PhD Thesis Defense

Robustness and Structure of Complex Networks

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Overview: Projects and Publications

- Percolation of localized attack on complex networks:

1. Percolation of localized attack on complex networks (NJP 17(2), 023409)
2. The influence of the broadness of the degree distribution on network's robustness: comparing localized attack and random attack (arXiv: 1504.06900)

- Networks with clustering structure:

1. The robustness of interdependent clustered networks (EPL 101(1), 18002)
2. Robustness of a partially interdependent network formed of clustered networks (PRE 89(3), 032812)
3. Network of interdependent networks: overview of theory and applications (Networks of Networks: The Last Frontier of Complexity, 3-36)

- Empirical study of real-world networks:

1. Dynamic motifs in socio-economic networks (EPL 108(5), 58001)

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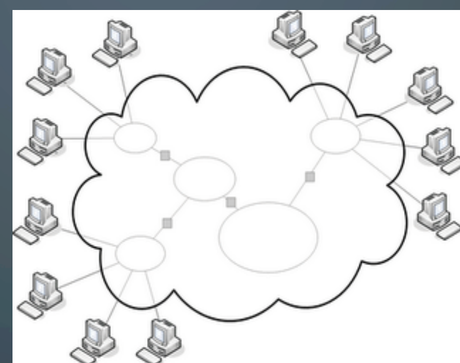
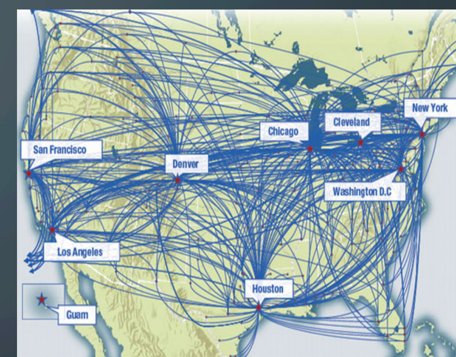
Outline

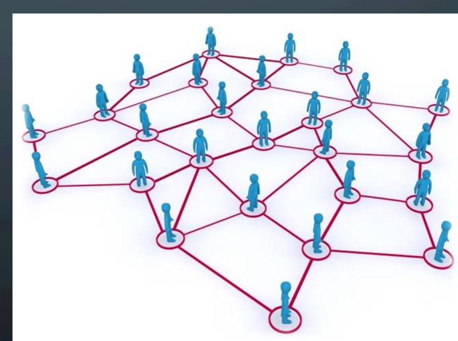
- **Background**
- Part I: Robustness of complex networks under localized attack
- Part II: Robustness of interdependent networks with clustering
- Summary

Background: Complex Networks

Complex networks are present in almost every aspect of our life:
The Internet
Airline Networks
Social Networks
Neural Networks
…







Background: Terminology

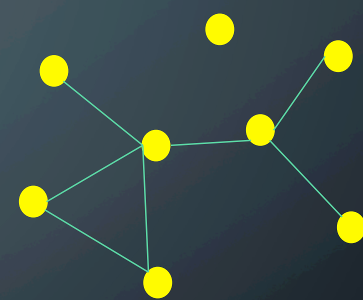
Network: a set of nodes connected with links

Degree k: number of links connected to a node

Degree distribution p(k): the probability that a randomly chosen node has degree k

Generating function of degree distribution:

$$G_0(x) = \sum_{k=0}^{\infty} p(k) x^k$$

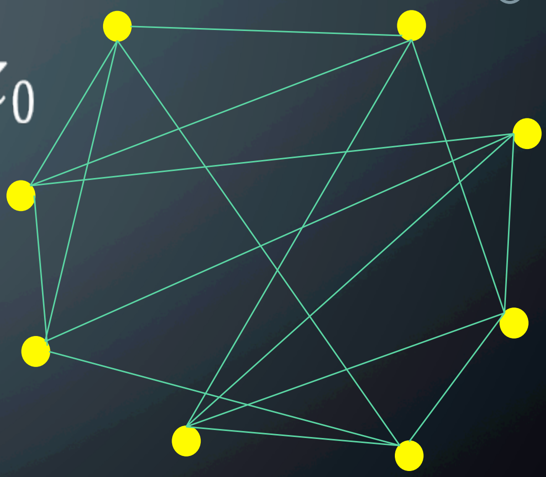


p(0)=1/8
p(1)=3/8
p(2)=2/8
p(3)=1/8
p(4)=1/8

Background: Random-regular Network (RR)

