Traffic Bottlenecks in Networks

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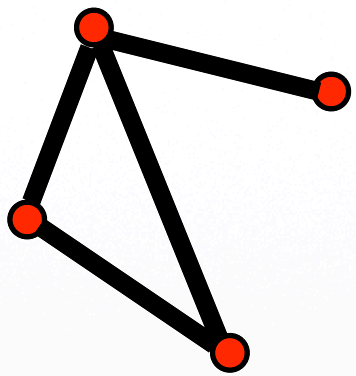
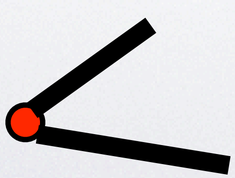
NETWORK



NODES

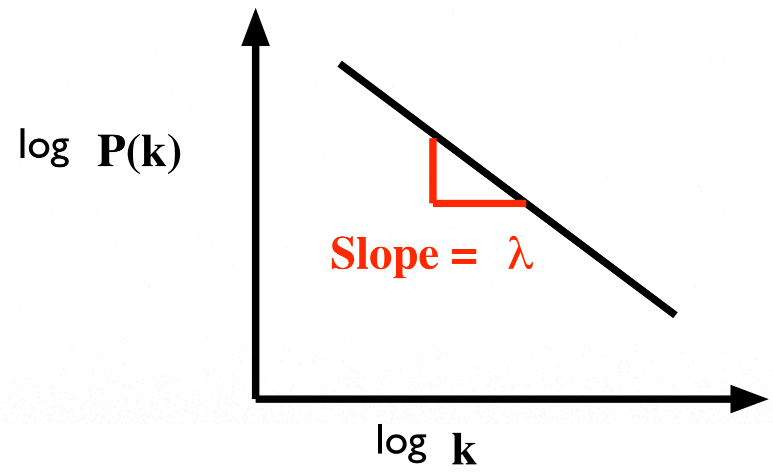
LINKS

DEGREE: Number of links per node.
Denoted by “k”



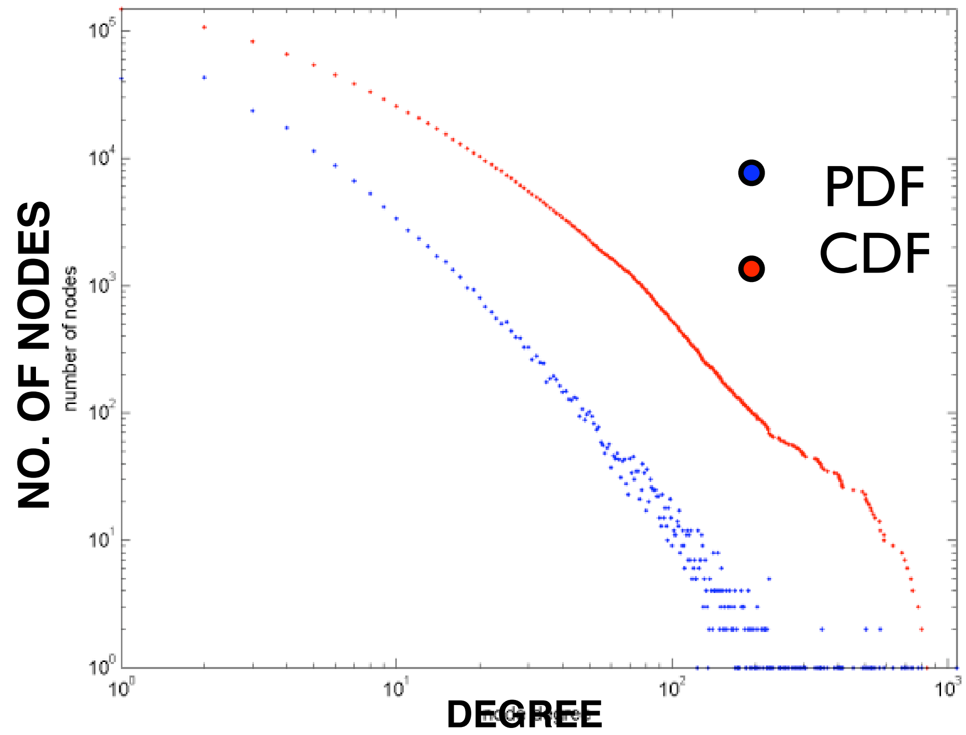
$k = 2$

SCALE FREE NETWORK:

