A small network with around N=100 nodes: in the hysteresis region we get switching between the two phases:

Sudden transition!

1. Why?? How??

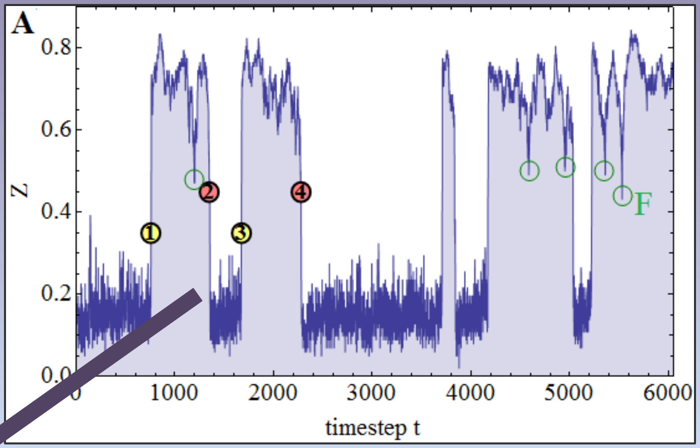
2. Is there any prewarning?



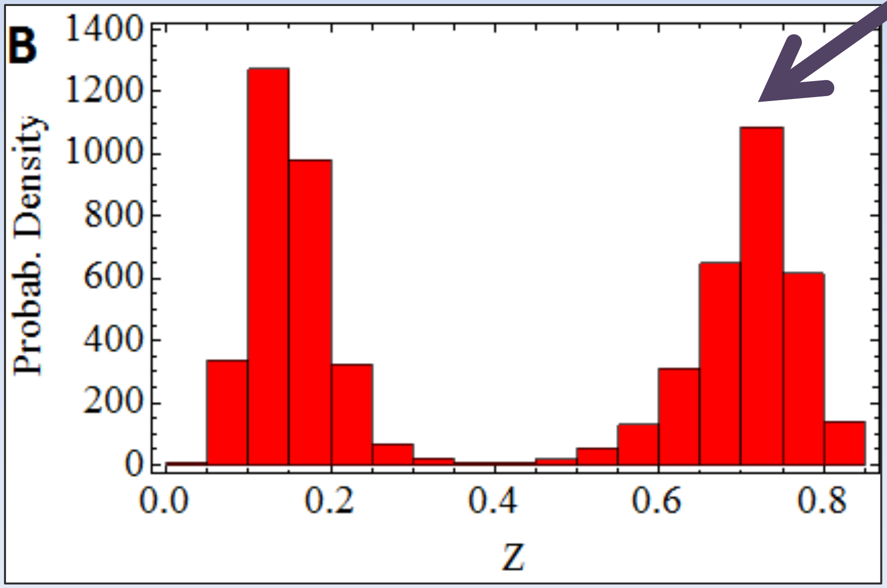
Z= Fraction of active nodes measured in time.

Finite size effects: qualitatively different physics

Time dynamics:

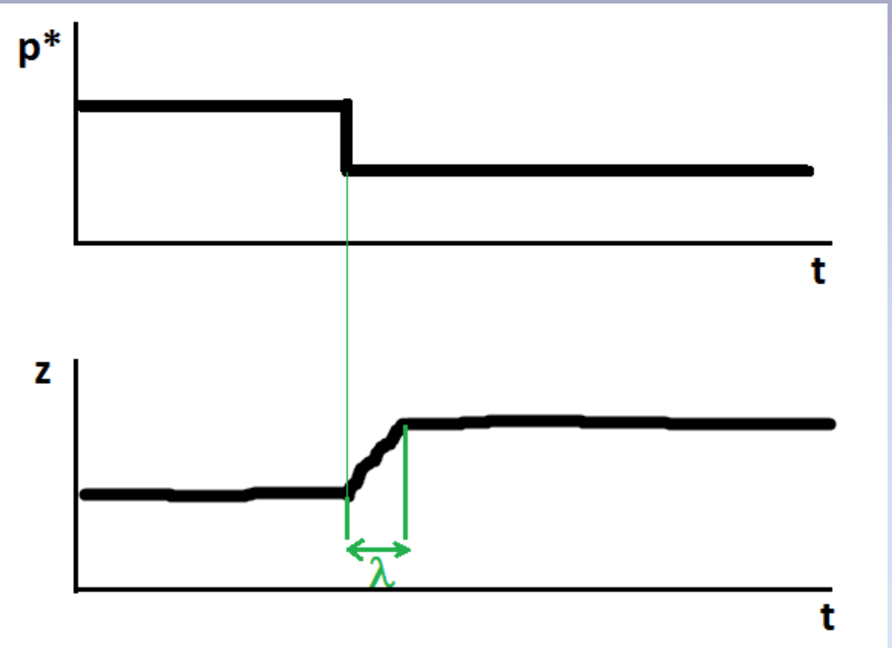


Probab. distribution:



