Opinion Models on Complex Networks

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- **Statistical Analysis of Bankrupting and Non-Bankrupting Stocks**

What is Sociophysics ?

- Quantitative laws in the collective properties of a large number of people.
- Quantitatively understand how statistical laws arise out of the free will of single individuals
- Recently transformed from a philosophical declaration of principles to a concrete research.
- Opinion dynamics, cultural dynamics, language dynamics, crowd behavior
- Using methods from statistical physics to solve social problems.

Map Opinion Dynamics to Physics Problems

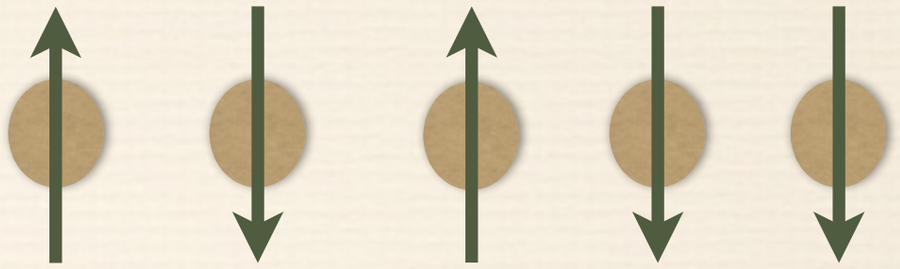
The Basic Questions

Disorder

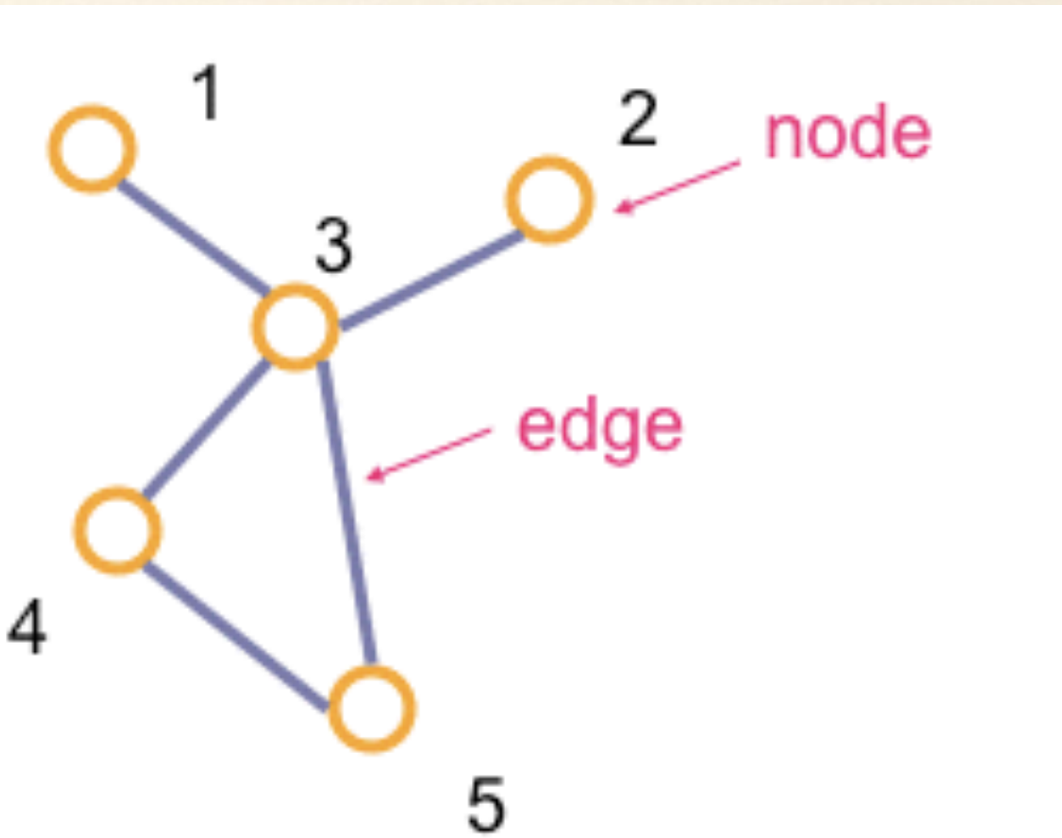
Order

Map Opinion Dynamics to Physics Problems

Opinions

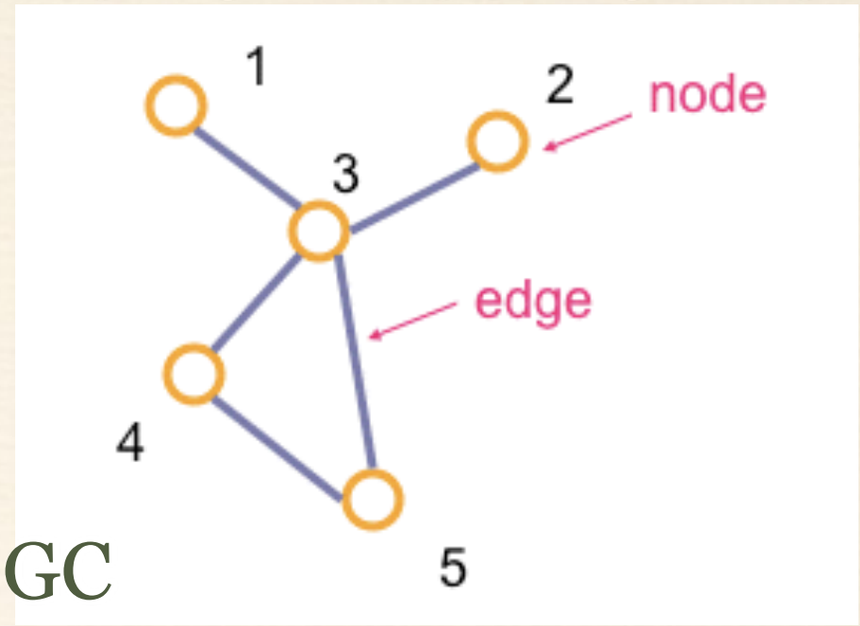


Human Relationship



Complex Network

Some Important Definitions for networks



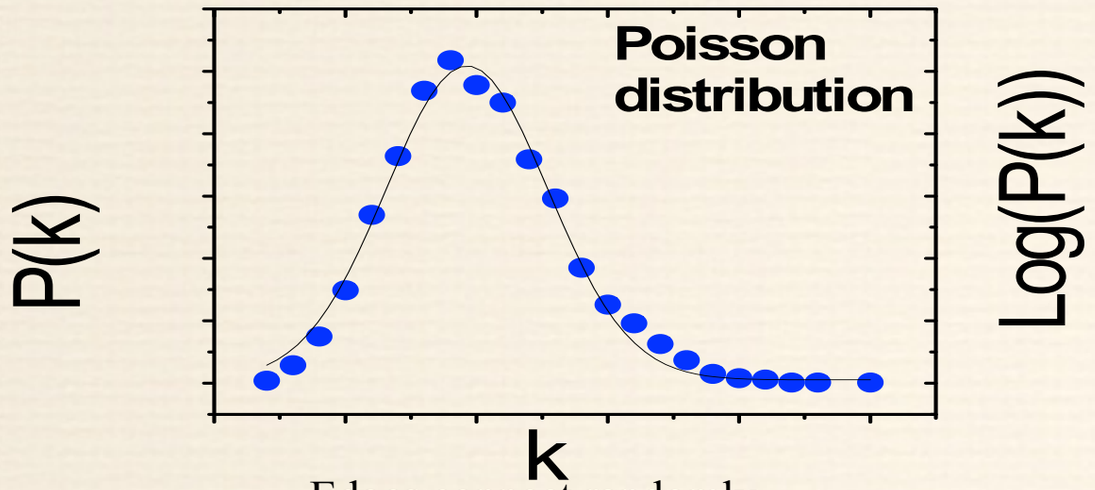
- Network size, N = total # of nodes

- Giant Component (GC):
  - The largest cluster of connected nodes
  - Size of the GC S_1 : total # of nodes in the GC
  - $s_1 = S_1/N$, fraction of nodes in the GC
  - S_2 size of the second largest cluster