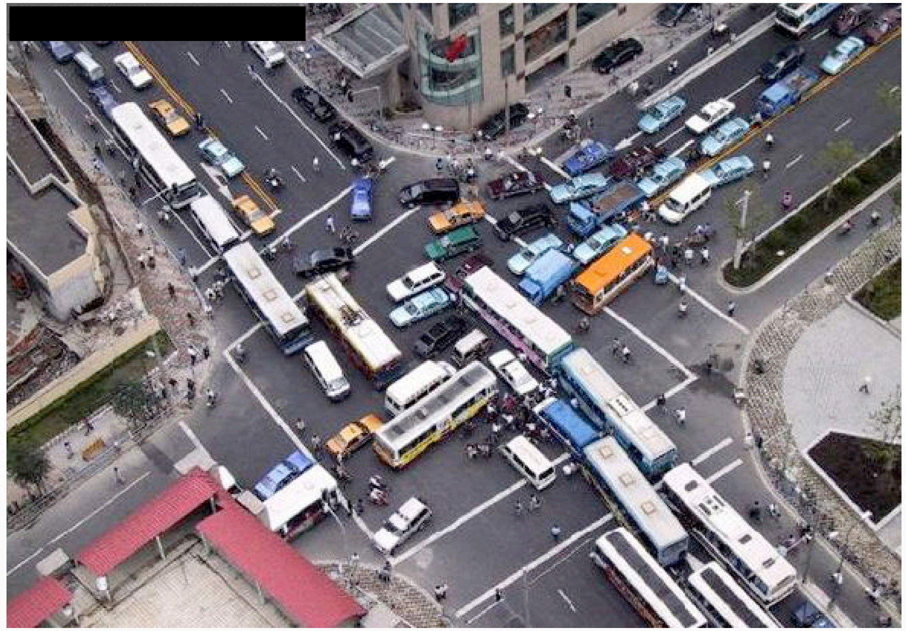


- Characterized by a power law in the degree distribution i.e P(k) ~ $k^{-\lambda}$. λ is called the degree exponent.

- A suitable abstraction of several networks, including the router-level internet.



QUESTIONS



Can we quantify the traffic bottlenecks in a network?

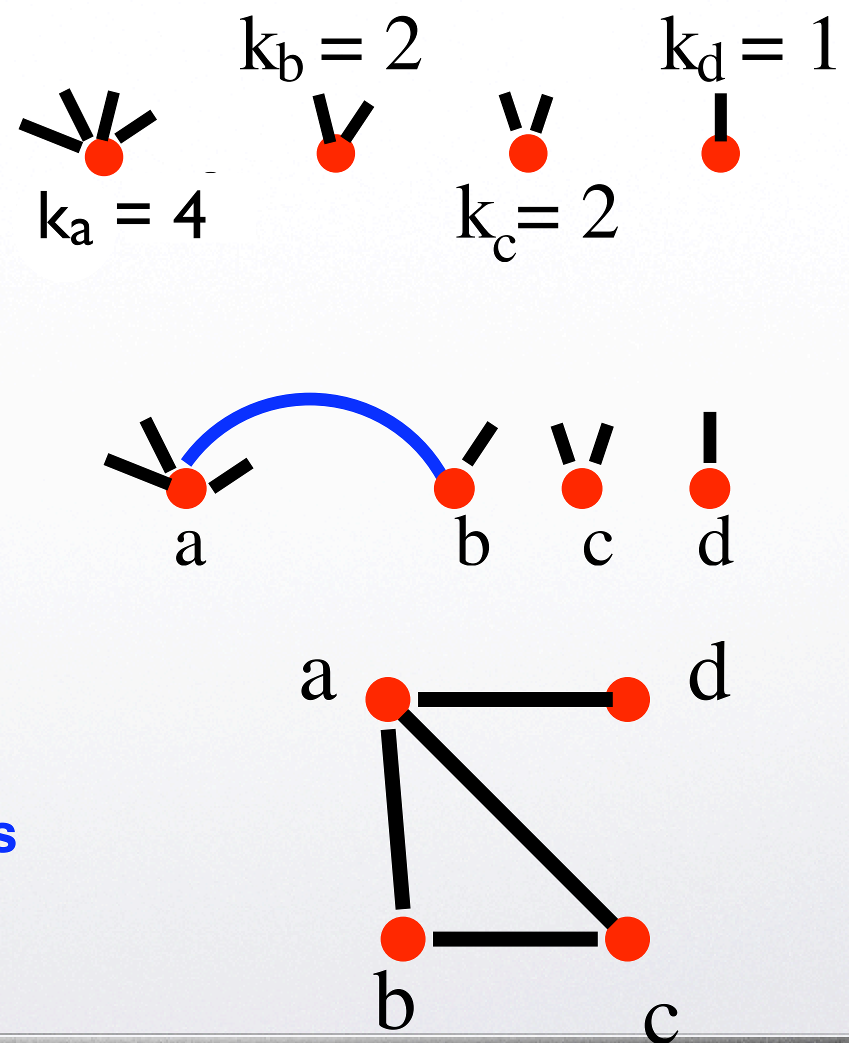
How does the bottleneck depend on the choice of paths for traffic flow ?

What is the inherent bottleneck due to network structure?

GENERATING SCALE-FREE NETWORKS:

- **To each node i, assign a degree k_i drawn from the degree distribution P(k). The node now has k_i stubs.**

- **Randomly match pairs of stubs until no stubs remain.**

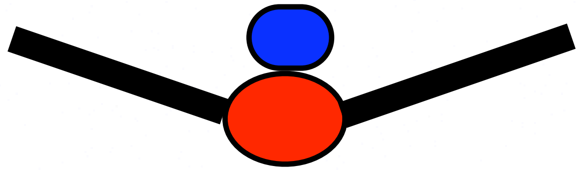


Studies done on ensemble of graphs generated in this way.

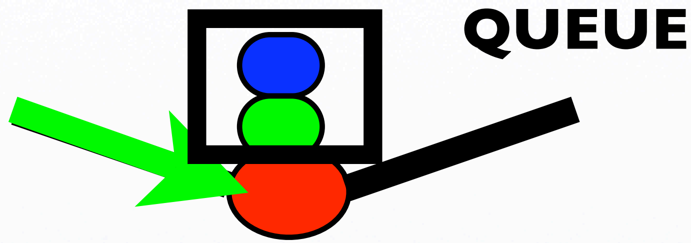
M. Molloy et al. Random Structures and Algorithms,6,161-180 (1995)

A SIMPLE NETWORK TRAFFIC MODEL

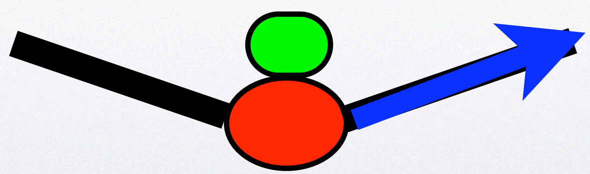
In a time step t :



Packet created with probability ρ.
Destination node of packet chosen uniformly at random.