RR: each node is connected to exactly k_0 other nodes randomly

$p(k) = \delta_{k,k_0}$ $\quad\quad$ $G(x) = x^{k_0}$



$k_0 = 4$

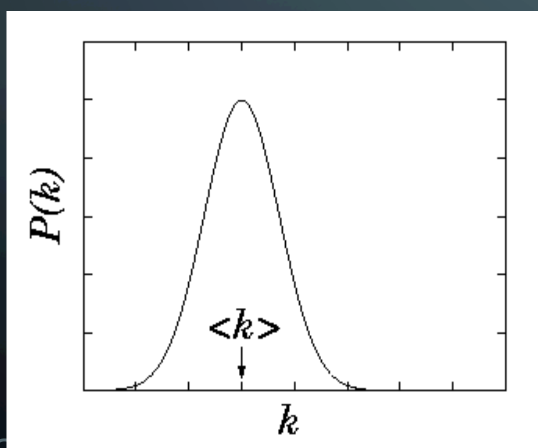
Background: Erdős-Rényi Network (ER)

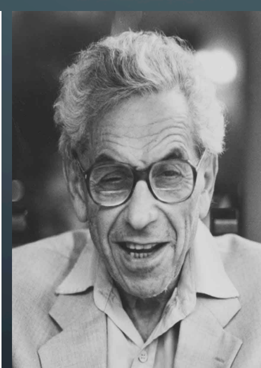
ER: each pair of nodes is connected with the same probability q (0<q<1)

$$p(k) = e^{-<k>} \frac{<k>^k}{k!}$$

Poisson distribution

$$G(x) = e^{<k>(x-1)}$$





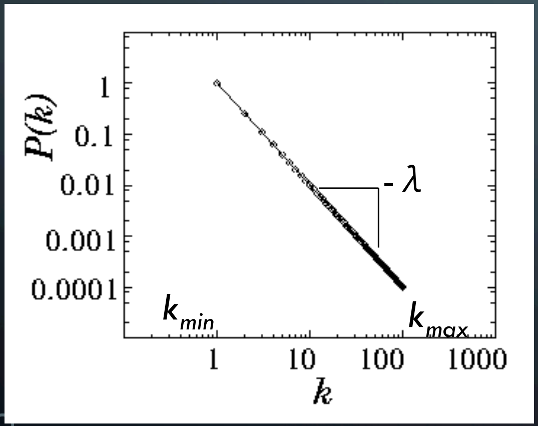
Paul Erdős

Alfréd Rényi

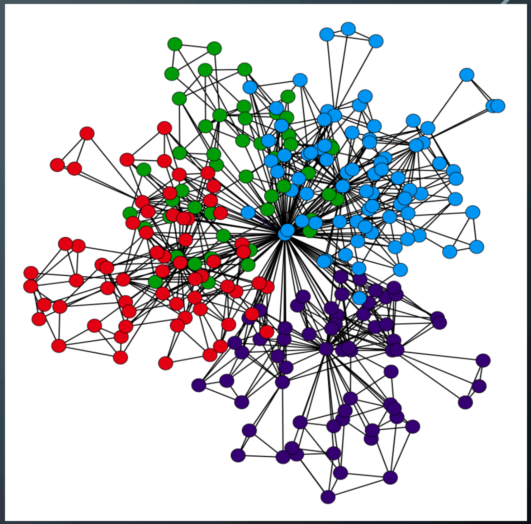
Background: Scale-free Network (SF)

SF: Most real-world networks are represented by scale-free graphs.