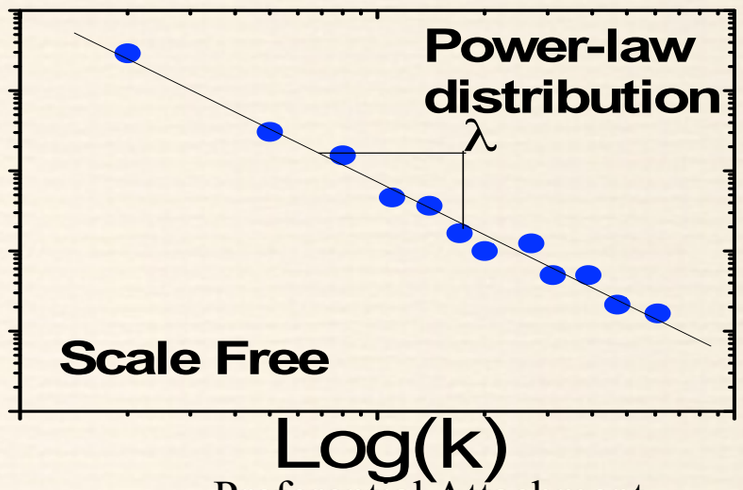
- Critical threshold f_c , such that the GC dose not exist below f_c , and GC begin to exist above f_c

Different Networks

- ER networks: random graphs, degree follows a poisson distribution
- SF networks: degree follows a power law distribution



Edges connect randomly



Preferential Attachment

Albert-László Barabási

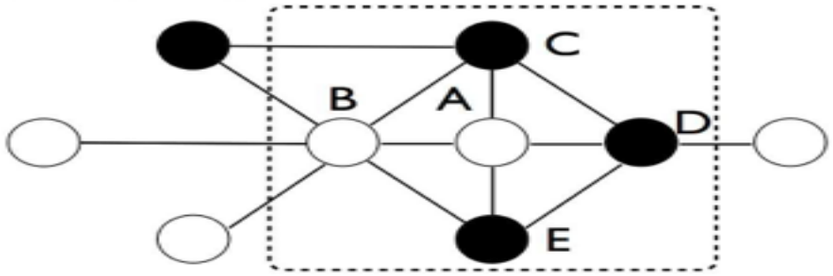
Motivations & Questions

Why NCO Model and What is the NCO rule?

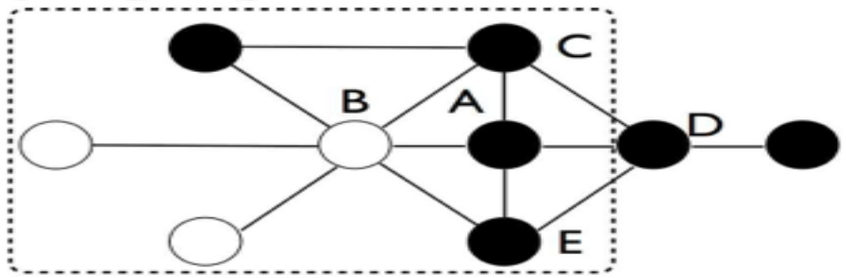
In the NCO model, an agent will follow the opinion of its local majority which include agent's own opinion