Packet may be received from neighbor.



First packet in queue forwarded towards destination node.

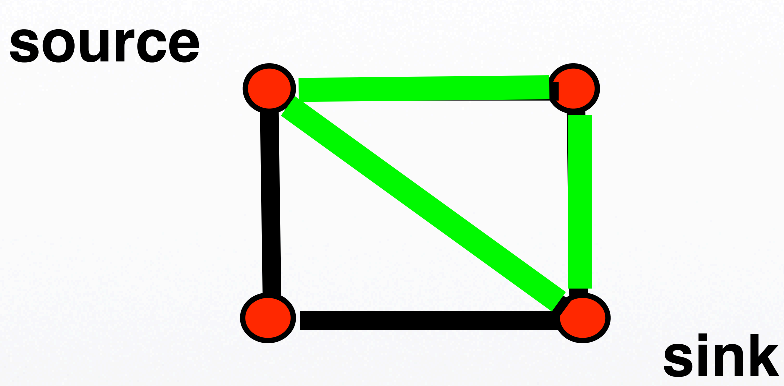
T. Ohira et al. Phys . Rev. E 58, 193 (1998)

CHOICE OF PATHS

ROUTING PROTOCOL assigns a path for each pair of nodes.

EXAMPLE:

SHORTEST PATH PROTOCOL= Assign the shortest path .

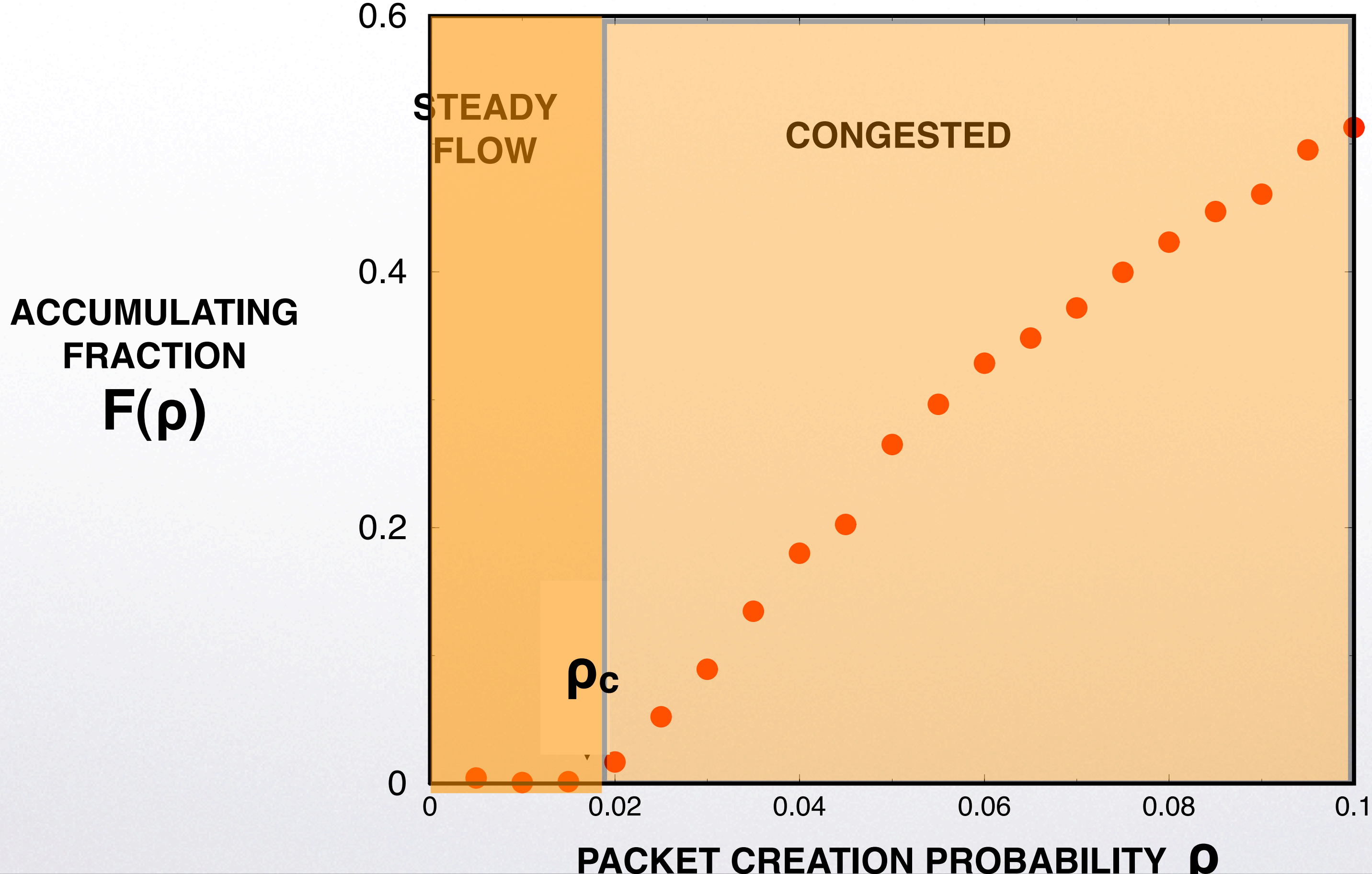


Assumption: The routing protocol does not change in time.