$p(k) = ck^{-\lambda}$, k=m,m+1,...,M



Power-law distribution



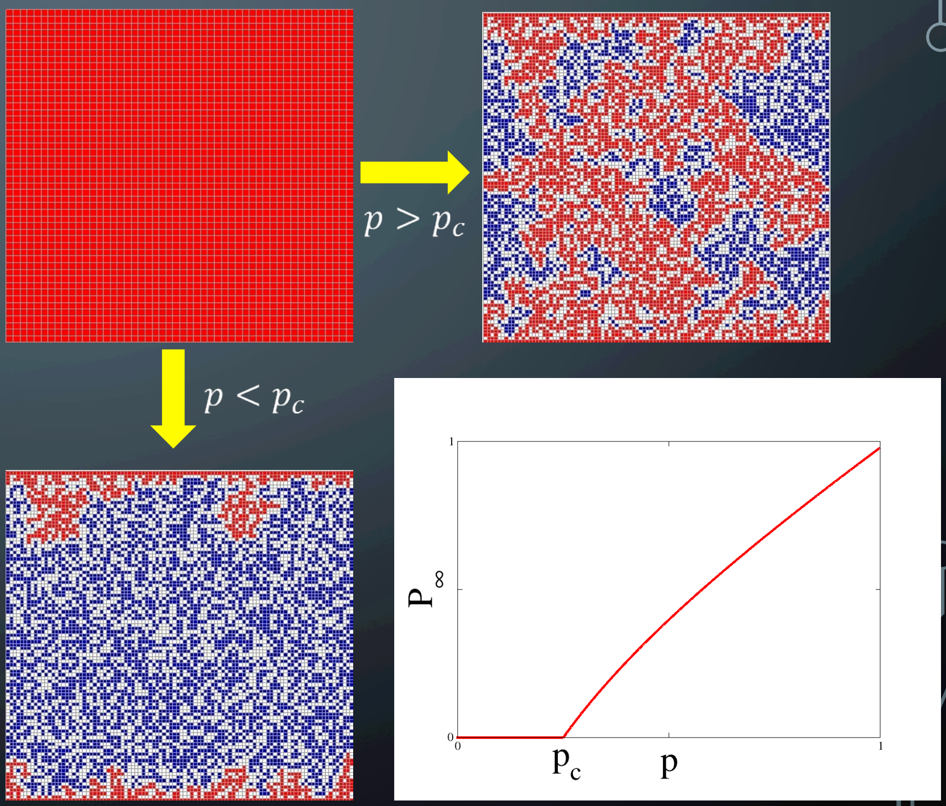
Network	Type	n	m	z	l	γ λ	$C^{(1)}$	$C^{(2)}$	r	References
Social										
film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	[ASBS00, WS98]
Technological										
Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	−0.189	[CCGJ02, FFF99]
Biological										
metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	−0.240	[JTA+00]
protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	−0.156	[JMBO01]

R. Cohen and S. Havlin, Complex networks: structure, robustness and function

Robustness of networks under attacks

1. If some nodes in the network are attacked and disabled, will the network keep functional (P_∞)?

2. How many nodes should be attacked in order to collapse the entire network (p_c)?



Robustness of networks under attacks

1. Type of the attack: random attack, targeted attack, localized attack (a new type of attack)

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2. Structure of the network: RR networks, ER networks, SF networks, clustered networks…

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Localized Attack: a new type of attack

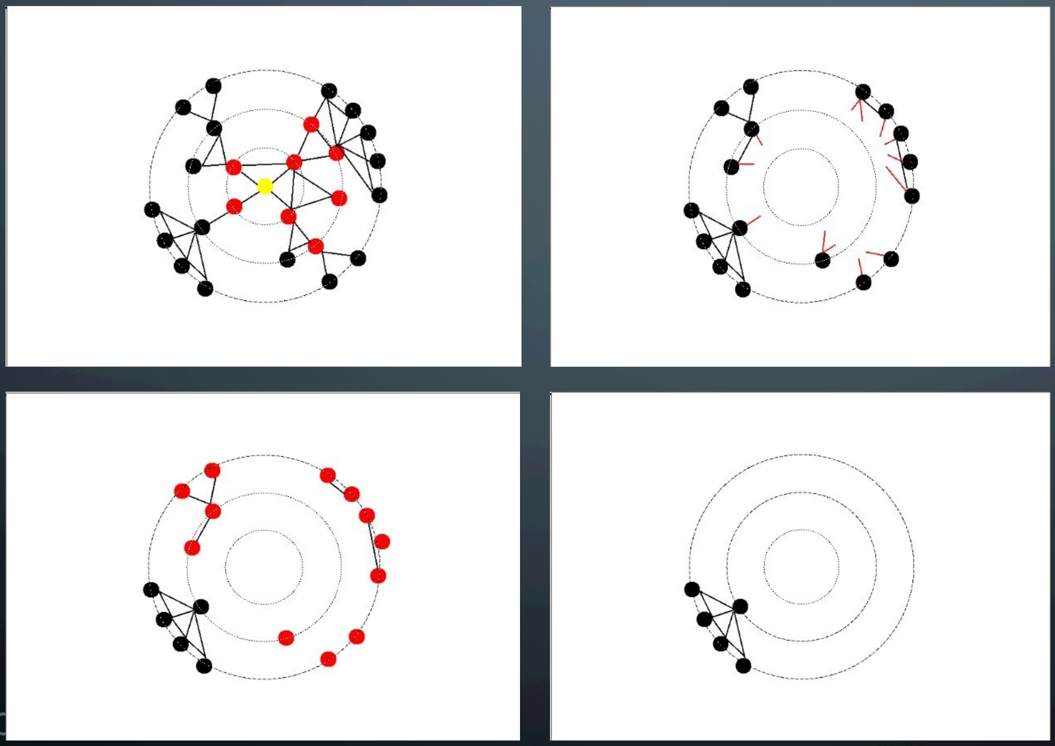
Motivation:
Most of percolation problems assume attacks being random or targeted, which is not the case for many realistic attacks.