Majority Model .VS. NCO Model

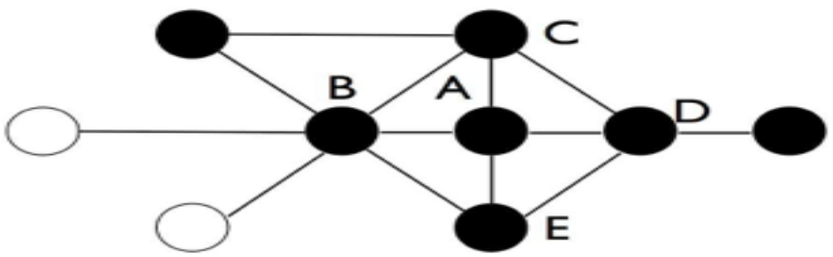
(a) Majority t=0



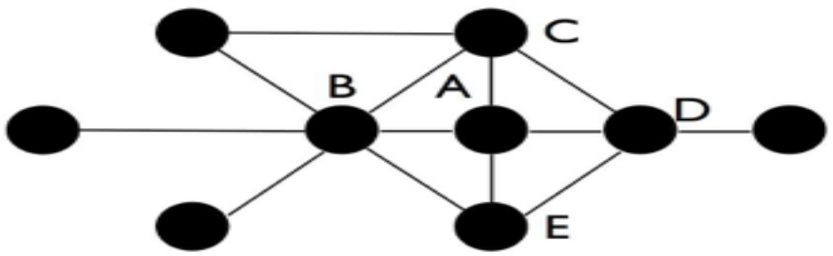
(b) Majority t=1



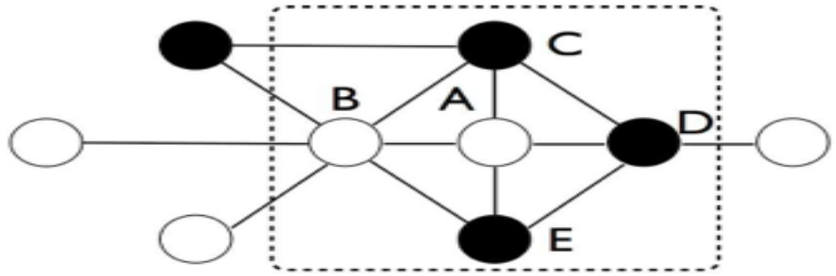
(c) Majority t=2



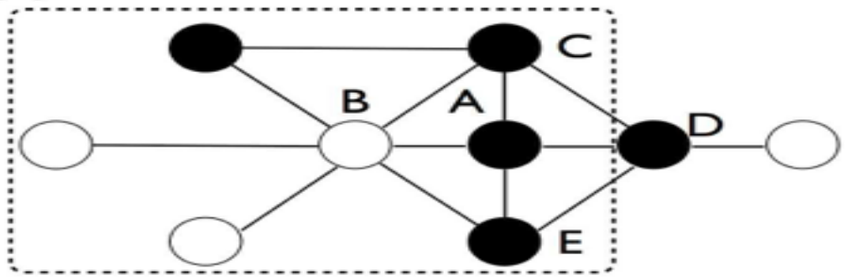
(d) Majority t=3



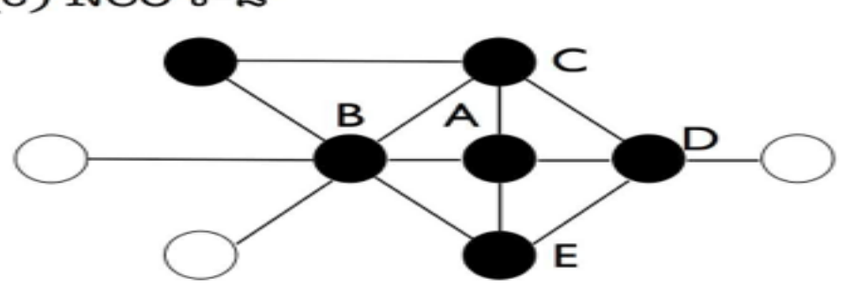
(a) NCO t=0



(b) NCO t=1



(c) NCO t=2

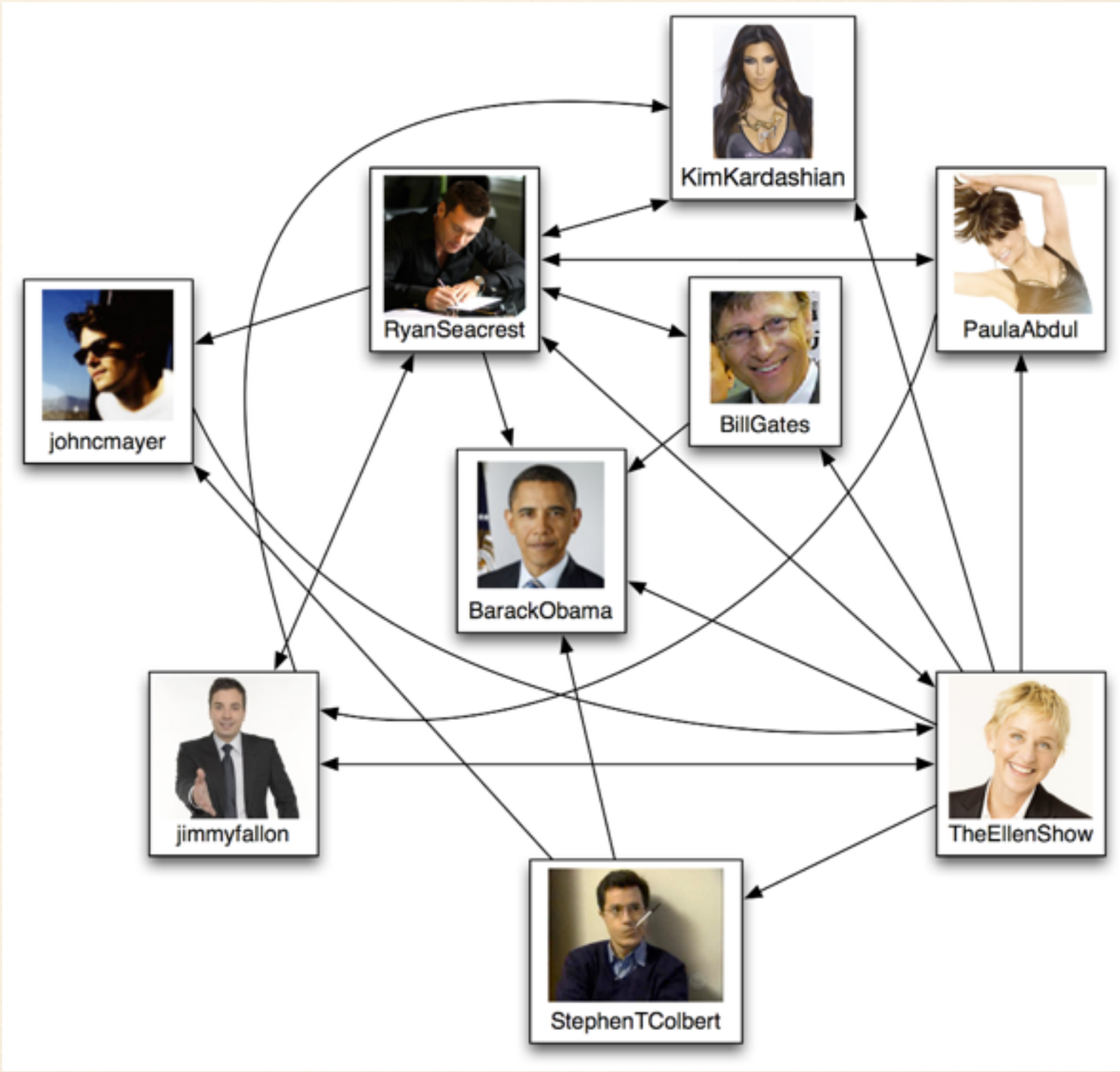


Consider the agent's own opinion leads to non-consensus state

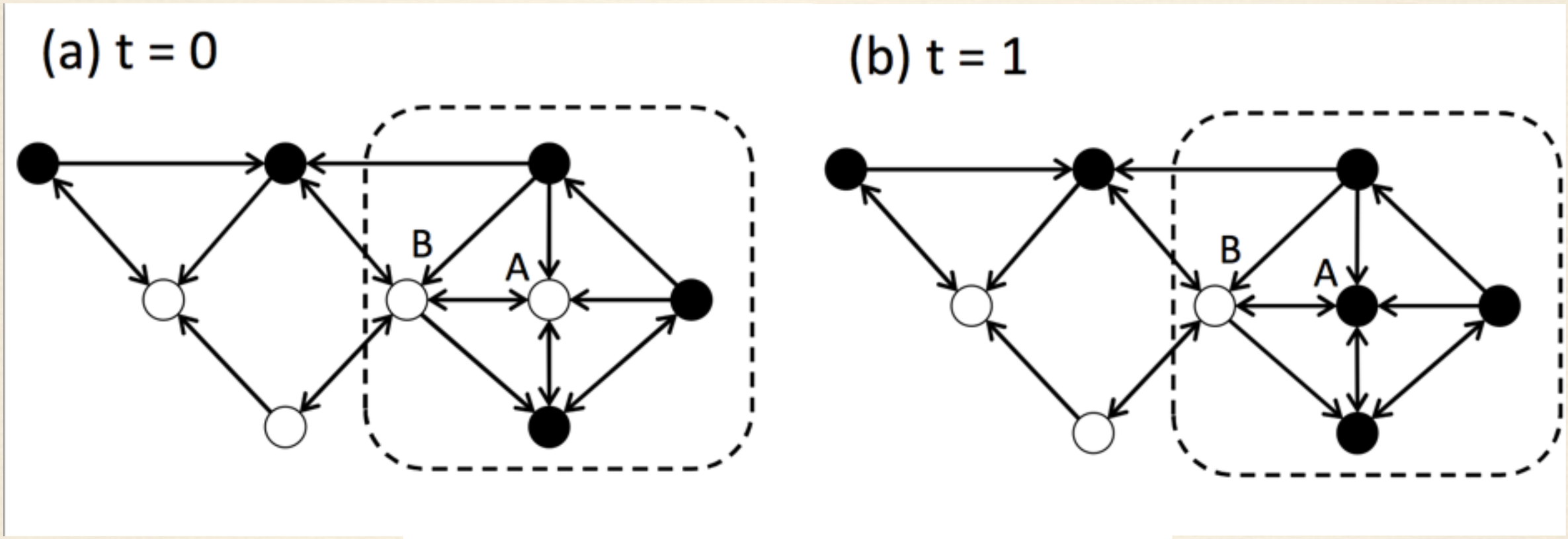
Motivations & Questions

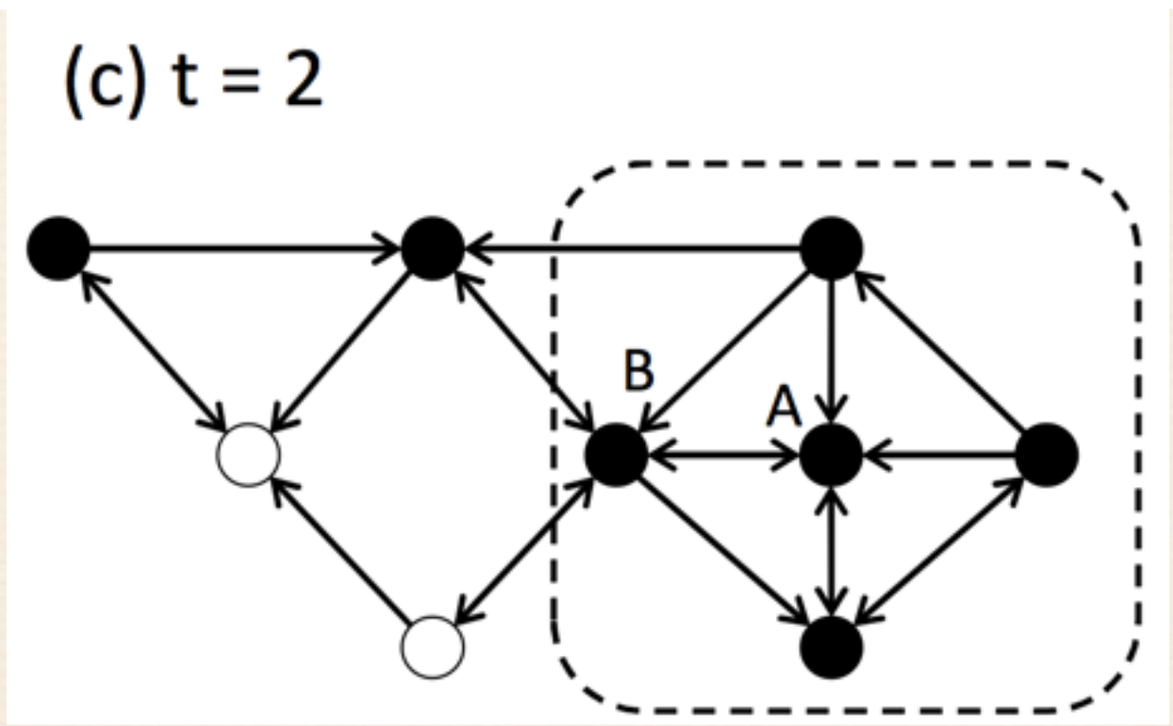
Why Directed Network?

How Directionality Influence the Opinion Dynamics ?