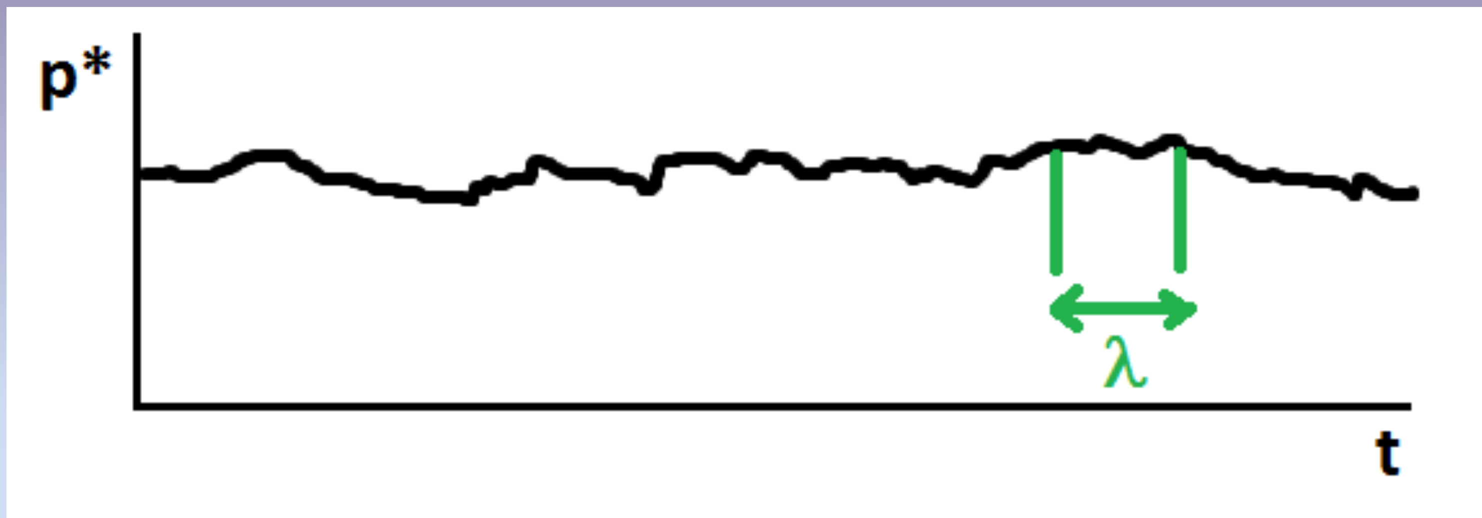
We find the exact mechanism of the phase switching.

How does the network react when p* is abruptly changed?



The system has a specific relaxation time, λ.

Because of the **stochastic nature of internal and external failures**, fraction of internally (or externally) failed nodes is actually fluctuating around the equilibrium values: p* and r.



It is natural to define the moving average: $\boxed{{p_\lambda}^*(t)}$

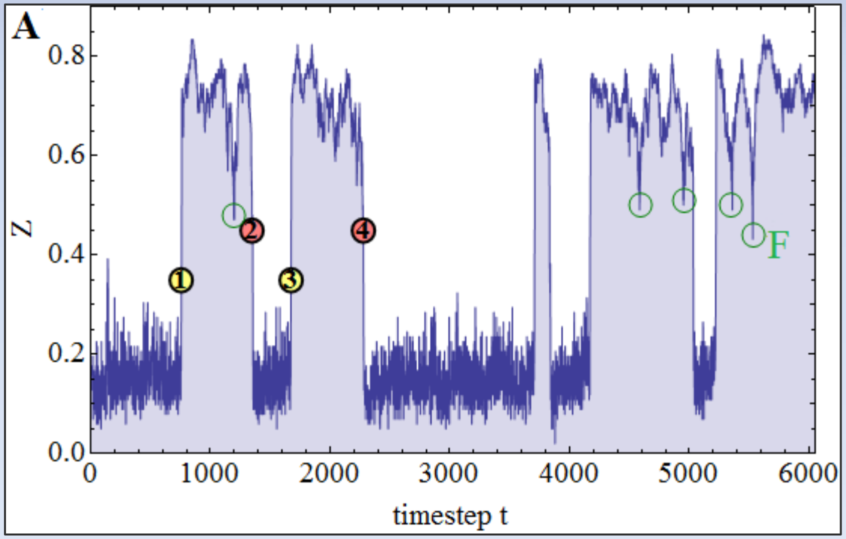
We hypothesize that the "true" value of p* that the system sees, is this moving average
→ **slow, adiabatic change**.