Localized attack:
Attacking nodes by shell structure.
Reflecting localized damages: computer viruses, earthquakes, floods…

Develop a theoretical framework for understanding the robustness of networks under localized attack?

$$P_\infty = ?,\ p_c = ?$$

Localized Attack: a new type of attack



S Shao, X Huang, H E Stanely, and S Havlin (paper submitted)

Can percolation properties of localized attack be solved?

	Lattice(Square)	RR	ER	SF
Random attack	$p_c = 0.593$	$p_c = (k_0 - 1)^{-1}$	$p_c = \frac{1}{< k >}$	$p_c = 0$ for $2 < \lambda < 3$
A new type of attack: Localized attack	$p_c = 0$	$p_c = (k_0 - 1)^{-\frac{k_0}{k_0 - 2}}$?	$p_c = \frac{1}{< k >}$?	$p_c > 0$ for $2 < \lambda < 3$?

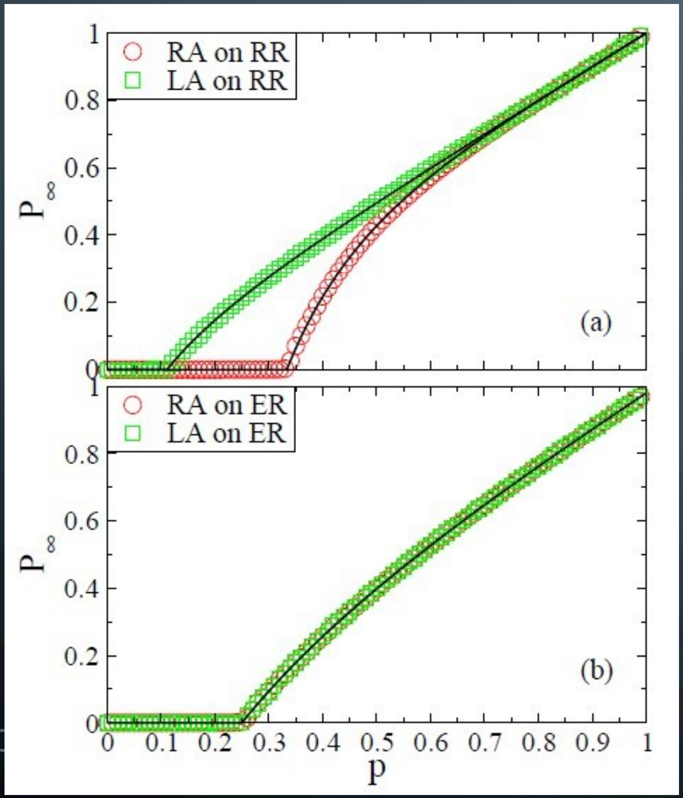
Theoretical framework: generating function method

$$G_0(x) = \sum_{k=0}^{\infty} p(k) x^k$$

$$G_{p0}(x) = \frac{1}{G_0(f)} G_0\left[f + \frac{G_0'(f)}{G_0'(1)}(\mathrm{x} - 1)\right]$$, where $$f = G_0^{-1}(p)$$

Relative size of the largest cluster P_∞ and percolation threshold p_c can be solved from $G_{p0}(x)$