Let n(t) = number of packets on network at time t,
N = Number of nodes in entire network.

Fraction of created packets that accumulate:

$$F(\rho) = \lim_{t \to \infty} \frac{n(t + \Delta t) - n(t)}{N\rho\Delta t}$$



TRANSITION FROM A STEADY FLOW PHASE TO A CONGESTED PHASE

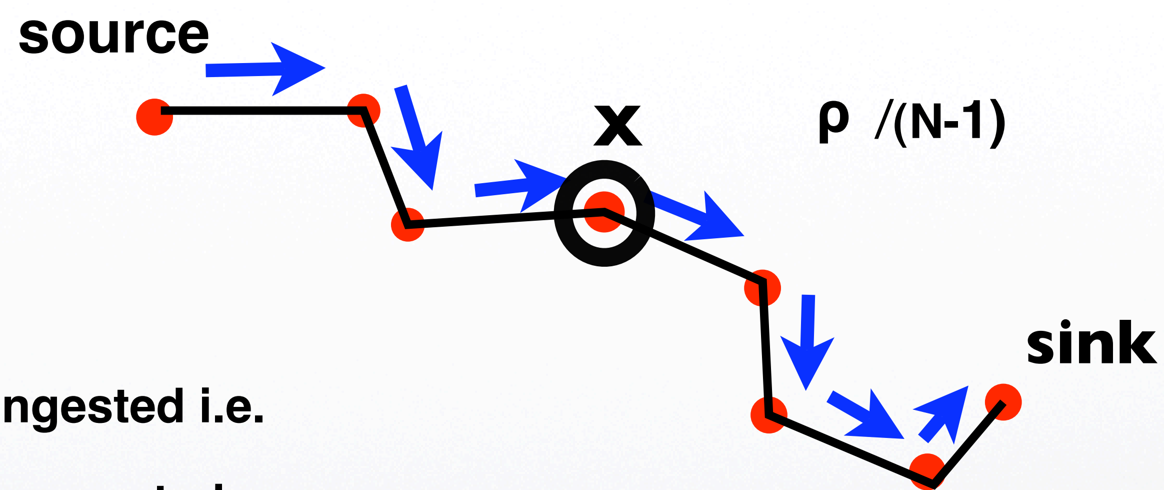
For 2D Lattices: T. Ohira et al. Phys . Rev. E 58, 193 (1998), R.V. Sole et al.Physica A 289,595

BETWEENESS B = number of paths using the node

Flow into the node ∝ BETWEENESS of the node

Flow in: $F_{in} = \frac{\rho B}{N-1}$

Flow out: $F_{out} = 1$



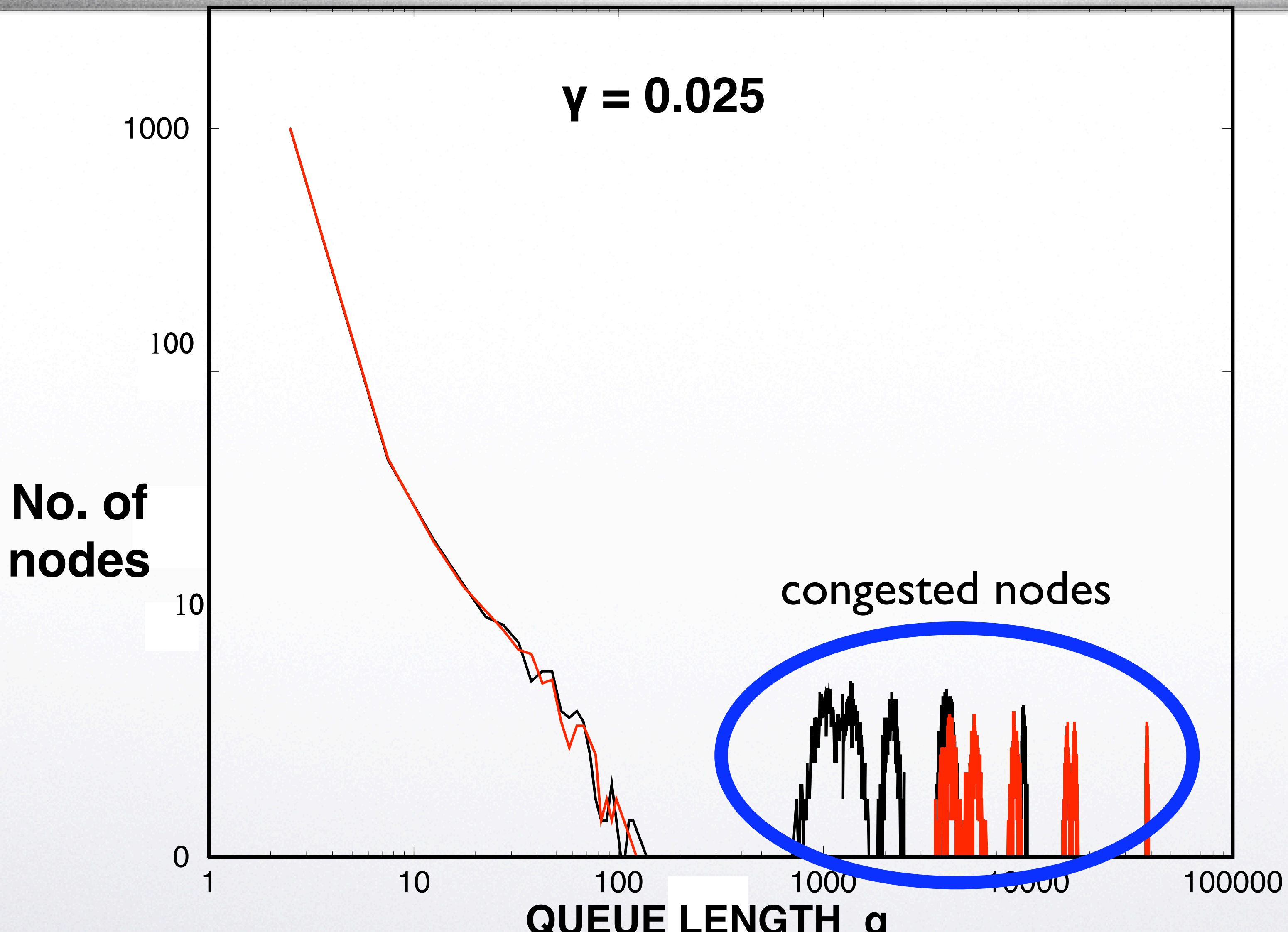
⇒ All nodes with $F_{in} > F_{out}$ get congested i.e.