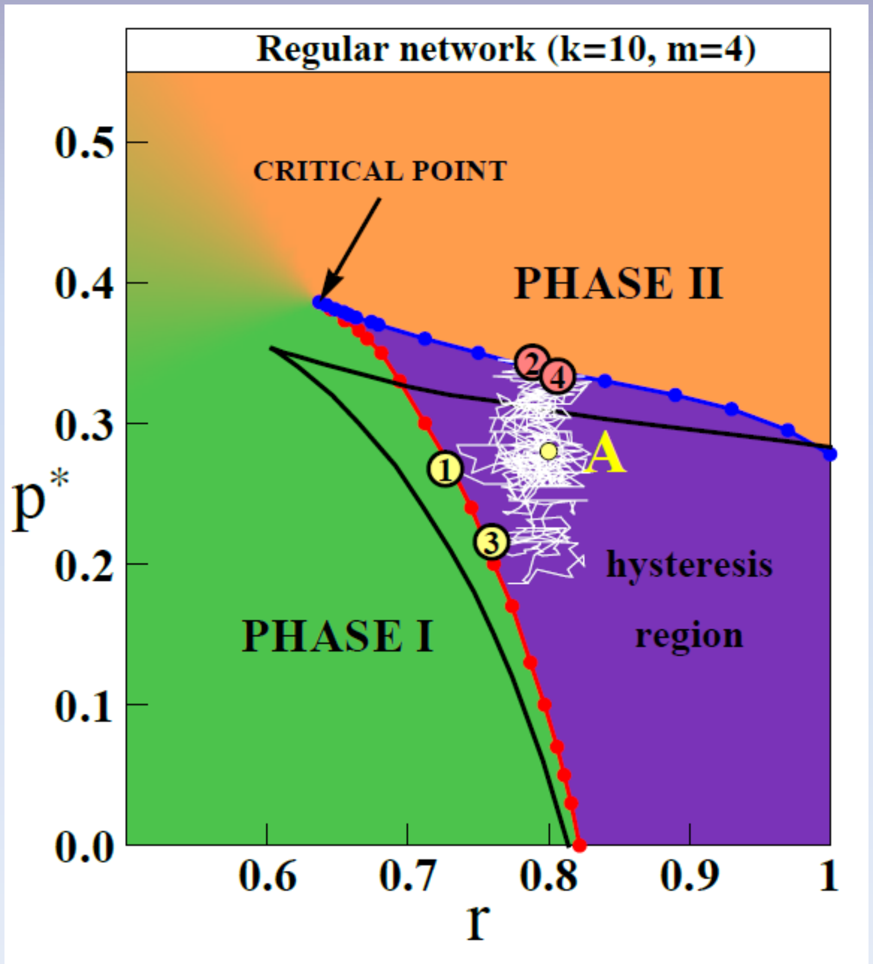
For the external failures we can define an analogous moving average: $\boxed{r_\lambda(t)}$

This defines the trajectory $\boxed{(r_\lambda(t),\ p_\lambda{}^*(t))}$!

Let's observe this trajectory in the phase diagram (white line, see below).

The trajectory crosses the spinodals (critical lines) interchangeably, and causes the phase flipping.