Percolation of localized attack: RR and ER



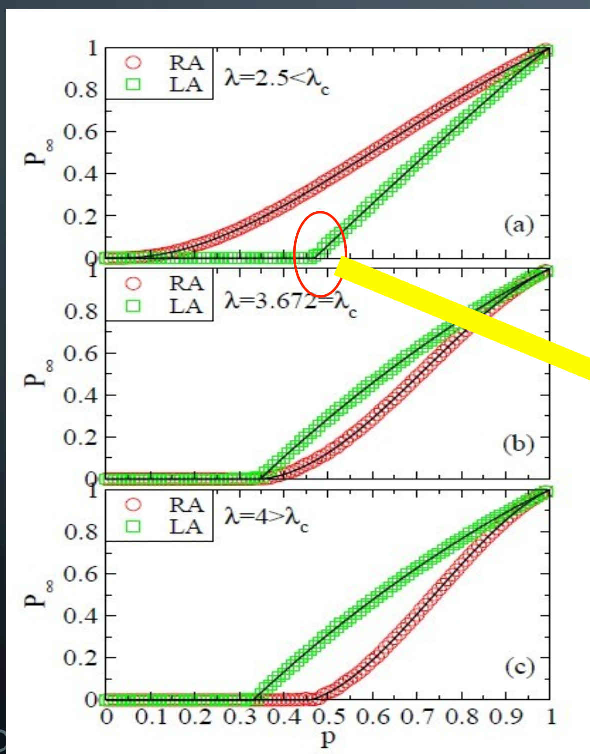
RR: $p_c(LA) < p_c(RA)$

Random regular networks are more robust against local attacks compared with random attacks