all nodes with $B > (N-1)/\rho$ get congested.

The first node to get congested is the one with highest value of betweeness, B_{max}.

The congestion threshold : $\rho_c = \frac{N-1}{B_{max}}$

QUEUE SIZE DISTRIBUTION



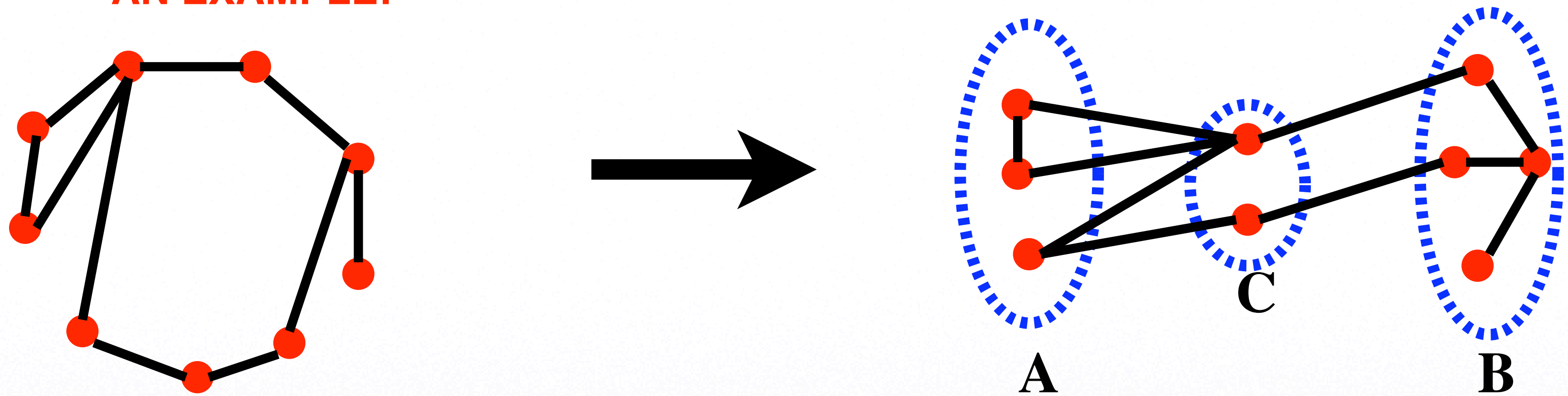
For a given γ only nodes with B > (N-1)/γ have growing queues.

RECAP & SPECIFIC QUESTIONS

- The bottleneck is the node with $B = B_{max}$.
- Q1: How does B_{max} scale with N when the shortest path routing protocol is used ?
- Q2: How “good” is shortest path routing for a scale-free network ?
- Q3 : Is there an “inherent bottleneck” in the network ?

INHERENT BOTTLENECK IN A NETWORK

AN EXAMPLE:

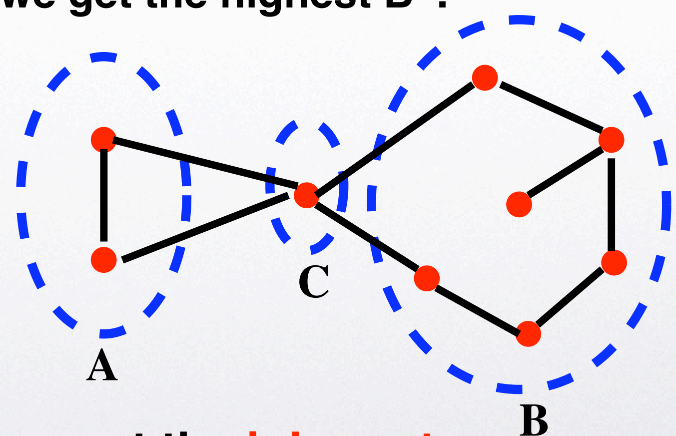


No. of paths that **must** pass through C ≥ 3x4 = 12

⇒ Highest Betweeness in C : $B \geq B^C = 12/2 = 6$

For one particular choice of C, we get the highest B^C:

$_{max}(B^c) = 12.$



For this network, for any routing protocol, $B_{max} \geq max(B^C) = 12$

Thus, the node(s) with $B_I = {}_{max}(B^c) = 12$ represent the **inherent bottleneck** in the network.