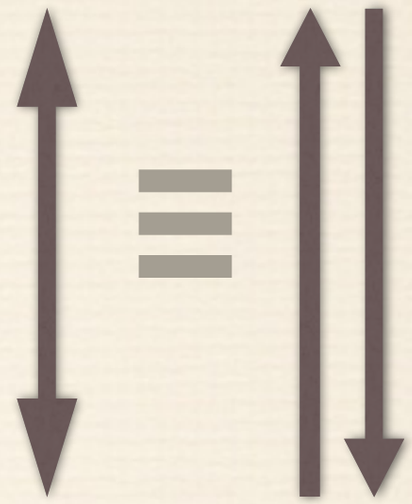
Demo of NCO on Directed Network





The directionality ξ and in-degree out-degree correlation ρ

$$\xi = \frac{L_{unidirectional}}{L_{total}}$$

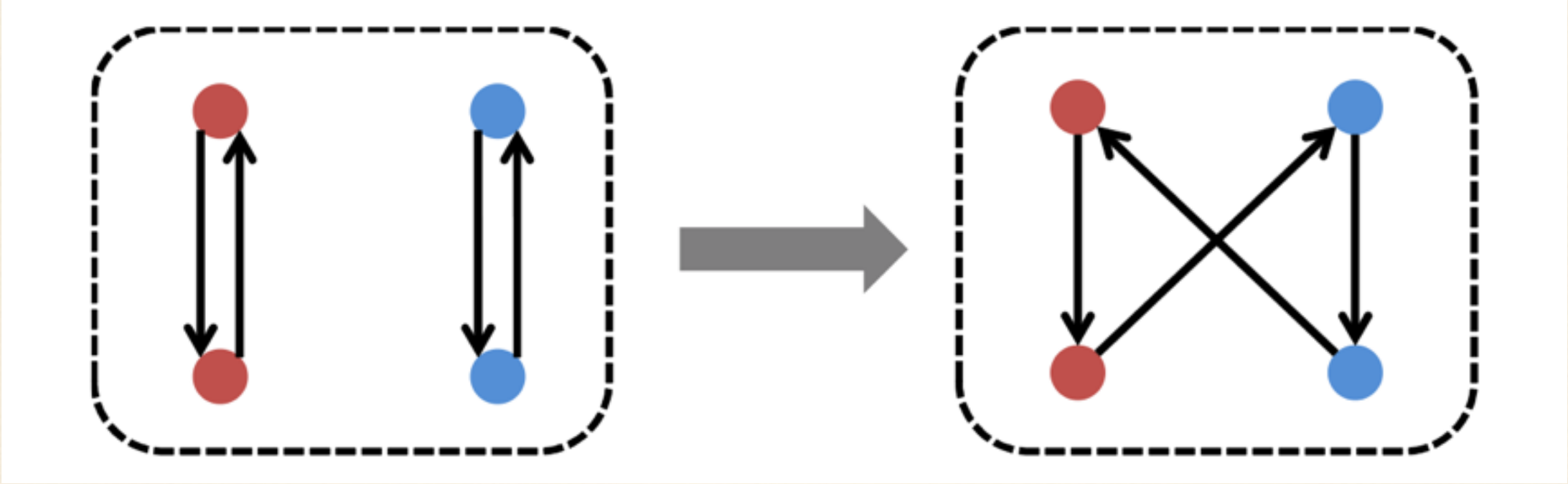


$$L_{total} = L_{unidirectional} + 2 * L_{bidirectional}$$

$$\rho = \frac{\sum_{i=1}^{N}(k_{i,\mathrm{in}} - \langle k \rangle)(k_{i,\mathrm{out}} - \langle k \rangle)}{\sqrt{\sum_{i=1}^{N}(k_{i,\mathrm{in}} - \langle k \rangle)^2}\sqrt{\sum_{i=1}^{N}(k_{i,\mathrm{out}} - \langle k \rangle)^2}}$$

Directionality-increasing rewiring (DIR)

A rewiring approach that changes the directionality but does not change the in-degree and out-degree of any node



Constructing an asymmetric in-degree and out-degree network and rewiring it to decrease its directionality (ANC-DDR)

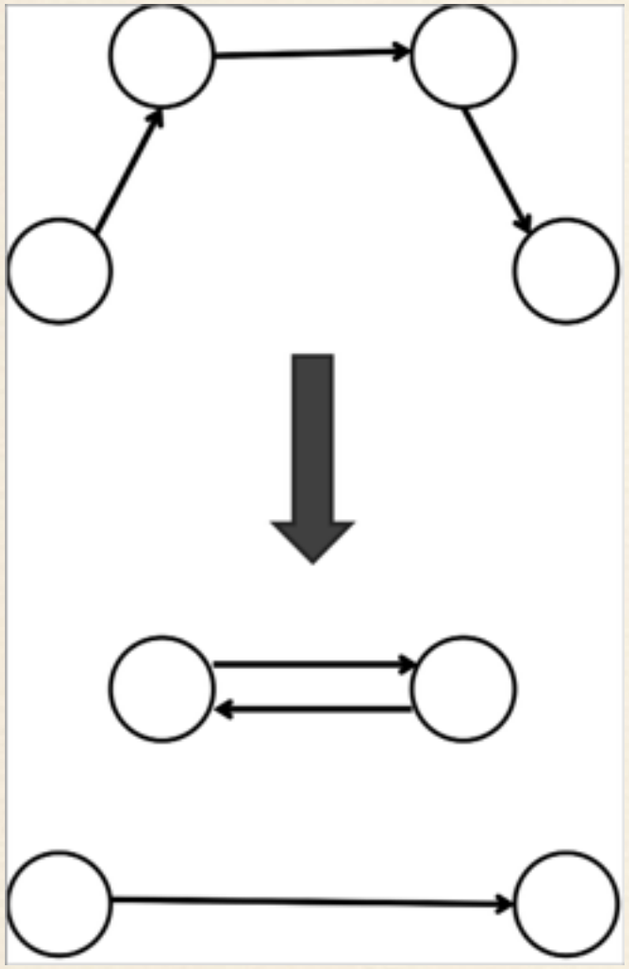
Asymmetric in-degree and out-degree Network Construction

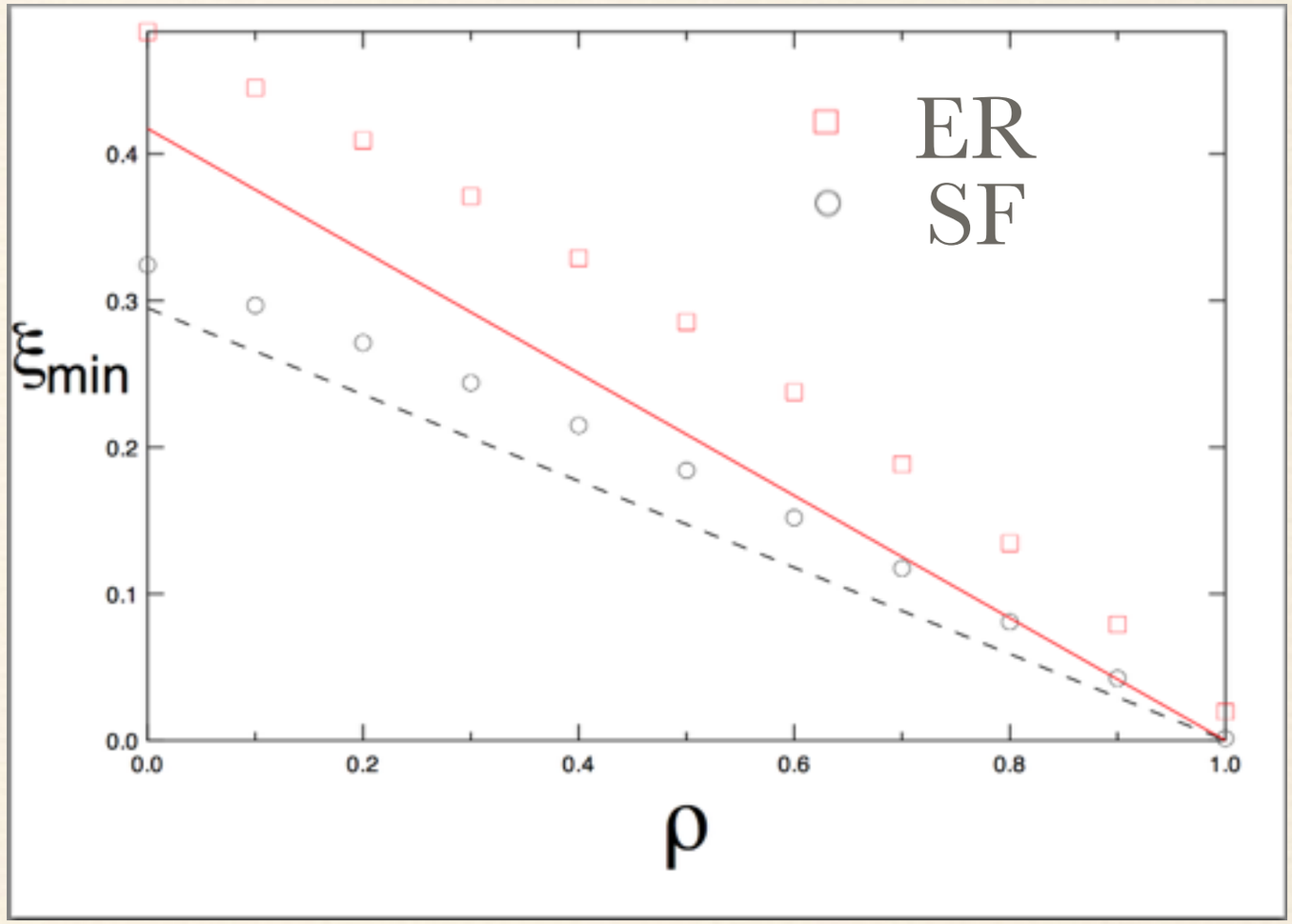
1. Generate an in-degree sequence and a null out-degree sequence
2. Copy a fraction ρ of the in-degree sequence to the out degree sequence
3. Shuffle the fraction 1-ρ of the in-degree sequence as the rest of the out-degree sequence
4. Randomly connecting all nodes based on their in and out degree