ER: $p_c(LA) = p_c(RA)$

Local attacks and random attacks have the same effects on Erdős-Rényi networks

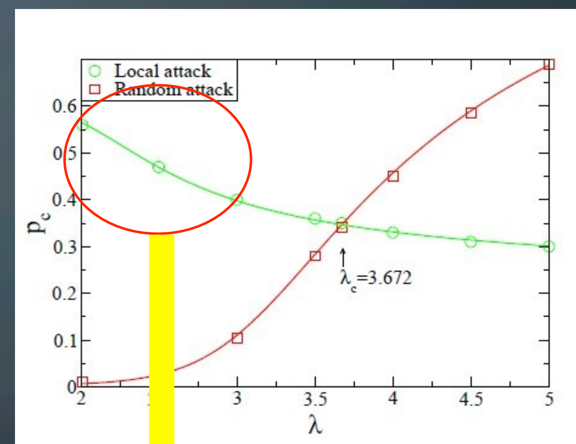
Percolation of localized attack: SF



$p_c(LA) > p_c(RA)$
$when\ \lambda < \lambda_c$

$p_c(LA) = p_c(RA)$
$when\ \lambda = \lambda_c$

$p_c(LA) < p_c(RA)$
$when\ \lambda > \lambda_c$

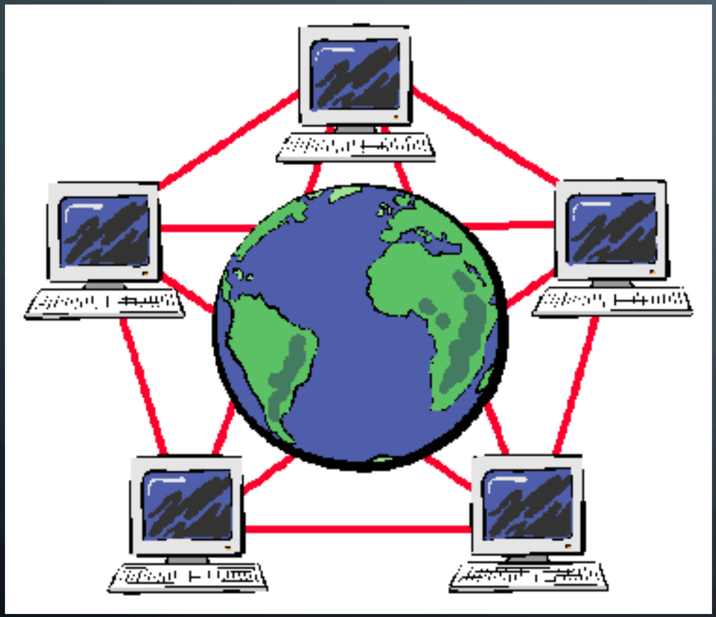


$p_c > 0\ even\ for\ 2 < \lambda < 3$

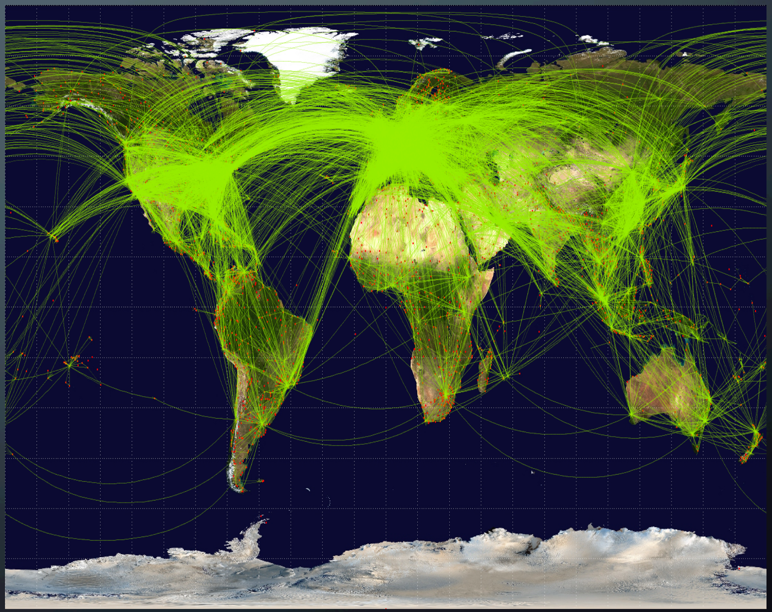
Real world networks are not robust against local attacks!

Real-world Networks

Internet

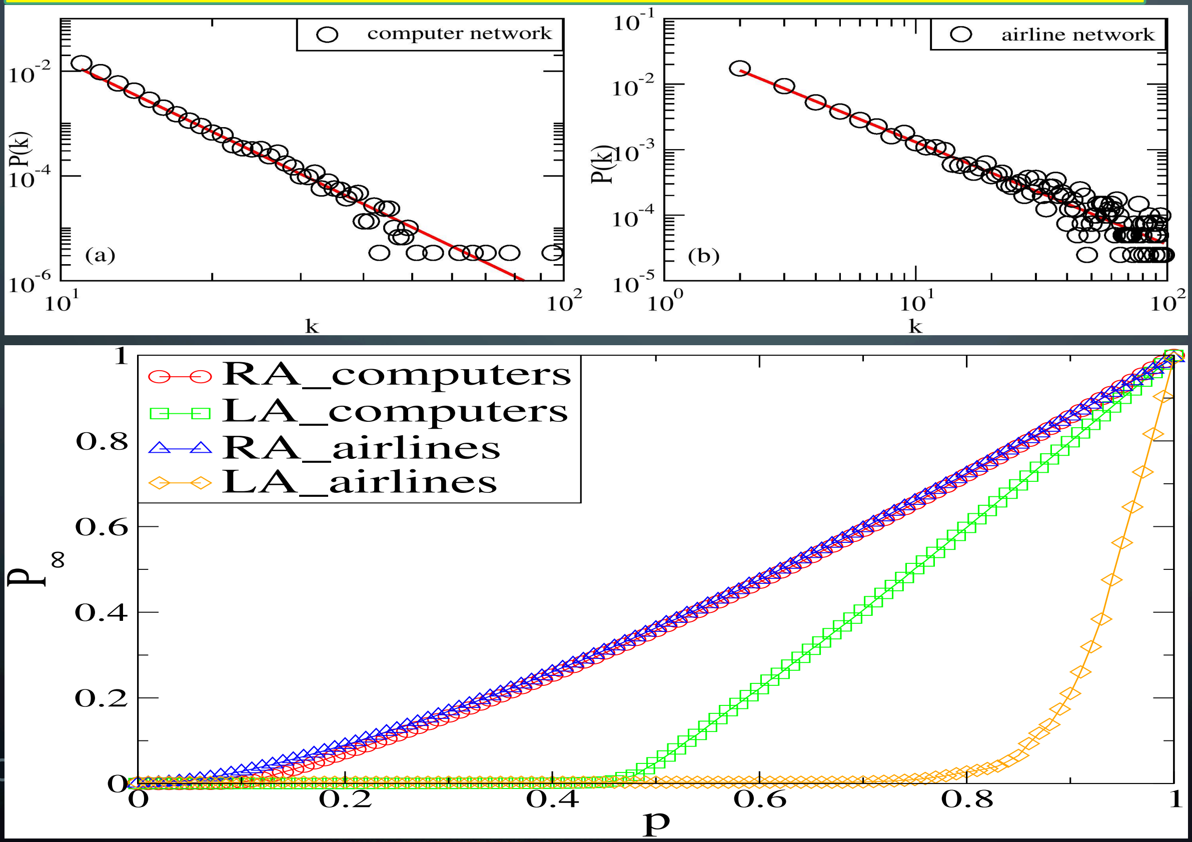


Airline Network



http://openflights.org/data.html
http://informationtechnologypramod.blogspot.com/2010/10/p2p.html