INTERESTED IN THE SCALING OF B_I WITH N.

B_I for a scale-free network.

For a scale-free network with degree distribution
$P(k) \sim k^{-\lambda}$

Using analytical arguments we obtain :

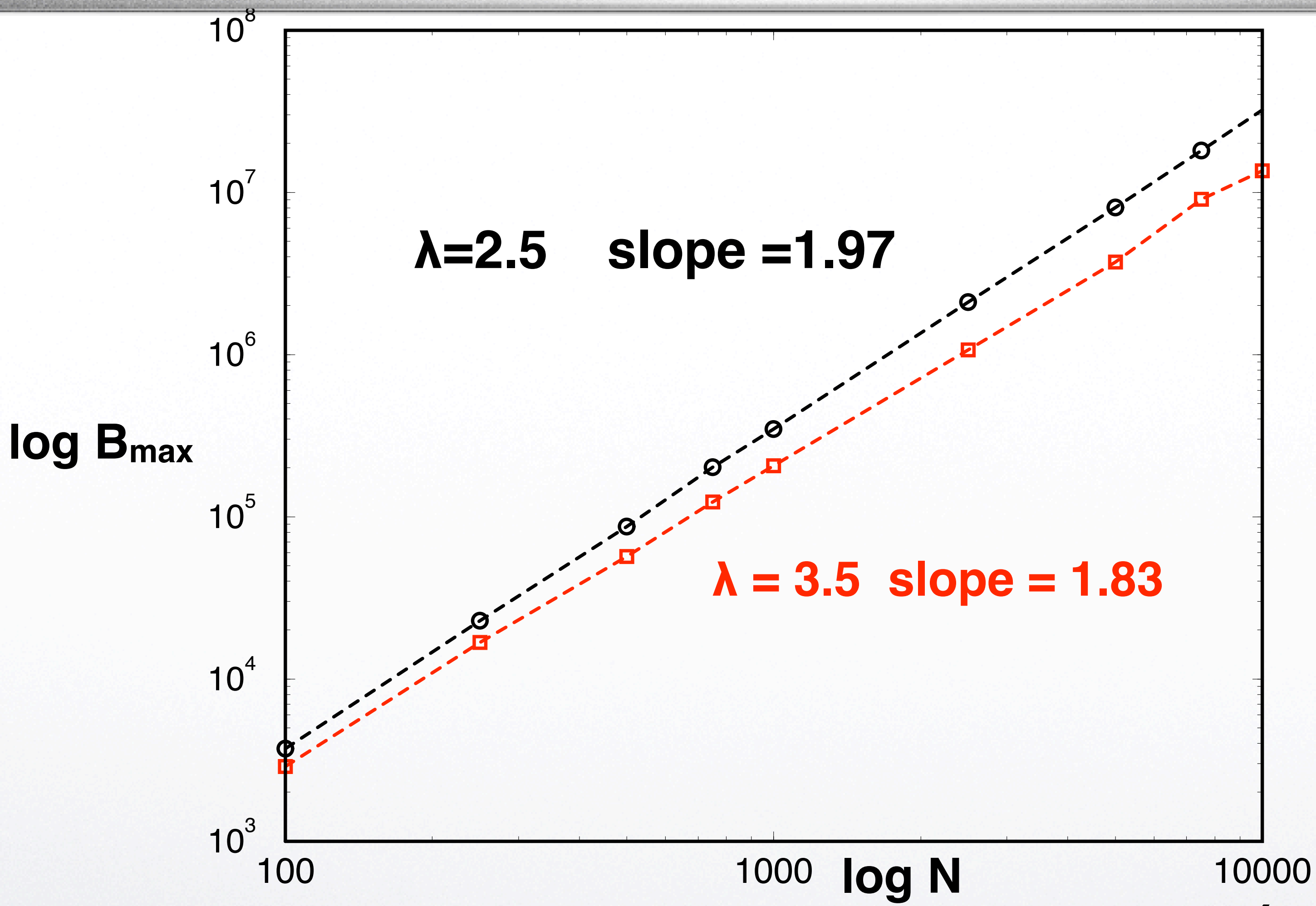
$$B_I = O\left(N^{\lambda/(\lambda-1)}\right)$$

How does this compare with the bottleneck B_{max} induced by shortest path routing ?

↓

Quantitative way of checking how "good" the shortest path protocol is.