A network with a directionality ξ —>1 and in and out degree correlation close to ρ

Constructing an asymmetric in-degree and out-degree network and rewiring it to decrease its directionality (ANC-DDR)

Directionality Decreasing Rewiring

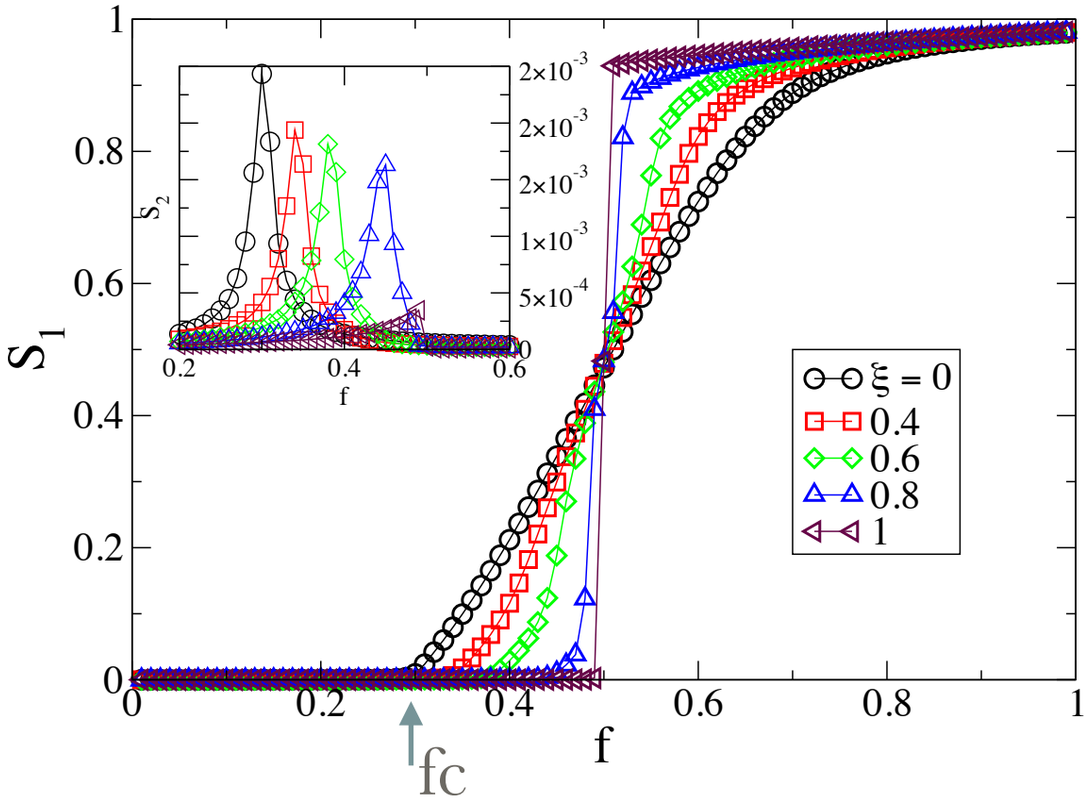




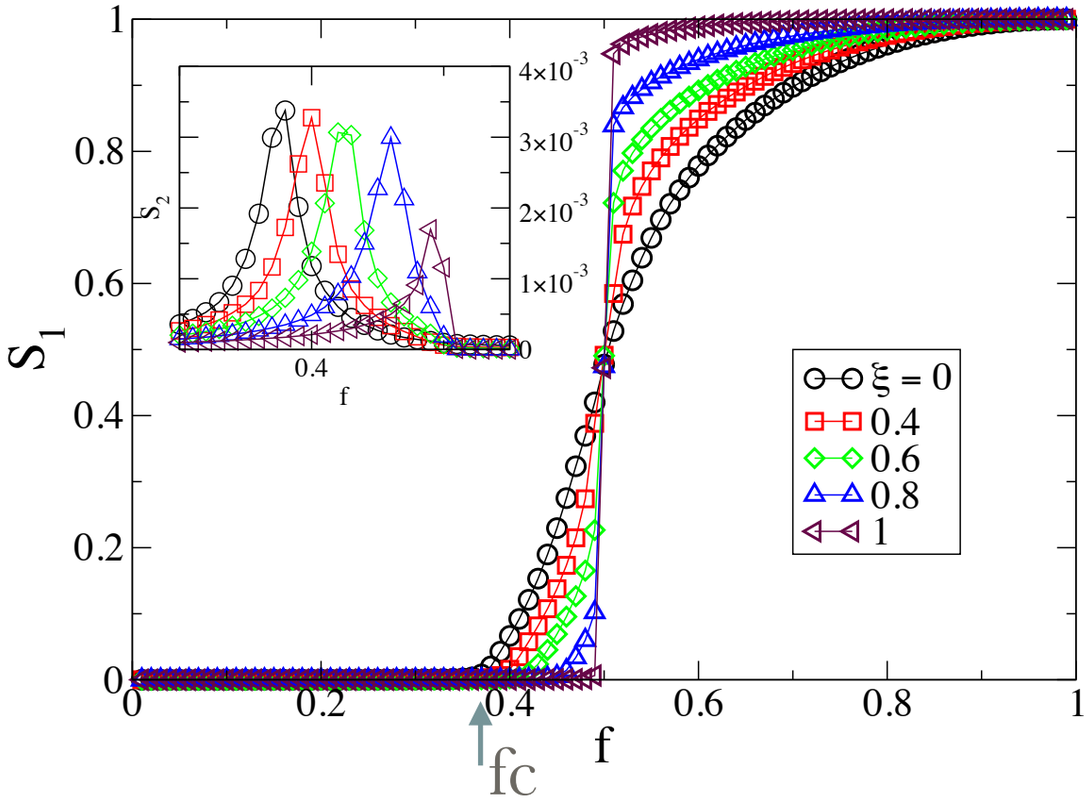
$$E(\xi_{\min}) = \frac{1-\rho}{\langle k \rangle} \sum_{k=0}^{N-1} kP(k) \left(\sum_{i=0}^{k} P(i) - \sum_{i=k}^{N-1} P(i) \right).$$

NCO Model on Directed Networks

- The influence of directionality (in-degree=out-degree)



ER Network

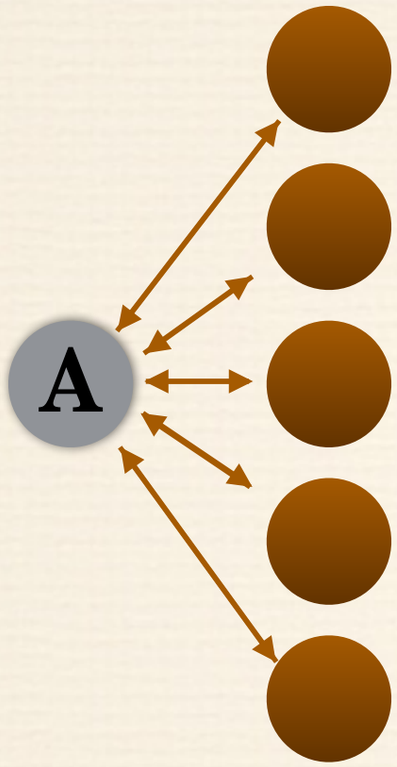


SF Network

Increasing directionality helps the majority to eliminate the minority and pushing the system undergoes an abrupt phase transition