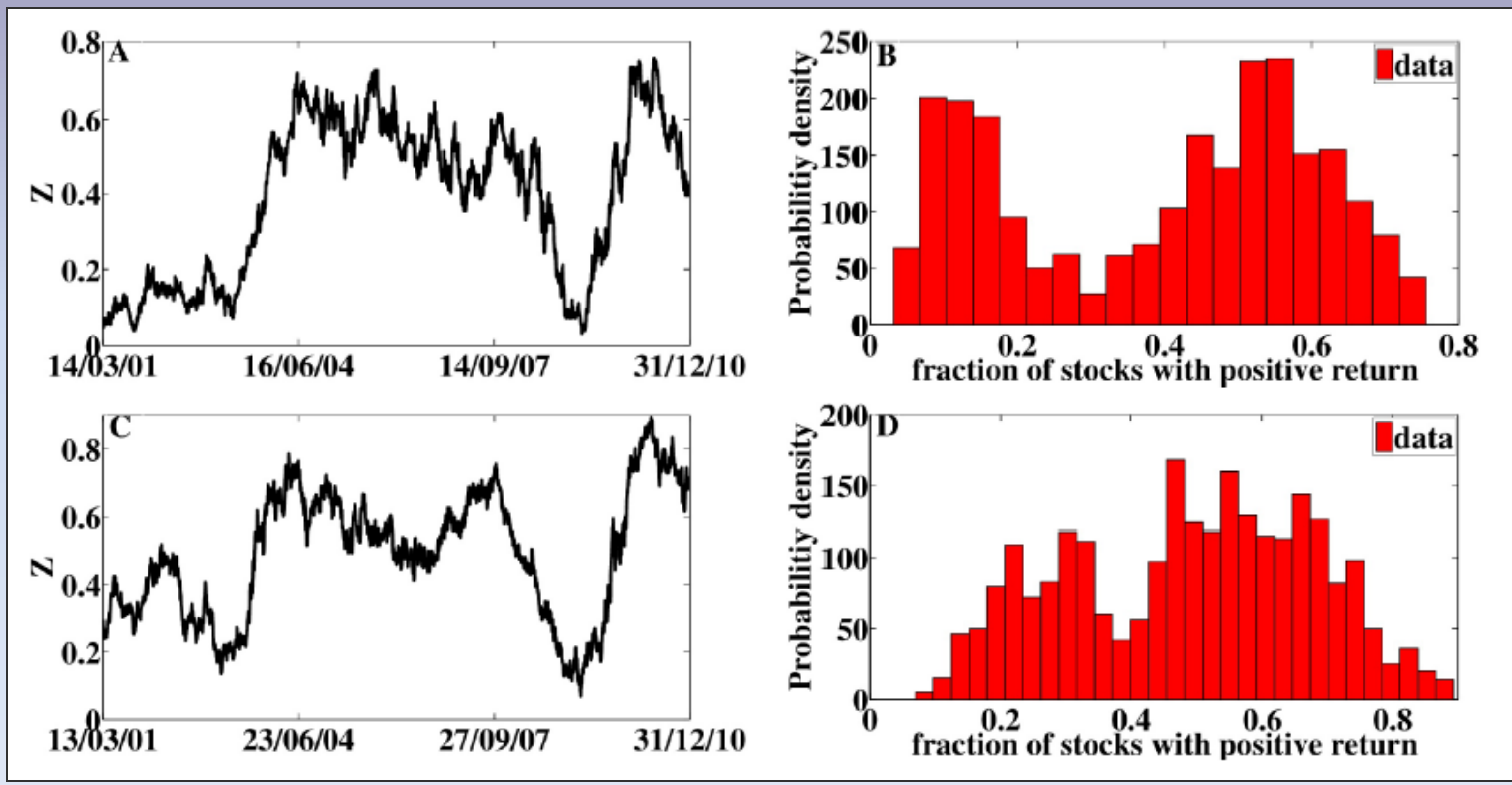


Real networks: empirical support

Economic networks: Networks of companies.