SCALING OF BOTTLENECK INDUCED BY SHORTEST PATH ROUTING B_{max}

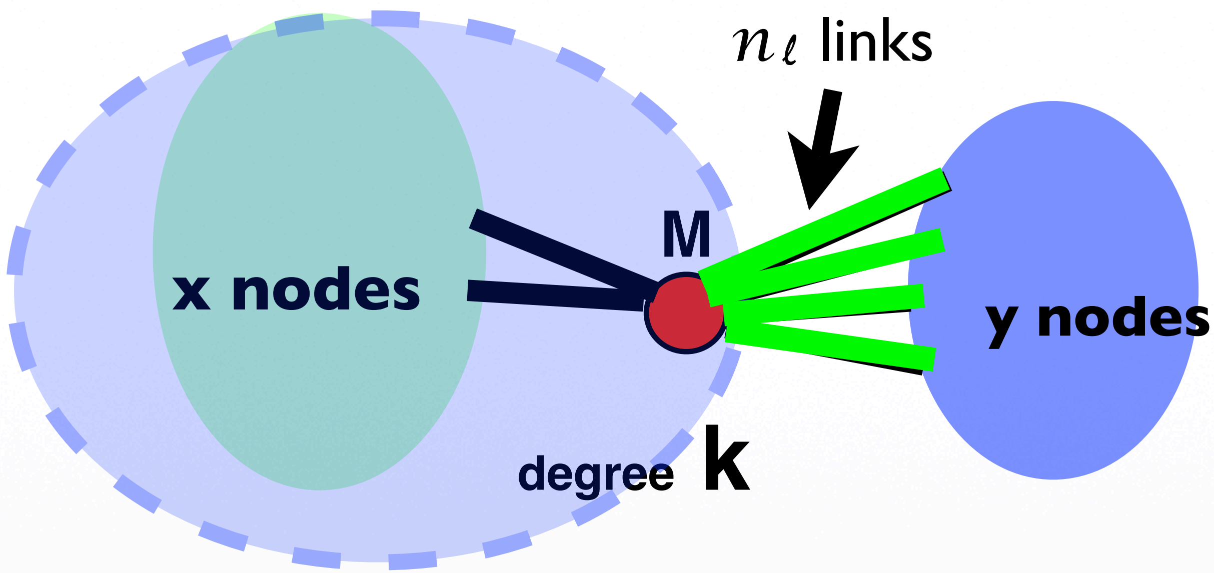


Inherent Bottleneck in the network $B_I = O\left(N^{\lambda/(\lambda-1)}\right)$.

$\lambda = 2.5$ $B_I = O\left(N^{5/3}\right) \cong O(N^{1.67})$

$\lambda = 3.5$ $B_I = O\left(N^{7/5}\right) = O(N^{1.4})$

S. Sreenivasan et al. (to be submitted)



Assume x > y $\Rightarrow$ x = O(N)

Betweenness of M = x y

Theorem* : Number of links n_ℓ between components x and y of a partition for a scale-free network:

$n_\ell \geq O(y)$, with Probability = 1 - o(1).

The largest that y can be is O(k);

Therefore, betweenness of M, is at most B = O(Nk).

The largest k is $O(N^{1/(\lambda-1)})$ and hence

The inherent bottleneck has betweenness $B_I = O(N^{\lambda/(\lambda-1)})$

*C. Gkantsidis et al. Proc. *SIGMETRICS* (2003)