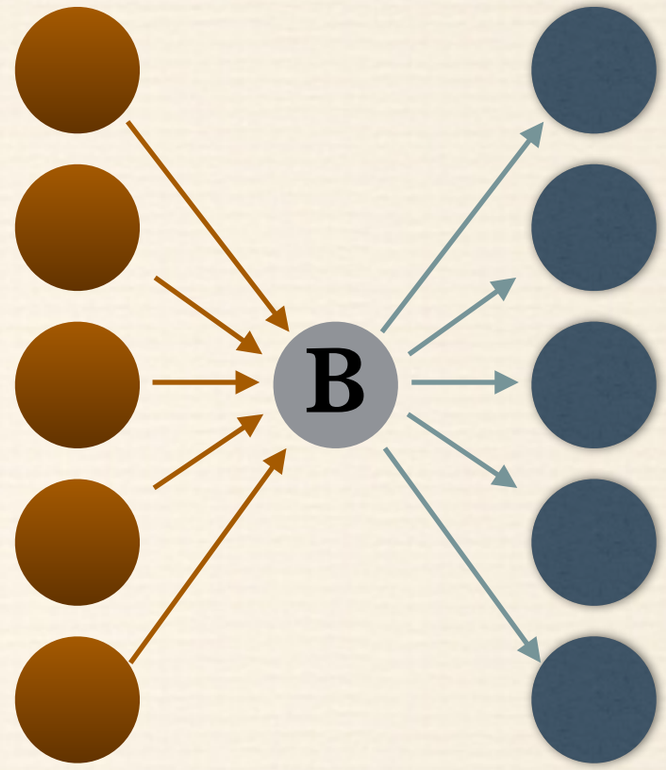
NCO Model on Directed Networks

- Directionality helps the opinion to spread



ξ=0

Opinion is confined within a group

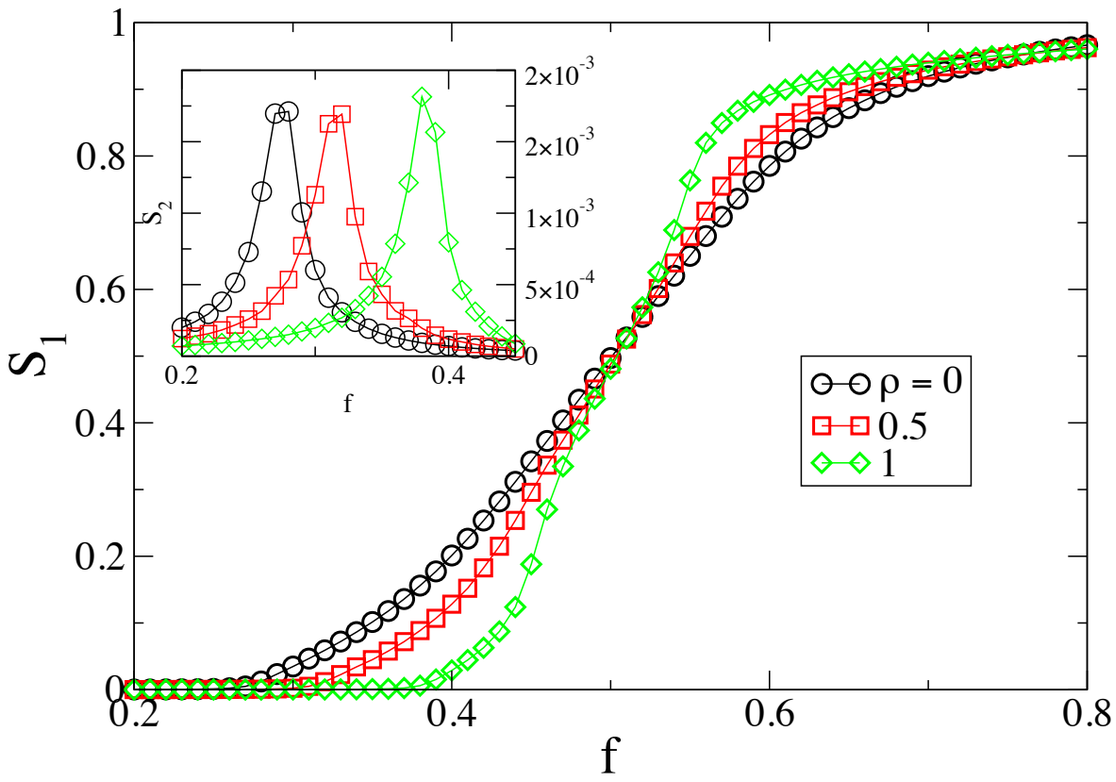


ξ=1

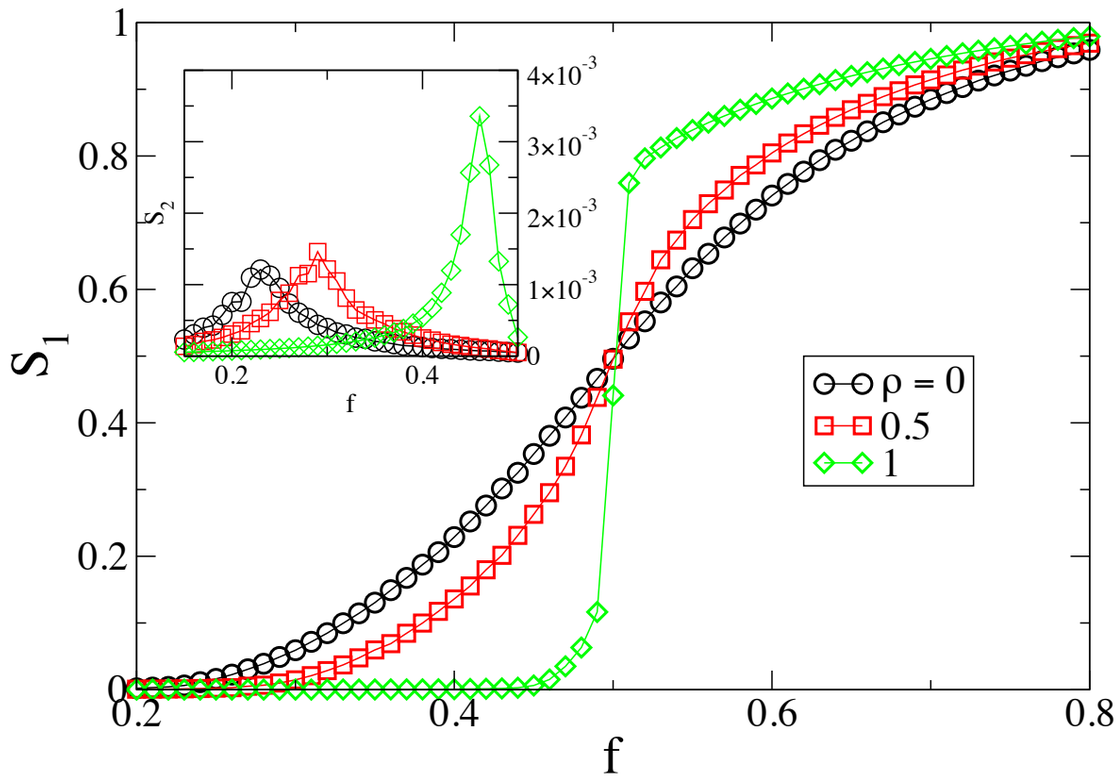
Opinion is spread out

NCO Model on Directed Networks

- The influence of in-degree and out-degree asymmetry



ER network with ξ=0.6

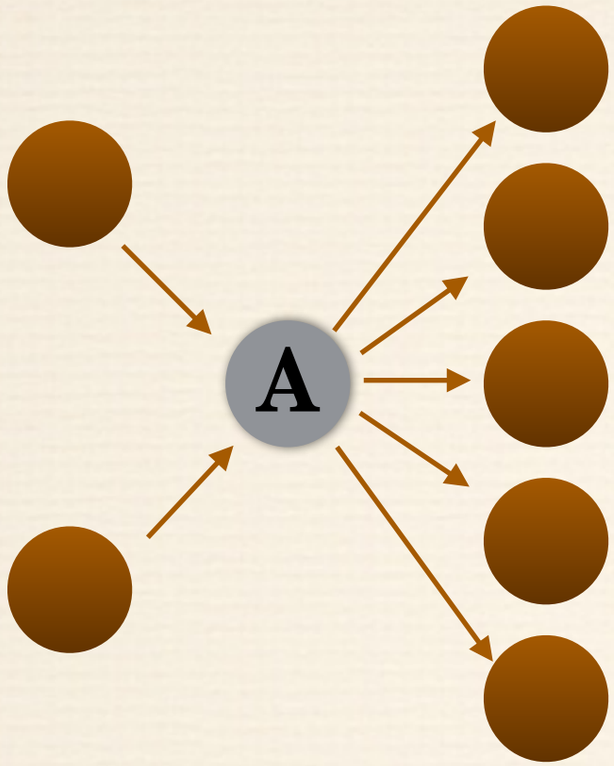


SF network with ξ=0.6

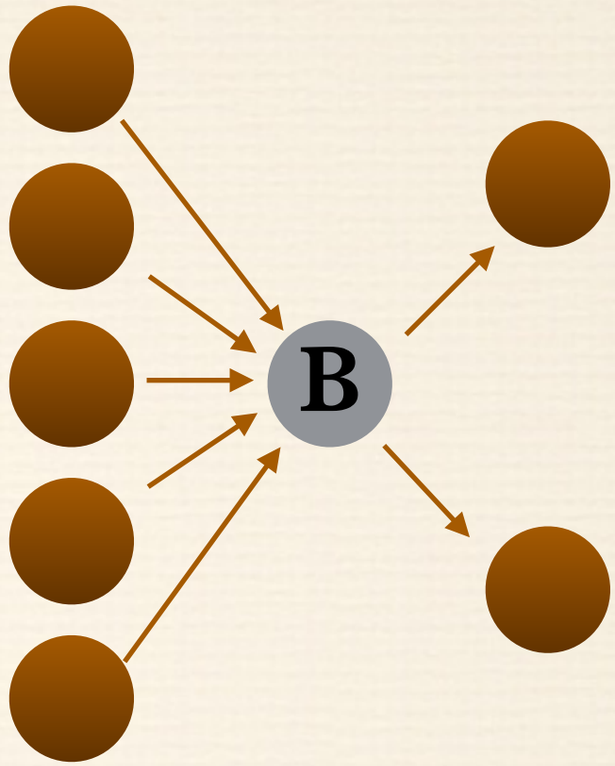
Decreasing the in-degree and out-degree correlation helps the minority to survive

NCO Model on Directed Networks