Real-world Networks



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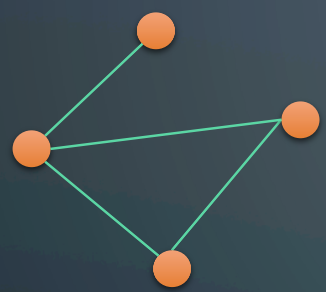
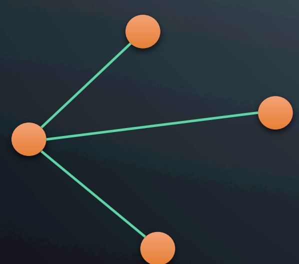
Structure of the network: Clustering



C = 1/3



C = 0

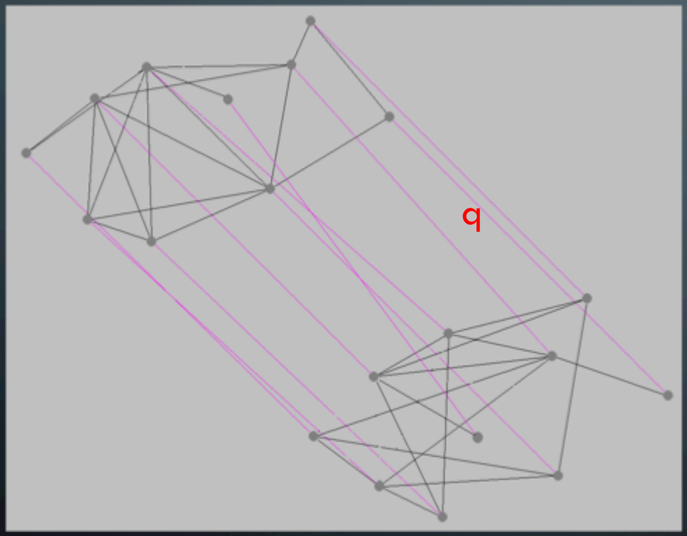
$$c = \frac{3 \times (\text{number of triangles in network})}{\text{number of connected triples}}$$

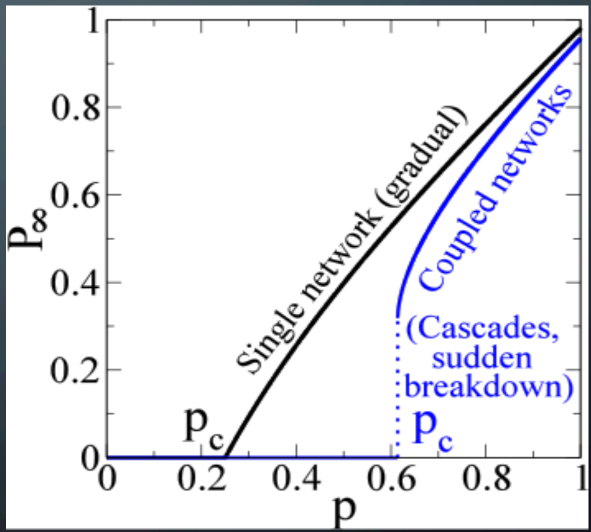
$$= \frac{N \sum_{st} t P_{st}}{N \sum_k \binom{k}{2} P_k}.$$

network	n	z	clustering coefficient C measured	clustering coefficient C random graph
Internet (autonomous systems)[a]	6 374	3.8	0.24	0.00060
World-Wide Web (sites)[b]	153 127	35.2	0.11	0.00023
power grid[c]	4 941	2.7	0.080	0.00054
biology collaborations[d]	1 520 251	15.5	0.081	0.000010
mathematics collaborations[e]	253 339	3.9	0.15	0.000015
film actor collaborations[f]	449 913	113.4	0.20	0.00025
company directors[f]	7 673	14.4	0.59	0.0019
word co-occurrence[g]	460 902	70.1	0.44	0.00015
neural network[c]	282	14.0	0.28	0.049
metabolic network[h]	315	28.3	0.59	0.090
food web[i]	134	8.7	0.22	0.065