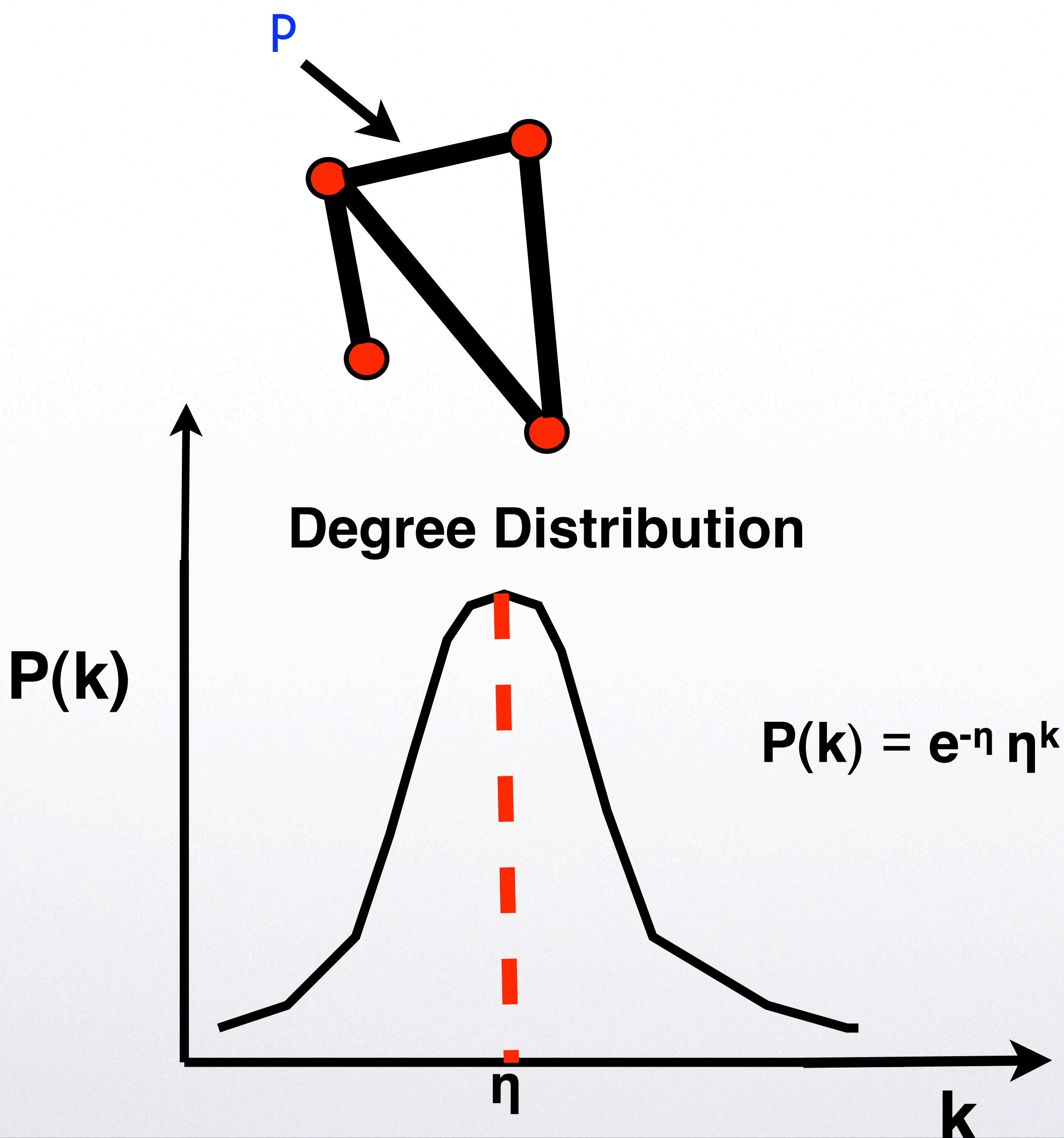
CONCLUSIONS

- The inherent bottleneck in scale-free networks have betweeness $B_I = O(N^{\lambda/(\lambda-1)})$.

- For scale free networks, the bottleneck induced by shortest path routing scales far worse with N than the inherent bottleneck due to network topology.

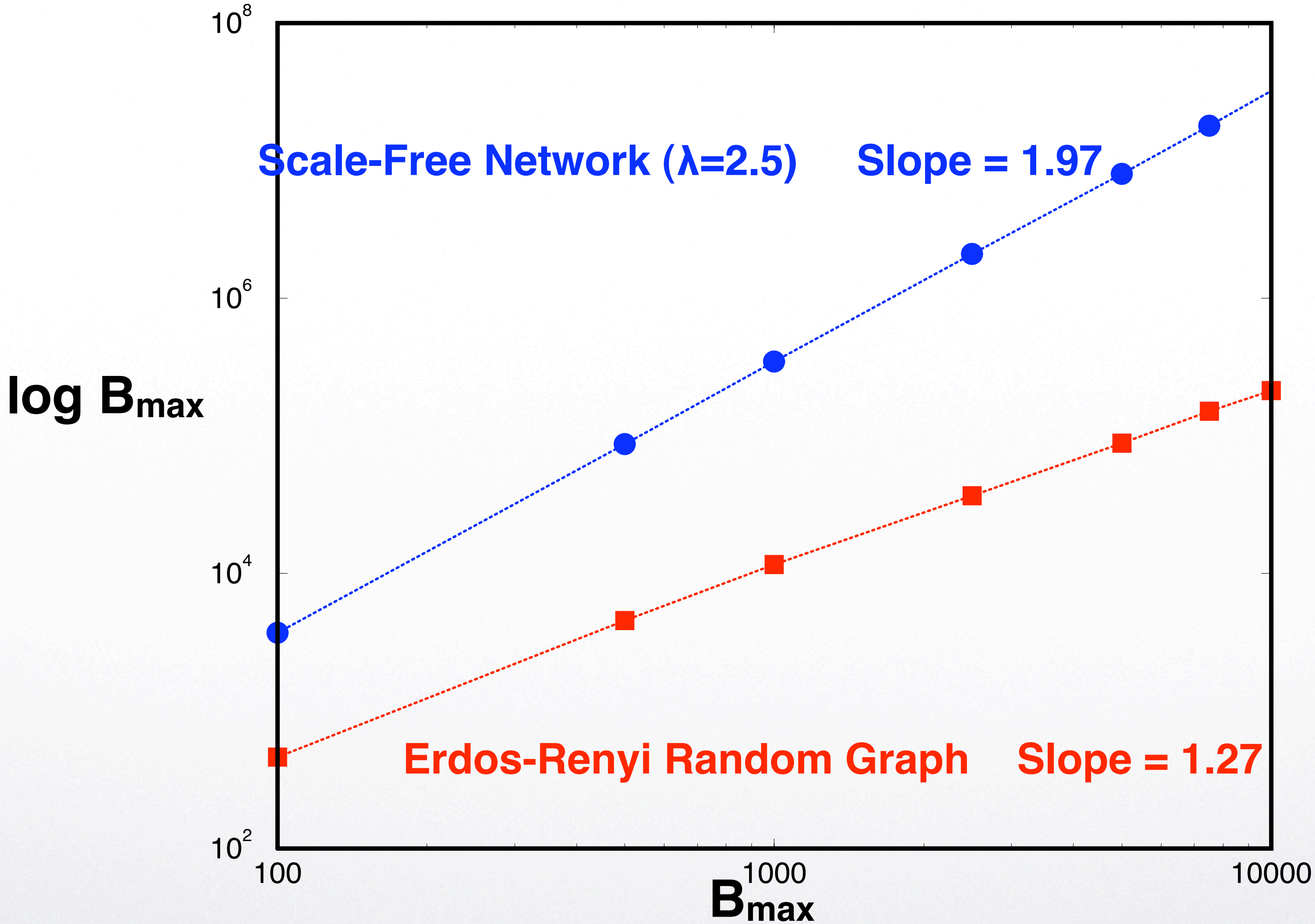
- There may exist better routing protocols than shortest path routing.

Erdos-Renyi Random Graphs



$$P(k) = e^{-\eta}\,\eta^{k} / k!$$

B_{max} for a Scale-Free Network and an Erdös-Rényi Random Graph



For an Erdös Rényi Random Graph

$$B_I = O(N \ln N)$$