- The influence of in-degree and out-degree asymmetry



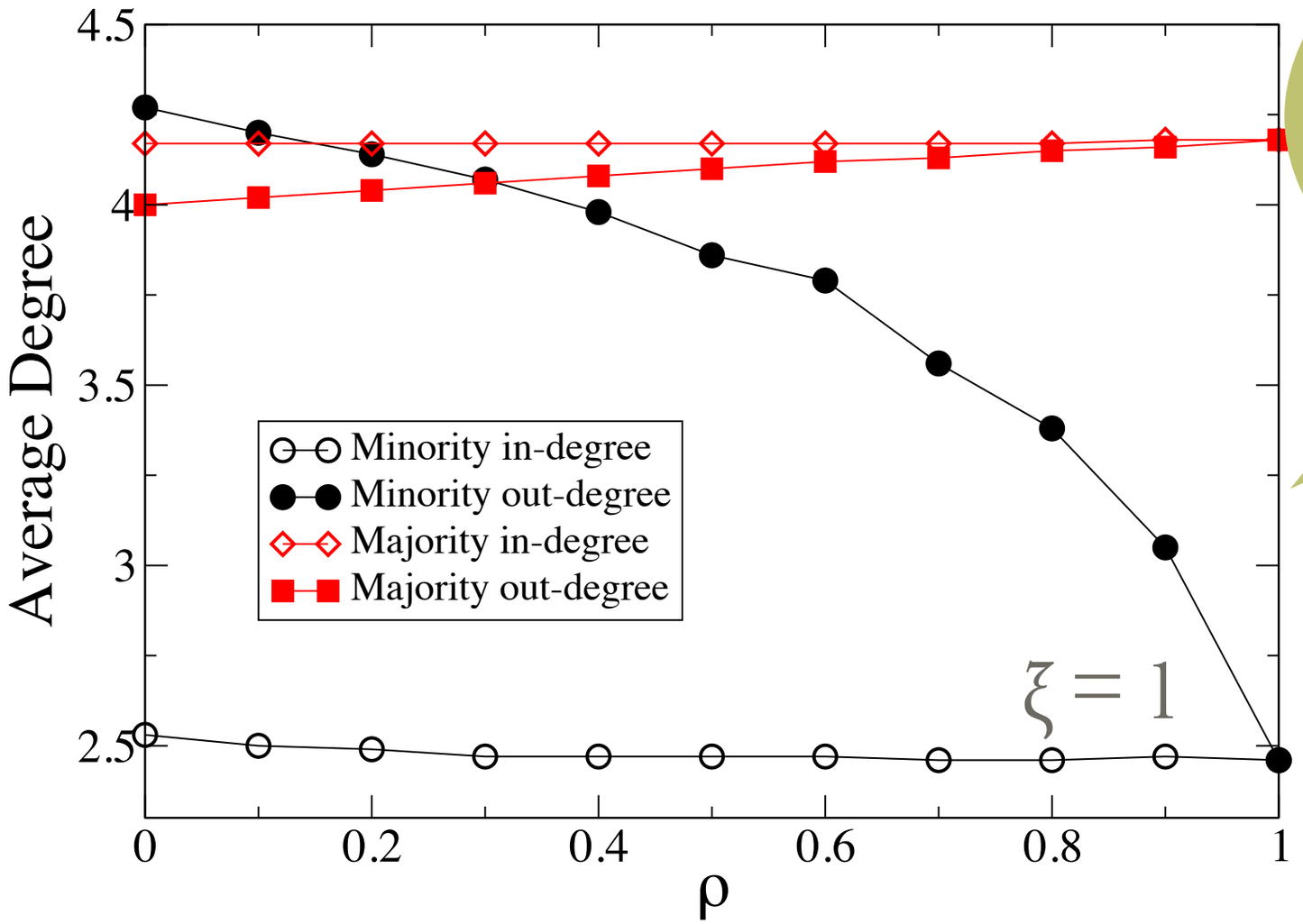
Help to hold the minority opinion and spread its opinion



More likely be influenced by majority opinion but hardly to spread

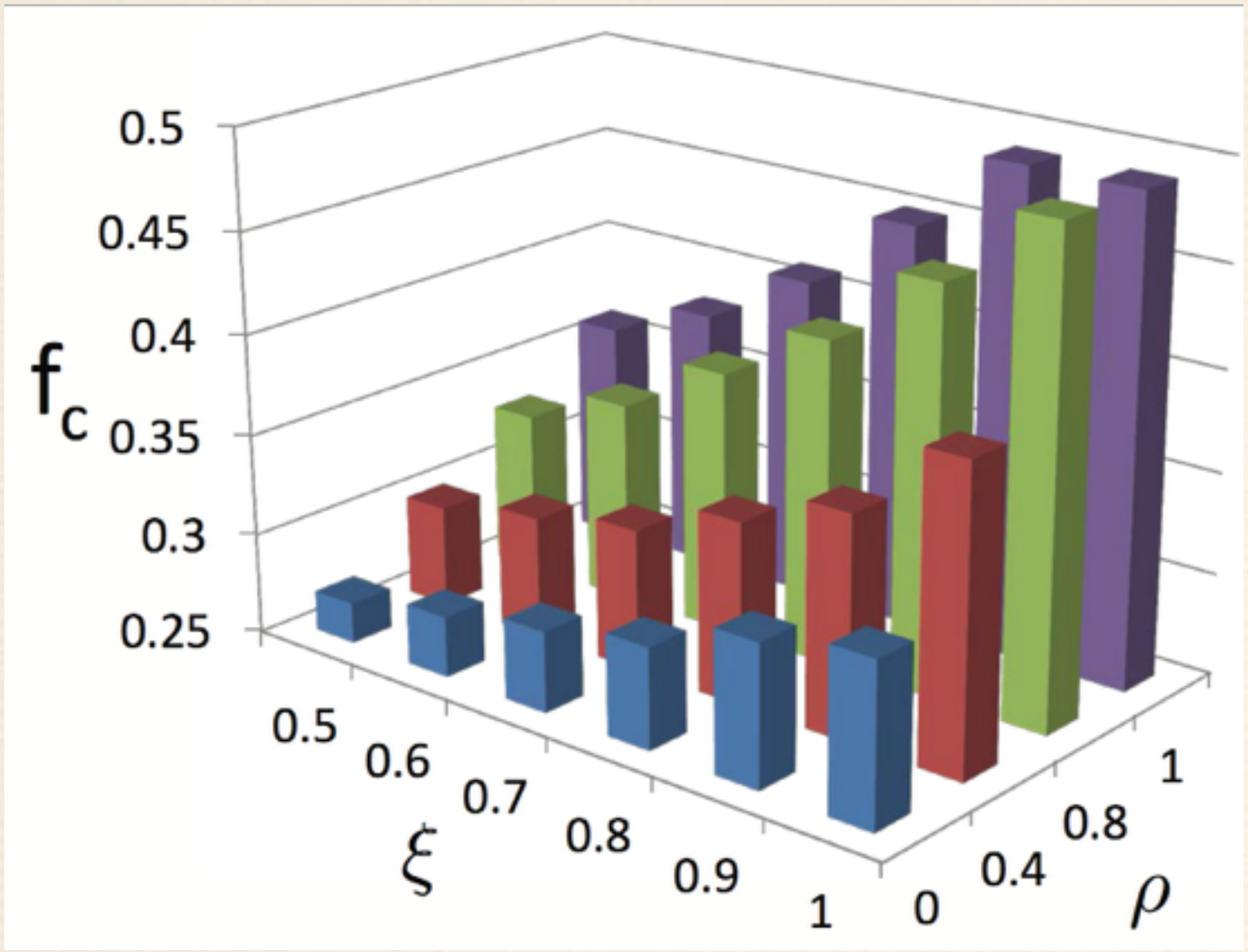
NCO Model on Directed Networks

- The influence of in-degree and out-degree asymmetry



Minority
Out-degree > In-degree
Majority
In-degree > Out-degree

NCO Model on Directed Networks



Take Home Message

- **Increasing directionality helps the majority opinion to spread**
- **Increasing the asymmetric between in and out degree helps the the minority to survive**