Indian
BSE 200

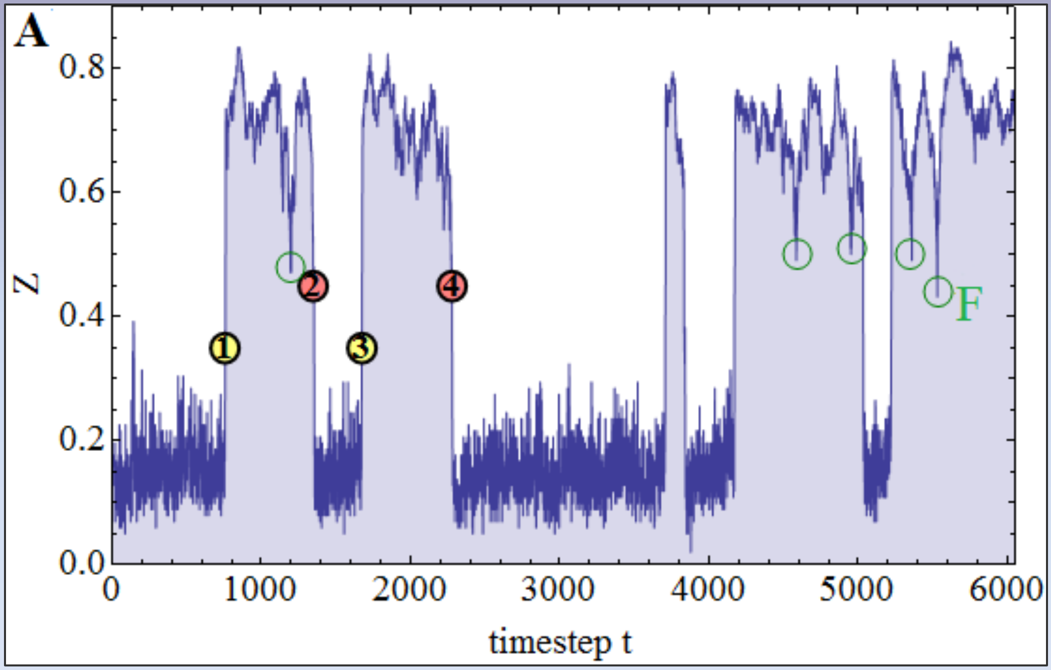
S&P 500



Mapping: z is defined as a fraction of companies with positive returns, measured in moving intervals to capture fundamental changes rather then speculations.

“Bonus” phenomenon: Flash crashes

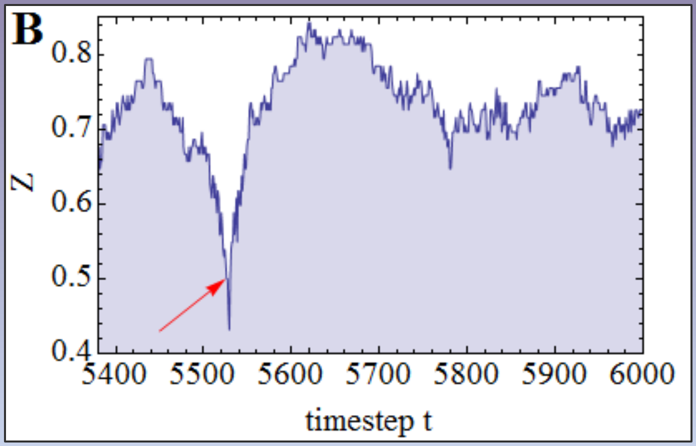
An interesting by-product produced by the model:



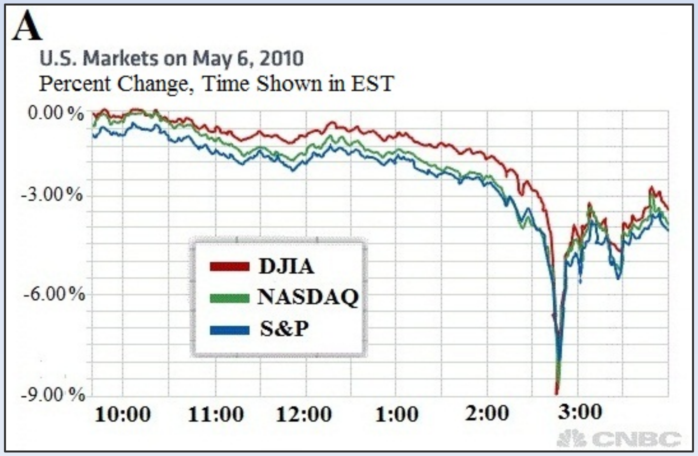
Sometimes the network rapidly crashes, and then quickly recovers.

“Bonus” phenomenon: Flash crashes

Model predicts the existance of “flash crashes”.



Real stock markets also show a similar phenomenon.
Q: Possible relation?



“Flash Crash 2010”

Future work:

-We are extending the model on interdependent networks.

Preliminary results show a complicated phase diagram with 6 critical lines and two critical points.

Mean field theory self-consistent equation for Z:

$$z(p^*, r) = 1 - p^* - r(1 - p^*) \sum_k f_k \sum_{j=0}^{m} \binom{k}{j} z^j (1 - z)^{k-j}$$