Interdependent Networks

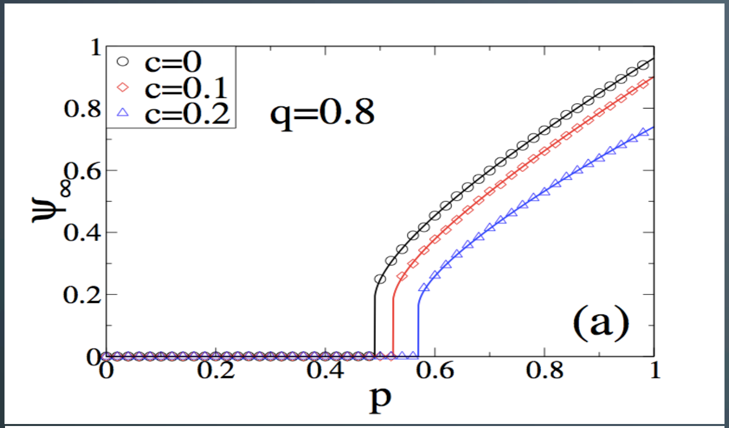
connectivity links (grey) + dependency links (purple)

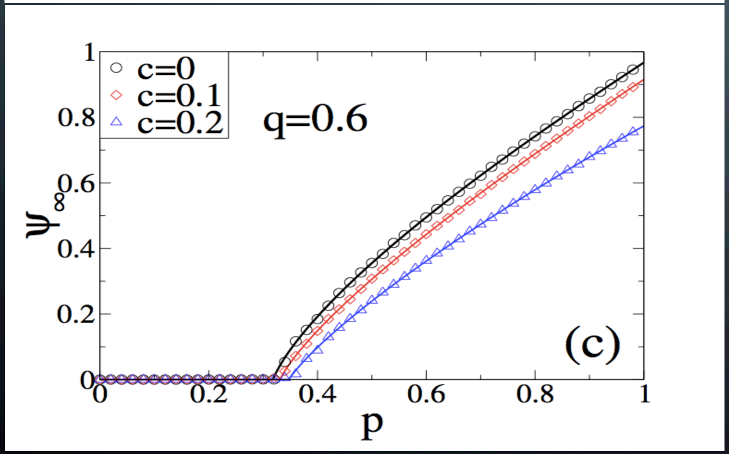


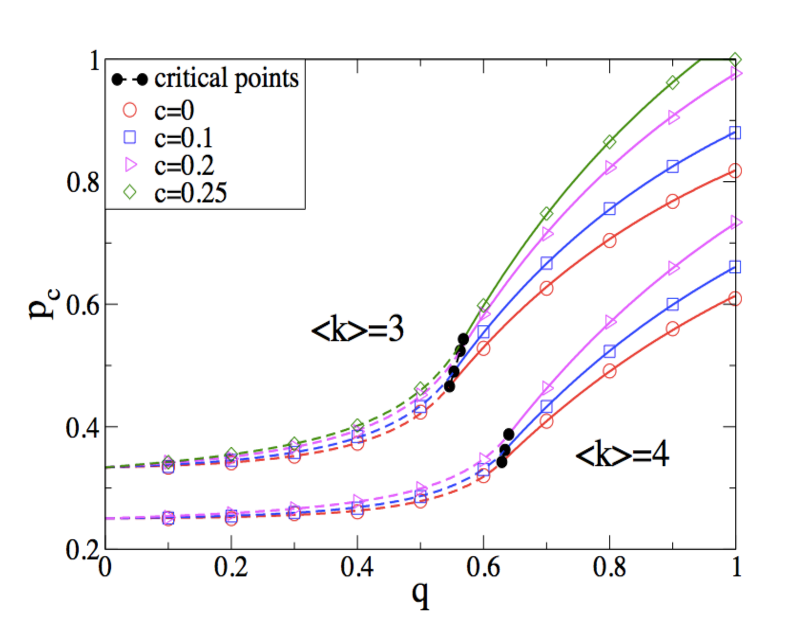


Nature 464, 1025 (2010)

Interdependent Networks with clustering







S Shao, X Huang, H E Stanley and S Havlin
Phys. Rev. E 89, 032812 (2014)
X Huang, S Shao, H Wang, S V Buldyrev, H E Stanley, and S Havlin,
EPL 101, 18002 (2013)

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Summary

1. Proposed the theoretical framework for the robustness of complex networks under localized attack and solved the percolation properties

2. Compared localized attack and random attack on ER, RR and SF networks:

- ER networks are equivalently robust under two types of attack
- RR networks are more robust under localized attack
- Robustness of SF networks depends crucially on the scale parameter λ

3. Real-world networks are much more fragile to localized attack than to random attack

4. Clustering structure in interdependent networks makes the system less robust under attack.

THANK YOU !