Publications

- "Strategy of Competition between Two Groups Based on an Inflexible Contrarian Opinion Model," Phys. Rev. E 84, 066101 (2011)
- "Statistical Analysis of Bankrupting and Non-Bankrupting Stocks," Europhys. Lett. (EPL) 98, 28005 (2012).
- "Non-Consensus Opinion Models on Complex Networks," J. Stat. Phys. 151, 92-112 (2013)
- "Effect of the Interconnected Network Structure on the Epidemic Threshold," Phys. Rev. E 88, 022801 (2013)
- "Non-consensus Opinion model on directed networks," Submitted to PRE
- "Susceptible-Infected-Susceptible Model on Interdependent Networks," Submitted to PRE
- "Centrality metrics and their application to the opinion model," Under construction

Thanks!

Proof of minimum directionality

$$E(\xi_{\min}) = \frac{1-\rho}{\langle k \rangle} \sum_{k=0}^{N-1} kP(k) \left(\sum_{i=0}^{k} P(i) - \sum_{i=k}^{N-1} P(i) \right).$$

$$\xi_{\min} = \frac{\sum_{i=1}^{N} |K_{i,\text{in}} - K_{i,\text{out}}|}{\sum_{i=1}^{N} (K_{i,\text{in}} + K_{i,\text{out}})}.$$

$$\begin{aligned} E(\xi_{\min}) &= E\left(\frac{\sum_{i=\rho N+1}^{N} |K_{i,\text{in}} - K_{i,\text{out}}|}{\sum_{i=1}^{N} (K_{i,\text{in}} + K_{i,\text{out}})}\right) \\ &= (1-\rho) E\left(\frac{N|K_{\text{in}} - K_{\text{out}}|}{\sum_{i=1}^{N} (K_{i,\text{in}} + K_{i,\text{out}})}\right) \\ &= (1-\rho) E(\xi_{min,\rho=0}) \end{aligned}$$

Proof of minimum directionality

$$E(\xi_{\min}) = \frac{1-\rho}{\langle k \rangle} \sum_{k=0}^{N-1} kP(k) \left(\sum_{i=0}^{k} P(i) - \sum_{i=k}^{N-1} P(i) \right).$$

$$E(\xi_{\min,\rho=0}) = \frac{E[\mathrm{Max}(K_{\mathrm{in}}, K_{\mathrm{out}})] - E[\mathrm{Min}(K_{\mathrm{in}}, K_{\mathrm{out}})]}{2\langle k \rangle}$$

$$\begin{aligned} &Pr[\mathrm{Min}(K_{\mathrm{in}}, K_{\mathrm{out}}) = k] \\ &\quad = Pr[K_{\mathrm{in}} = k]Pr[K_{\mathrm{out}} \geq k] + Pr[K_{\mathrm{out}} = k]Pr[K_{\mathrm{in}} \geq k] \\ &\quad = 2P(k) \sum_{i=k}^{N-1} P(i) \end{aligned}$$

$$Pr[\mathrm{Max}(K_{\mathrm{in}}, K_{\mathrm{out}}) = k] = 2P(k) \sum_{i=1}^{k} P(i)$$

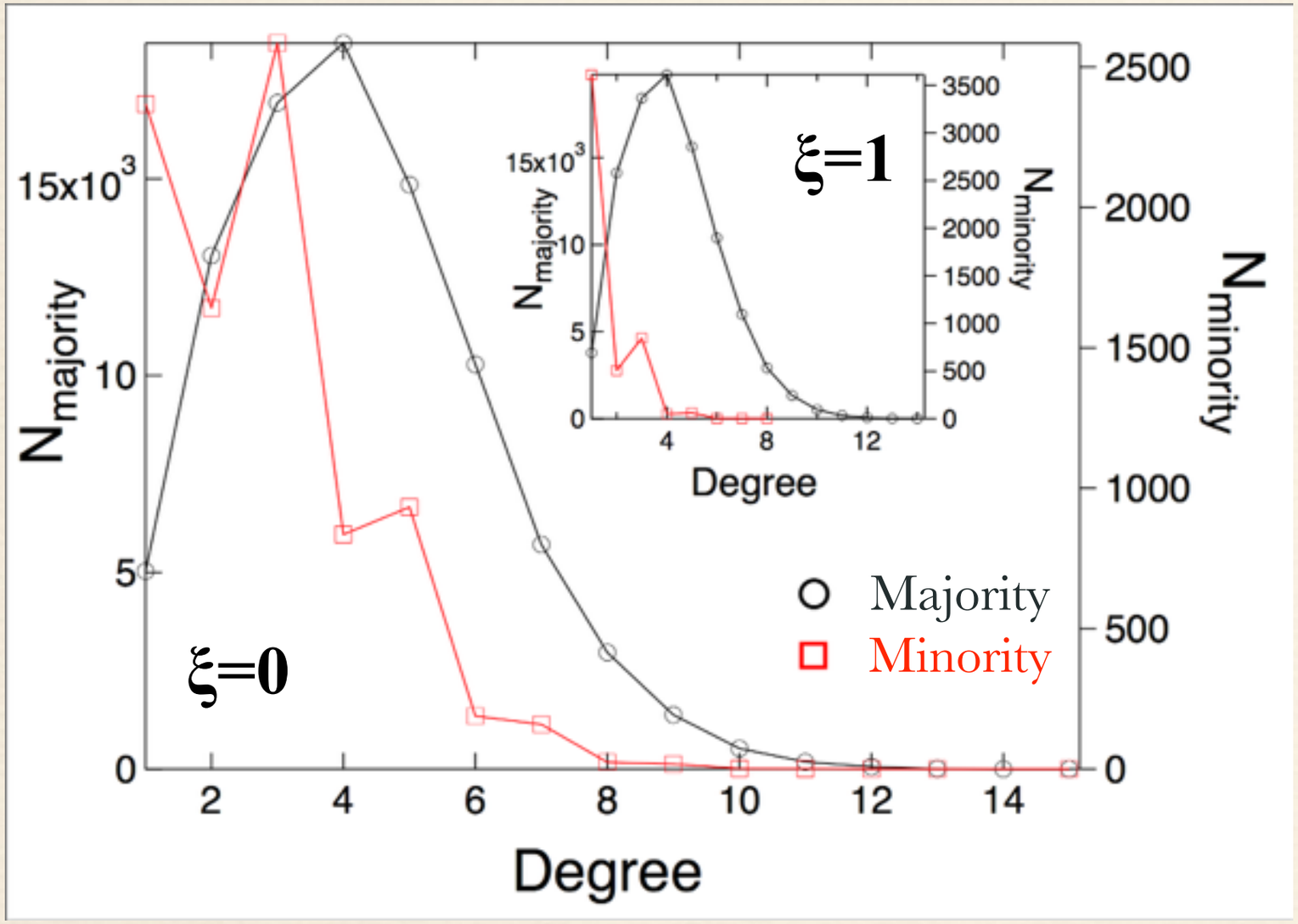
Proof of minimum directionality

$$E(\xi_{\min}) = \frac{1-\rho}{\langle k \rangle} \sum_{k=0}^{N-1} kP(k) \left(\sum_{i=0}^{k} P(i) - \sum_{i=k}^{N-1} P(i) \right).$$

$$E(\xi_{\min,\rho=0}) = \frac{1}{\langle k \rangle} \sum_{k=0}^{N-1} kP(k) \left(\sum_{i=0}^{k} P(i) - \sum_{i=k}^{N-1} P(i) \right)$$

NCO Model on Directed Networks

- The influence of directionality



Increasing directionality mainly changes larger degree minority nodes' opinion

As directionality increases, only the low-degree agents are able to keep the minority opinion

OUTLINE

- What is Sociophysics?
- Motivations & Questions
- NCO Model on Directed Networks
- Conclusions